Abstract—Smart antenna technologies are very important for the system implementation. Smart Antennas serve different users by radiating narrow beams. The same frequency can be reused even if the users are in the same cell or the users are well separated. Thus the capacity of the system is increased by implementing this additional intra cell reuse. This paper discusses algorithms developed for smart antenna applications to WCDMA. The Minimum mean-square error (MMSE) Algorithm and Maximum likelihood (ML) algorithms are the two adaptive beam forming algorithms used in smart antennas.

Index Terms—Minimum mean-square error (MMSE), Maximum likelihood (ML), Adaptive beam forming, smart antenna.

I. INTRODUCTION

Adaptive Beam forming is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arrival from a desired direction (in the presence of noise) while signals of the same frequency from other directions are rejected. This is achieved by varying the weights of each of the sensors (antennas) used in the array. It basically uses the idea that, though the signals emanating from different transmitters occupy the same frequency channel, they still arrive from different directions. This spatial separation is exploited to separate the desired signal from the interfering signals. In adaptive beam forming the optimum weights are iteratively computed using complex algorithms based upon different criteria [1],[2].

II. THEORY

Antennas (and antenna arrays) often operate in dynamic environments, where signals (both desired and interfering) arrive from changing directions and with varying powers. As a result, adaptive antenna arrays have been developed. These antenna arrays employ an adaptive weighting algorithm that adapts the weights based on the received signals to improve the performance of the array [3].

Figure (1) shows a diagram of an adaptive array. It consists of the sensor array, the beam forming network, and the adaptive processor that adjusts the variable weights in the beam forming network. The array design depends on the propagation medium in which the array operates the frequency spectrum of interest, and the user’s knowledge of the operational signal environment [4].

The array consists of N sensors designed to receive (and transmit) signals in the propagation medium. The output of each of the N elements goes to the beam forming network, where the output of each sensor element is first multiplied by a complex weight (having both amplitude and phase) and then summed with all other weighted sensor element outputs to form the overall adaptive array output signal. The weight values within the beam forming network then determine the overall array pattern. It is the ability to shape this overall array pattern that in turn determines how well the specified system requirements can be met for a given signal environment[5].

![Figure (1): Functional diagram of an N-element adaptive array](image)

Figure (1): Functional diagram of an N-element adaptive array [6].

The weight values within the beam forming network then determine the overall array pattern. It is the ability to shape this overall array pattern that in turn determines how well the specified system requirements can be met for a given signal environment[7].

There are type of arrays have proven useful in different applications is called the conventional array illustrated in Figure (2).

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The outputs of the array illustrated in Figures (2) as:

\[ y(t) = w^T x(t) \]  

(1)

Where \( x(t) = s(t) + n(t) \) is the vector of received signals that are complex valued functions. The signal vector \( s(t) \) induced at the sensor elements from a single directional signal source is assumed to be:

\[ s(t) = \sqrt{S} e^{j\omega_0 t} \]  

(2)

Where \( \omega_0 \) is the (radian) carrier frequency and \( S \) represents the signal power. Assuming identical antenna elements, the resulting signal component in each array element is just a phase-shifted version of the signal appearing at the first array element encountered by the directional signal source. It therefore follows that the signal vector \( s(t) \) is written as

\[ s^T(t) = [s(t), e^{j\theta_1}, ..., e^{j\theta_{N-1}}] \]  

(3)

Where \( v \) is defined to be the array propagation vector

\[ v^T = [1, e^{j\theta_1}, ..., e^{j\theta_{N-1}}] \]  

(4)

Consequently, the received signal vector for the conventional array

\[ x(t) = s(t)u + n(t) \]  

(5)

In developing the optimal solutions for selected performance measures, four correlation matrices will be required. These correlation matrices are defined as follows for narrowband uncorrelated signal processes:

\[ R_{xx} = E\{x^*(t)x^T(t)\} = R_{ss} + R_{nn} \]  

(9)

Suppose the desired directional signal \( s(t) \) is known and represented by a reference signal \( d(t) \). This assumption is never strictly met in practice because a communication signal cannot possibly be known a priori if it is to convey information; hence, the desired signal must be unknown in some respect. Nevertheless, it turns out that in practice enough is usually known about the desired signal that a suitable reference signal \( d(t) \) is obtained to approximate \( s(t) \) in some sense by appropriately processing the array output signal. For example, when \( s(t) \) is an amplitude modulated signal, it is possible to use the carrier component of \( s(t) \) for \( d(t) \) and still obtain suitable operation. Consequently, the desired or “reference” signal concept is a valuable tool, and one can proceed with the analysis as though the adaptive processor had a complete desired signal characterization [8], [9].

The difference between the desired array response and the actual array output signal defines an error signal as shown in Figure (3).

\[ y(t) = w^T x(t) \]  

(10)

\[ e(t) = d(t) - y(t) = d(t) - w^T(t)x(t) \]  

(21)

Figure (3): Basic adaptive array structure with known desired signal [10].

Since \( d(t) = s(t) \), it follows that the optimum choice for the weight vector must satisfy

\[ R_{xx}w_{opt} = r_{xd} \quad \text{or} \quad w_{opt} = R_{xx}^{-1}r_{xd} \]  

(32)

1. Minimum mean-square error (MMSE)

The signal \( d(k) \) is the reference signal. Preferably the reference signal is either identical to the desired signal \( s(k) \) or it is highly correlated with \( s(k) \) and uncorrelated with the interfering signals \( i(k) \). If \( s(k) \) is not distinctly different from the interfering signals, the minimum mean square technique will not work properly. The signal \( e(k) \) is the
error signal such that [10]:
\[ \epsilon(k) = d(k) - \bar{\omega}^H \tilde{x}(k) \]  
(43)

\[ |\epsilon(k)|^2 = |d(k)|^2 - 2d(k)\bar{\omega}^H \tilde{x}(k) + \bar{\omega}^H \tilde{x}(k)\bar{\omega} \]  
(54)

Taking the expected value of both sides and simplifying the expression we get
\[ E[|\epsilon|^2] = E[|d|^2] - 2\bar{\omega}^H \bar{r} + \bar{\omega}^H \bar{R}_{xx} \bar{\omega} \]  
(65)

Where the following correlation are defined
\[ \bar{r} = E[d^* \cdot \tilde{x}] = E[d^*(\tilde{x} + \tilde{x}_1 + \tilde{n})] \]  
(16)
\[ \bar{R}_{ss} = E[\tilde{x} \tilde{x}^H] = \bar{R}_{xx} + \bar{R}_{uu} \]  
(17)
\[ \bar{R}_{uu} = \bar{R}_{ii} + \bar{R}_{nn} \]  
(19)
\[ \bar{R}_{xx} = E[\tilde{x}_1 \tilde{x}_1^H] \]  
(20)
\[ \bar{R}_{uu} = \bar{R}_{ii} + \bar{R}_{nn} \]  
(21)

In general, for an arbitrary number of weights, we can find the minimum value by taking the gradient of the MSE with respect to the weight vectors and equating it to zero. Thus the Wiener-Hopf equation is given as
\[ \nabla_{\bar{\omega}}(E[|\epsilon|^2]) = 2\bar{R}_{xx} \bar{\omega} - 2\bar{r} = 0 \]  
(22)

Simple algebra can be applied to yield the optimum Wiener solution given as
\[ \hat{\bar{\omega}}_{MSE} = \bar{R}_{xx}^{-1} \bar{r} \]  
(23)

If we allow the reference signal d to be equal to the desired signal s, and if s is uncorrelated with all interferers, we may simplify the correlation. So the simplified correlation is
\[ \bar{r} = E[s \cdot \tilde{x}] = S \cdot \bar{a}_0 \]  
(24)
\[ S = E[|s|^2] \]  
(25)

The optimum weights can then be identified as
\[ \hat{\bar{\omega}}_{MSE} = S \bar{R}_{xx}^{-1} \bar{a}_0 \]  
(26)

2. Maximum likelihood

The maximum likelihood (ML) method is predicated on the assumption that we have an unknown desired signal \( \tilde{x}_c \) and that the unwanted signal \( \tilde{n} \) has a zero mean Gaussian distribution. The goal of this method is to define a likelihood function which can give us an estimate on the desired signal. It should be noted that no feedback is given to the antenna elements. The input signal vector is given by [11]:
\[ \tilde{x} = \bar{a}_0 s + \tilde{n} = \tilde{x}_c + \tilde{n} \]  
(27)

The overall distribution is assumed to be Gaussian but the mean is controlled by the desired signal \( \tilde{x}_c \). The probability density function can be described as the joint probability density \( p(\tilde{x} | \tilde{x}_c) \). This density can be viewed as the likelihood function that can be used to estimate the parameter \( \tilde{x}_c \). The probability density can be described as
\[ p(\tilde{x} | \tilde{x}_c) = \frac{1}{\sqrt{2\pi \sigma_n^2}} e^{-((\tilde{x} - \bar{a}_0 \tilde{x}_c)^H \bar{R}_{nn}^{-1} (\tilde{x} - \bar{a}_0 \tilde{x}_c))} \]  
(28)

Where \( \sigma_n \) = noise standard deviation
\[ \bar{R}_{nn} = \sigma_n^2 \bar{I} \] = noise correlation matrix

Thus, we can define the log-likelihood function as
\[ L[\tilde{x}] = -\ln(p(\tilde{x} | \tilde{x}_c)) = C(\tilde{x} - \bar{a}_0 \tilde{x}_c)^H \bar{R}_{nn}^{-1} (\tilde{x} - \bar{a}_0 \tilde{x}_c) \]  
(29)

Where \( C = \) constant
\[ \bar{R}_{nn} = E[\bar{a} \bar{a}^H] \]  
(30)

Let us define our estimate of the desired signal, called \( \hat{\tilde{x}} \), that maximizes the log-likelihood function. The maximum of \( \hat{\tilde{x}} \) is found by taking the partial derivative with respect to \( \tilde{x} \) and setting it equal to zero.

Thus
\[ \frac{\partial L[\tilde{x}]}{\partial \tilde{x}} = 0 = -2 \bar{d}^H \bar{R}_{nn}^{-1} \hat{\tilde{x}} + 2 \]  
(31)

Solving for \( \hat{\tilde{x}} \) we get
\[ \hat{\tilde{x}} = \frac{\bar{d}^H \bar{R}_{nn}^{-1} \bar{a}_0}{\bar{a}_0^H \bar{R}_{nn} \bar{a}_0} \]  
(32)

Thus
\[ \bar{w}_{ML} = \frac{\bar{R}_{nn}^{-1} \bar{a}_0}{\bar{a}_0^H \bar{R}_{nn} \bar{a}_0} \]  
(33)

III. RESULTS AND DISCUSSION

1- Minimum mean-square error (MMSE)

Let \( N = 8 \) element array with \( d = 0.5 \lambda \) has a received signal \( S = 1 \) arriving at \( \theta_0 = 30^\circ \), and interferer arriving at angle \( \theta_1 = -60^\circ \), with noise variance 0.001. Calculate the optimum weights and plot the pattern.

Matlab code:
\[ a0 = \exp(1j*2*pi*d*(n-1)*sin(th0)); \]
\[ a1 = \exp(1j*2*pi*d*(n-1)*sin(th1)); \]
\[ Rss = a0*a0'; \]
\[ Rnn = 0.001*eye(N); \]
\[ Rii = a1*a1'; \]
\[ Rnn = 0.001*eye(N); \]
\[ a1 = \exp(1j*2*pi*d*(n-1)*sin(th1)); \]
\[ w = \text{inv}(Rxx)*a0; \]
For $N=8$, $d=0.5$

signal arriving at the angle $\theta_0 = 30^\circ$

an interferer at $\theta_1 = -60^\circ$.

2- Maximum likelihood

Let $N = 5$ element array with $d = 0.5\lambda$ has a received signal arriving at the angle $\theta_0 = 30^\circ$, with noise variance $\sigma_n^2=0.001$. Calculate the optimum weights and plot pattern.

Matlab code:

$$\bar{R}_{nn} = \sigma_n^2$$

$$\bar{\alpha}_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The array steering vector is given by

$$\bar{a}_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The calculated array weights are given as

$$\bar{\omega}_{ML} = \frac{\bar{R}_{nn}^{-1} \bar{a}_0}{\bar{a}_0^H \bar{R}_{nn}^{-1} \bar{a}_0} = 2\bar{a}_0$$

For $N=8$, $d=0.5$

signal arriving at the angle $\theta_0 = 30^\circ$

an interferer at $\theta_1 = -60^\circ$.

For $N=32$, $d=0.5$

For $N=8$, $d=1$
IV. CONCLUSION

1- In this project, two of adaptive algorithms have been discussed and simulated using Matlab. These algorithms were found differ in their complexity and convergence. The adaptive algorithm in 3G must have low computational complexity and hardware implementation.

2- For (MMSE) system, it was found that any increasing in the number of elements (N) will enhance the efficiency of received signal and decrease the effect of noise, but at the same time this increasing in (N) will make the beam width of radiated pattern narrower more due to reduction in the radiated energy signal.

3- (ML) algorithm will be efficient more than other algorithms when treated with more than two interference signals, and it's similar to (MMSE) when the number of antenna (N) increases.

REFERENCES


