

Optimization of Hamilton Path Tournament Schedule

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Abstract: A Hamilton path tournament design involving n teams and $n/2$ stadiums, is a round robin schedule on $n - 1$ days in which each team plays in each stadium at most twice, and the set of games played in each stadium induce a Hamilton path on n teams [1]. We have created an algorithm to calculate the cost of all pairs which is generated using Hamilton path tournament Design [1] that work on n teams, $n/2$ stadiums and $n - 1$ days schedule in which each team played at most twice in each stadium [1]. In our algorithm, we have calculated the cost of each combination of pairs and choose the Optimize Hamilton path tournament.

Index Terms – Cost, Combination, Schedule, Path.

1 INTRODUCTION

There are number of algorithms for scheduling balanced tournament designs [2], [6], [7]. Yoshiko T. Ikebe and Akihisa Tamura [1] introduced a Hamilton path tournament design and have proved it for all $n = 2p \geq 8$ teams. Gelling and Odeh introduced the balanced tournament design [4], and Schellenberg, van Rees and Vanstone proved its existence for all even $n \geq 6$ [9]. In this paper, we have introduced cost effect approach for tournament design using Hamilton path. We have used cost constant to find the optimized combination of pairs of balance tournament. Several authors have recently have proposed a schedule closely related to balanced tournament design [3], [5], [8], [10]. We have designed an optimized Hamilton path tournament in which there are n teams, $n-1$ days and $n/2$ stadiums. Cost for each and every team in each combination of pairs is calculated and then add these costs in a single one for each pair and compare with last minimum cost and at last we got the optimize combination of Hamilton path tournament.

II CONSTRUCTIONS

First, we have taken combination of pair in which we have n team means $n/2$ stadium. Then, we calculate the cost of each and every team traversal according to a given combination of pair. Suppose team t_1 play first match in stadium s_1 than we initiate the cost of team t_1 as 0 and check the next match play by team t_1 , if t_1 play second match in stadium s_2 the add the transportation cost of s_1 to s_2 into cost of team t_1 and check for the next match for team t_1 up to last match of t_1 and same

procedure apply for the all remaining teams and then add the cost of all teams which is called First Combination Cost.

Now this procedure applies for all the combinations of pairs and then we select the best Combination of pairs generated by Hamilton path tournament.

II ILLUSTRATION USING EXAMPLE

Table 1 indicate the one combination of pairs according to Hamilton path tournament for $n=8$. Let there are 8 team $t_1, t_2, t_3, t_4, t_5, t_6, t_7$ and t_8 , 4 stadiums s_1, s_2, s_3 and s_4 and 7 days $d_1, d_2, d_3, d_4, d_5, d_6$ and d_7 [1].

Days \ Stadium	s1	s2	s3	s4
d1	1,5	2,6	3,7	4,8
d2	1,6	5,7	2,8	3,4
d3	6,8	2,3	5,4	1,7
d4	7,4	6,3	1,8	5,2
d5	8,3	1,4	7,2	6,5
d6	4,2	8,5	1,3	7,6
d7	3,5	8,7	4,6	1,2

Table 1

And Table-2 indicate the transportation cost between stadiums such as transportation cost between s_1 to s_2 is a , s_2 to s_3 is b , s_3 to s_4 is c , s_1 to s_4 is d , s_1 to s_3 is e and s_2 to s_4 is f .

Stadiums	s1	s2	s3	s4
s1	0	a	e	d
s2	a	0	b	f
s3	e	b	0	c
s4	d	f	c	0

Table 2

Also show by figure1.

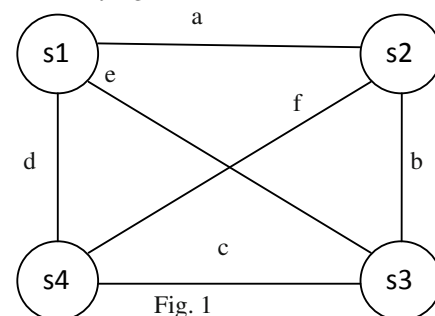


Fig. 1

And table3 indicate the transportation cost of each team day by day. D1 is the cost of traversal

between Stadiums for Team1 at Day1 and Day2. D2 is the cost of traversal between stadiums for Team1 at Day2 and Day3 and so on for D3, D4 and D5.

teams\ days	d1	d2	d3	d4	d4	d5
t1	0	d	c	b	b	c
t2	b	b	f	c	e	d
t3	c	f	0	a	e	e
t4	0	c	e	a	a	e
t5	a	b	c	0	f	a
t6	a	0	a	f	0	c
t7	b	f	d	e	c	f
t8	c	a	e	e	a	0

Table 3

So for the team t1 total transportation cost in the combination1 of this tournament is TCT1.

$$TCT1 = \sum_{i=1}^{n-2} t_1 D_i$$

TCT1= Transportation Cost of Team t1.

It means

$$TCT1 = 0 + d + c + b + b + c = 0 * a + 2 * b + 2 * c + 1 * d + 0 * e + 0 * f$$

Same for the team t2 total transportation cost in the combination1 of this tournament is TCT2.

$$TCT2 = \sum_{i=1}^{n-2} t_2 D_i$$

TCT2= Transportation Cost of Team t2

It means

$$TCT2 = b + b + f + c + e + d = 0 * a + 2 * b + 1 * c + 1 * d + 1 * e + 1 * f$$

And so on for all the remaining teams.

And the total transportation cost of Combination1 is TC1.

$$TC1 = \sum_{k=1}^n \sum_{i=1}^{n-2} t_k D_i$$

Or

$$TC1 = \sum_{i=1}^n TCT_i$$

If we will take the transportation cost between all stadiums as an unit cost then transportation cost of team t1 in Combination1 of this tournament is TCT1.

$$TCT1 = 0 + 2 + 2 + 1 + 0 + 0 = 5$$

$$TCT2 = 0 + 2 + 1 + 1 + 1 + 1 = 6$$

$$TCT3 = 5$$

$$TCT4 = 5$$

$$TCT5 = 5$$

$$TCT6 = 4$$

$$TCT7 = 6$$

$$TCT8 = 5$$

And TC1= 41

And for another combination of Hamilton path tournament we have calculated the total transportation cost as TC2. Table4 indicate the another combination of pairs according to Hamilton path tournament for n=8. If there are 8 team t1, t2, t3, t4, t5, t6, t7 and t8, 4 stadiums s1, s2, s3 and s4 and 7 days d1, d2, d3, d4, d5, d6 and d7.

Days \ Stadium	s1	s2	s3	s4
d1	1,5	2,6	3,7	4,8
d2	1,6	3,4	2,8	5,7
d3	5,4	1,7	6,8	2,3
d4	6,3	7,4	5,2	1,8
d5	7,2	8,3	1,4	6,5
d6	4,2	8,5	7,6	1,3
d7	8,7	1,2	3,5	4,6

Table 4

And table5 indicate the transportation cost of each team day after previous day. D1 is the cost of traversal between Stadiums for Team1 at Day1 and Day2. D2 is the cost of traversal between stadiums for Team1 at Day2 and Day3 and so on for D3, D4 and D5.

teams \ days	D1	D2	D3	D4	D4	D5
t1	0	a	f	c	c	f
t2	b	c	c	e	0	a
t3	b	f	d	a	f	c
t4	f	a	a	b	e	d

t5	d	d	e	c	f	b
t6	a	e	e	d	c	c
t7	c	f	0	a	e	e
t8	c	0	c	f	0	a

So for the team t1 total transportation cost in the combination1 of this tournament is TCT1.

$$TCT1 = \sum_{i=1}^{n-2} t_1 D_i$$

TCT1= Transportation Cost of Team t1.

It means

$$TCT1 = 0 + a + f + c + c + f \\ = 1 * a + 0 * b + 2 * c + 0 * d + 0 * e + 2 * f$$

Same for the team t2 total transportation cost in the combination1 of this tournament is TCT2.

$$TCT2 = \sum_{i=1}^{n-2} t_2 D_i$$

TCT2= Transportation Cost of Team t2

It means

$$TCT2 = b + c + c + e + 0 + a \\ = 1 * a + 1 * b + 2 * c + 0 * d + 1 * e + 0 * f$$

And so on for all the remaining teams.

And the total transportation cost of Combination2 is TC2.

$$TC2 = \sum_{k=1}^n \sum_{i=1}^{n-2} t_k D_i$$

Or

$$TC2 = \sum_{i=1}^n TCT_i$$

If we will take the transportation cost between all stadiums as an unit cost then transportation cost of team t1 in Combination1 of this tournament is TCT1 and value of TCT1 is

$$TCT1 = 1 + 0 + 2 + 0 + 0 + 2 \\ = 5$$

$$TCT2 = 1 + 1 + 2 + 0 + 1 + 0 \\ = 5$$

$$TCT3 = 6$$

$$TCT4 = 6$$

$$TCT5 = 6$$

$$TCT6 = 6$$

$$TCT7 = 5$$

$$TCT8 = 4$$

And the total transportation cost of the second combination of same tournament is TC2.

$$TC2 = 43$$

And so on for the all remains combinations and then we select minimum cost and optimized combination for Hamilton path tournament.

IV ALGORITHM

Optimize_Hamilton_Path_Tournament(A,TC,n, m)
A is the 4 dimension array used to store all the combination of pairs generated by Hamilton path tournament and n is the number of team played in this tournament. m is indicate the number of total combination and TC used to store the Transportation cost of combinations.

- I. Set $p \leftarrow 1$ $\min \leftarrow \text{MAX_VALUE}$
Best_Combination $\leftarrow 1$
- II. Repeat while $p \leq m$
 1. Set $i \leftarrow 1$
 2. Repeat while $i \leq n$
 - (a) Set $\text{sum} \leftarrow 0$ $D1 \leftarrow 0$ $D2 \leftarrow 0$
 $j \leftarrow 1$
 - (b) Repeat while $j \square n$
 - (i) Set $k \leftarrow 1$
 - (ii) Repeat while $k \leq n/2$
 - (A) If $A[j][k][0] = i$ or $A[j][k][1] = i$ then do
 - a. If $D1 = 0$ then set $D1 \leftarrow k$
 - b. Set $D2 \leftarrow k$
 - c. If $D1 = D2$ then
 set $\text{sum} \leftarrow \text{sum} + 0$
 Otherwise set $\text{sum} \leftarrow \text{sum} + 1$
 $D1 \leftarrow D2$
 - d. Go to step (b)
 - (B) Set $k \leftarrow k + 1$
 - (iii) Set $j \leftarrow j + 1$
 - (c) Set $TCT[i] \leftarrow \text{sum}$
 - (d) Set $TC[p] \leftarrow TC[p] + \text{sum}$
 - (e) Set $i \leftarrow i + 1$
 3. If $\min > TC[p]$ then
set $\min \leftarrow TC[p]$
Best_Combination $\leftarrow p$
 4. Set $p \leftarrow p + 1$
- III. Return Best_Combination

V CONCLUSION

Using Cost constraint on various combinations of Hamilton path tournament design for n teams, we have selected the combination of teams and stadiums such that all teams have to travel minimum distance for whole tournament. As all the teams will travel minimum distance, so the cost of their transportation and other expenses associated with them will also be minimum. This algorithm

can be used to design tournaments in a cost effective manner and also if the teams will travel less, their performance will also increase. In future, time constraint can also be added to this algorithm.

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