

# Trajectory Tracking of Nonlinear Manipulator System Based an Independent PD Controller

Hang Li<sup>1</sup>, Lei Zhang<sup>2</sup>, Linyun Cheng<sup>3</sup>, Hao Luo<sup>4</sup>

**Abstract**—For the application of computational torque method to the nonlinear control of the manipulator arm, the parameter error of the dynamic model will cause the position error to increase and the stability of the system to decrease. The independent PD adjustment technology will be applied to the nonlinear control of the robot arm to establish the position error. The relationship between the parameters of the dynamic model and the error compensation term is proposed in the control system. This method is suitable for the robot arm that does repeated work. In the MATLAB software, the two-link manipulator model established by the S function module was used to evaluate the method's position error capability, and a trajectory following test was performed. The research results show that this method can effectively compensate the error of the dynamic model, reduce the position error of the robot arm, and improve the trajectory accuracy.

**Index Terms**—nonlinearity; PD; tracking

## I. INTRODUCTION

Due to the important role of robotic arms in the automated production process, the control of robotic arms has become a research hot spots issue in recent years. Among them, the method of torque calculation is widely used for trajectory tracking of a robot arm. However, with the improvement of the tracking accuracy and speed, the traditional linear robot control system can no longer meet. For robotic systems with strong coupling and nonlinearity, linear PID control is the simplest and most effective control method and has been widely used in industrial robots<sup>[1]</sup>. This paper selects two-link manipulators as the research object. In the analysis of the two-link manipulator system, its dynamic equations and its control problems are the basis for understanding the manipulator control technology. Since the two control arm loops are not completely independent of each other, there is a coupling between them. Therefore, the control loops of each joint cannot be designed individually and must be processed by a multiple-input multiple-output system. Such a system has severe nonlinear characteristics<sup>[2]</sup>. In this way, traditional linear mechanical control systems cannot be used for tracking. In this paper, based on the

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establishment of a nonlinear dynamic model, the PD parameters are set to track and control the position and moment values of the robot arm<sup>[3]</sup>.

## II. DESIGN AND EXPERIMENT

### A. Description of a Two Degree of Freedom Manipulator

Equations that characterize the robot dynamic are represented by a set of coupling differential equations, and, there are terms such as: varying inertia, centrifugal and Coriolis torques, load and gravity terms. The movement of the end-effector in a particular trajectory with constraint speed requires a complex set of torque functions to be applied to the actuators in the link of the robotic manipulator. Next, the description of the robot mathematical model is presented.

The manipulator model usually considers the representation of the robotic manipulator dynamic of n-links (in our case n = 2) governed by the following equation<sup>[4]</sup>.

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau \quad (1)$$

where  $M(\theta) \in R^{n \times n}$  is the positive definite matrix of the system,  $C(\theta, \dot{\theta}) \in R^{n \times 1}$  is the vector that represents the effects of centrifugal and Coriolis torques,  $G(\theta) \in R^{n \times 1}$  is the vector of the gravitational torque effect,  $\tau \in R^{n \times 1}$  the vector of the torque of the links, and  $\theta, \dot{\theta}$  and  $\ddot{\theta}$  are angular position, velocity and acceleration of the links. The dynamic model of robotic manipulator utilized for evaluation of the controllers is presented in Fig. 1

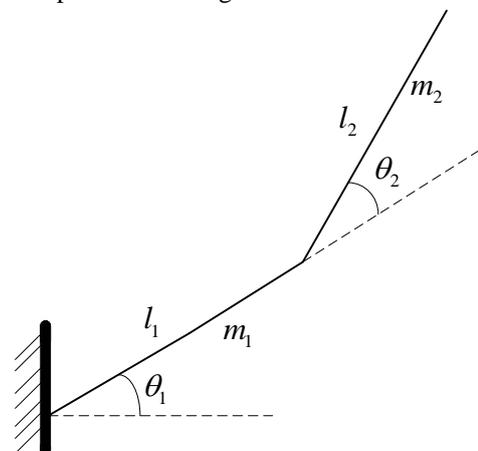


Fig.1. Geometry of robotic manipulator of two-degree of freedom

**B. Robotic arm dynamic model and its structural characteristics**

For the ideal model (1), it is usually for linear systems, but in practice, there are still uncertainties such as friction and disturbance in the robot arm. Consider an N-joint robotic arm whose arm whose dynamic performance can be described by the following second-order nonlinear differential equation<sup>[5]</sup>:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \omega = \tau \quad (2)$$

In the formula:  $q \in R^n$  is the angular displacement of the joint,  $D(q) \in R^{n \times n}$  is the inertial matrix of the robot,  $C(q, \dot{q}) \in R^n$  is the centrifugal force and Coriolis force,  $G(q) \in R^n$  is the gravity term,  $\tau \in R^n$  is the control moment, and  $\omega \in R^n$  is various errors and disturbances. The dynamic characteristics of the robotic system are as follows:

- Feature 1:  $D(q) - 2C(q, q)$  is a skew symmetric matrix.
- Feature 2: The inertial matrix  $D(q)$  is a symmetric positive definite matrix, There are positive numbers  $h_1, h_2$  that satisfy the following inequality:

$$h_1 \|x\|^2 \leq x^T D(q)x \leq h_2 \|x\|^2 \quad (3)$$

Feature:3 There is a parameter vector that depends on the robot parameters so that  $D(q), C(q), G(q)$  satisfy the linear relationship:

$$D(q)\dot{\theta} + C(q, \dot{q})\rho + G(q) = \Phi(q, q, \rho, \theta)P \quad (4)$$

Where  $\Phi(q, q, \rho, \theta)P \in R^{n \times m}$  is regression matrix of a known joint variable function, which is a known function matrix of the generalized coordinates of the robot arm and its derivatives;  $P \in R^n$  is unknown constant parameter vector describing the quality characteristics of the robot<sup>[6]</sup>.

**C. Control Law Design**

The independent PD control law is:

$$\tau = K_d \dot{e} + K_p e \quad (5)$$

Take the tracking error as  $e = q_d - q$ , when using fixed-point control, the  $q_d$  is constant, then  $\dot{q}_d = \ddot{q}_d = 0$ . At this point, the arm equation:

$$D(q)(\ddot{q}_d - \ddot{q}) + C(q, \dot{q})(\dot{q}_d - \dot{q}) + K_d \dot{e} + K_p e = 0 \quad (6)$$

That is:

$$D(q)\ddot{e} + C(q, \dot{q})\dot{e} + K_p e = -K_d \dot{e} \quad (7)$$

Take the Lyapunov function as:

$$\dot{V} = \dot{e}^T D(q)\dot{e} + \frac{1}{2} e^T K_p e \quad (8)$$

By the positive definiteness of  $D(q)$  and  $k_p, V$  is global positive definite, then

$$\dot{V} = \dot{e}^T D\ddot{e} + \frac{1}{2} \dot{e}^T \dot{D}\dot{e} + \dot{e}^T K_p e \quad (9)$$

Using the  $\dot{D} - 2C$  oblique symmetry of  $\dot{e}^T \dot{D}\dot{e} = 2\dot{e}^T C\dot{e}$ , then:

$$\dot{V} = \dot{e}^T D\ddot{e} + \dot{e}^T C\dot{e} + \dot{e}^T K_p e = \dot{e}^T (D\ddot{e} + C\dot{e} + K_p e) = -\dot{e}^T K_d \dot{e} \leq 0 \quad (10)$$

**D. Convergence analysis**

Since  $\dot{V}$  is semi-negative definite and  $K_d$  is positive definite, then when  $\dot{V} \equiv 0$ , there is  $\dot{e} \equiv 0$  and thus  $\ddot{e} \equiv 0$ . Substituting into equation (7) has  $K_p e = 0$ , then the reversibility of  $K_p$  is known as  $e = 0$ . According to LaSalle's theorem,  $(e, \dot{e}) = (0, 0)$  is global asymptotically stable equilibrium of the controlled robot, then, starting from any initial condition  $(q_0, \dot{q}_0)$ , there  $q \rightarrow q_d, \dot{q} \rightarrow 0$ .

For any nonlinear continuous function, the purpose of nonlinear compensation is achieved by tuning the PID parameters. Due to the complexity of its structure, the manipulator of practical application often has modeling errors, mainly uncertain friction and disturbance. The composite control system constructed in this paper first sets the parameters of the PD for the prevailing friction of the robot arm, and thus obtains the compensation control amount of the friction part. For the actual random disturbance<sup>[7]</sup>, a feedback control law is designed to overcome the disturbance. The impact; Finally, the two are combined with the control of the torque calculation to achieve tracking control of the desired trajectory. The controller consists of two parts, nonlinear PID feedback and calculated torque feedforward control. The structure of the composite control system is shown in Fig. 2.

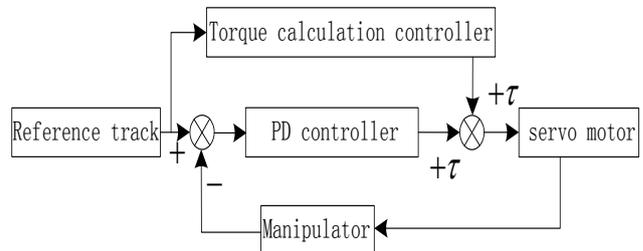


Fig.2. Robot control block diagram based on independent PD control

**E. S function introduction**

In this paper, the manipulator is simulated using SIMULINK based on S function. It is suitable for the mathematical description of complex dynamic systems, and the simulation parameters are more accurately described in the simulation process. In the SIMULINK simulation of the manipulator control system in this paper, Use the S function to implement the control rate, adaptive law, and description of the controlled object<sup>[8]</sup>.

The basic function and important parameter setting of S function:

(1) S function function module: Various function modules perform different tasks. These function modules (functions) are called simulation routines or callback functions, including initialization, derivative, and output.

(2) NumContStates represents the number of consecutive states in the module described by the S-function.

(3) NumDiscStates indicates the number of discrete states.

(4) NumOutputs and NumInputs represent the number of modules input and output, respectively.

(5) Direct feedthrough is the identification of whether the input signal appears at the output, and the value is 0 or 1.

(6) NumSampleTimes is the number of sampling cycles of the module. The S function supports systems with multiple sampling periods<sup>[9]</sup>.

In addition to sys, the system's initial state variable  $x_0$ , description variable str, and sampling period variable  $t_s$  should also be set. The  $t_s$  variable is a two-column matrix where each row corresponds to one sampling period. For continuous systems and systems with a single sampling period, the variable is  $[t_1, t_2]$ ,  $t_1$  is the sampling period,  $t_1 = -1$  represents the sampling period of the inherited input signal, and  $t_2$  is the offset, which is generally 0. For continuous systems,  $t_s$  is taken as  $[-1, 0]$ .

#### F. The setting of simulation parameters for a nonlinear two-degree-of-freedom manipulator

This paper selects a two-degree-of-freedom nonlinear robotic arm system as a simulation object, The nonlinear dynamic equation is as follows:

$$\tau_1 = m_2 l_2^2 (\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2 (2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \quad (11)$$

$$\tau_2 = m_2 l_1 l_2 c_2 \ddot{\theta}_1 + m_2 l_1 l_2 s_2 \ddot{\theta}_1^2 + m_2 l_1 g c_{12} + m_2 l_2^2 (\ddot{\theta}_1 + \ddot{\theta}_2) \quad (12)$$

where  $s_1 = \sin(\theta_1)$ ,  $s_2 = \sin(\theta_2)$ ,  $c_1 = \cos(\theta_1)$ ,  $c_2 = \cos(\theta_2)$ , and  $c_{12} = \cos(\theta_1 + \theta_2)$  and the subscript 1 and 2 denote the parameters of the links 1 and 2, respectively. Parameters utilized in all simulations were: links lengths  $l_1 = 0.8m$  and  $l_2 = 0.4m$ , links masses  $m_1 = m_2 = 0.1kg$ , and gravity acceleration <sup>[10]</sup>  $g = 9.81m/s^2$ . The sampling period is  $T_s = 10$  ms and the simulation period is 2 s ( $N = 200$  samples). The imposed constraint in torques  $\tau_1$  and  $\tau_2$  are  $[1000; 1000]$  N m. Signals  $\theta_{d,j}$  and  $\dot{\theta}_{d,j}$  are desired values of the angular position and velocity of the robotic links, respectively. The position and velocity error vectors are respectively defined by:

$$e_j(t) = \theta_j(t) - \theta_{d,j}(t), \quad j = 1, 2 \quad (13)$$

$$v_j(t) = \dot{\theta}_j(t) - \dot{\theta}_{d,j}(t), \quad j = 1, 2 \quad (14)$$

This paper calculates the form of multivariable PD control:

$$\tau_j(t) = K_{p,j} e_j(t) + K_{i,j} \int_0^t d(s) ds, \quad j = 1, 2 \quad (15)$$

$K_{p,j}$  and  $K_{i,j}$  are positive diagonal matrices with respect to positional and integration errors, respectively. From the previous discretization equation, the discrete equations of the controller are given

$$\Delta \tau_j(t) = (K_{p,j} + 0.5T_s K_{i,j}) e_j(t) - (K_{p,j} - 0.5T_s K_{i,j}) e_j(t-1), \quad j = 1, 2 \quad (16)$$

$$v_j = [\theta_{d,j} - \theta_{d,j}(t-1) - \theta_j(t) + \theta_j(t-1)] / T_s, \quad j = 1, 2$$

(17)

This paper selects the curve trajectory to verify the tracking performance of the two-degree-of-freedom manipulator. In order to obtain the trajectory in the joint space, the inverse kinematics equation is used to convert the path point into the joint angle value, and then map each joint variable into one. The smooth time function starts from the starting point, passes through all the path points in turn, and finally reaches the target point. The time function of each joint is independent of each other, but the total exercise time is the same. This section uses different smooth time functions to analyze the planning results and uses a cubic polynomial time function. Consider the problem of moving the arm end from the initial position to the target position within a certain period of time. Applying kinematics can relieve each joint angle that corresponds to the target pose. The initial position of the operating arm is known and described with a set of joint angles. Determine the equation of motion of each joint (18)-(20):

$$\theta_{d,j}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3, \quad j = 1, 2 \quad (18)$$

$$\dot{\theta}_{d,j}(t) = a_1 + 2a_2 t_f + 3a_3 t_f^2, \quad j = 1, 2 \quad (19)$$

$$\ddot{\theta}_{d,j}(t) = 2a_2 + 6a_3 t_f, \quad j = 1, 2 \quad (20)$$

where  $\theta_{d,j}(t)$  is the instantaneous desired position,  $\theta_0$  and  $\dot{\theta}_0$  are the adopted values for position, velocity and acceleration in initial time, respectively. In this context,  $\theta_{df,1} = 1$  rad and  $\theta_{df,2} = 2$  are the final desired values for the position ( $\theta_{df,1} = 1$  rad and  $\theta_{df,2} = 2$  rad in  $t = 2$  s and  $\theta_{df,1} = 0.5$  rad and  $\theta_{df,2} = 4$  rad in final time,  $t_f = 4$ ) and velocity ( $\dot{\theta}_{df,1} = \dot{\theta}_{df,2} = 0$  rad/s in  $t = 2$  s and  $t_f = 4$ ), respectively. The search range of PID parameters is  $[0; 350]$  for the gains  $K_{p,1}$  and  $K_{p,2}$ , and  $[0; 50]$  for gains  $K_{d,1}$ ,  $K_{d,2}$ .

S function is used to design the controller and the controlled object. The simulation model of the module is shown in Figure 3.

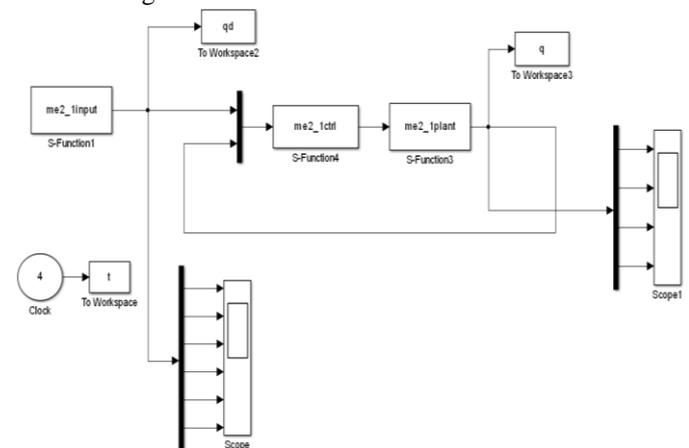


Fig.3. Simulink simulation diagram based on independent PD controller

The trajectory tracking curves of joint 1 and joint 2 in the MATLAB simulation diagram are shown in Fig. 4 to Fig. 5. From the results, the controller has good tracking characteristics.

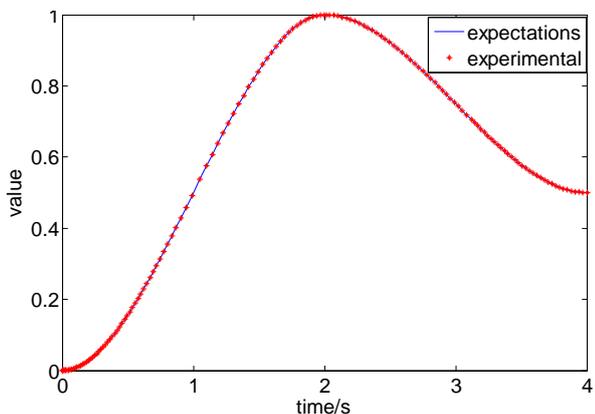


Fig.4. Joint 1 trajectory tracking curve

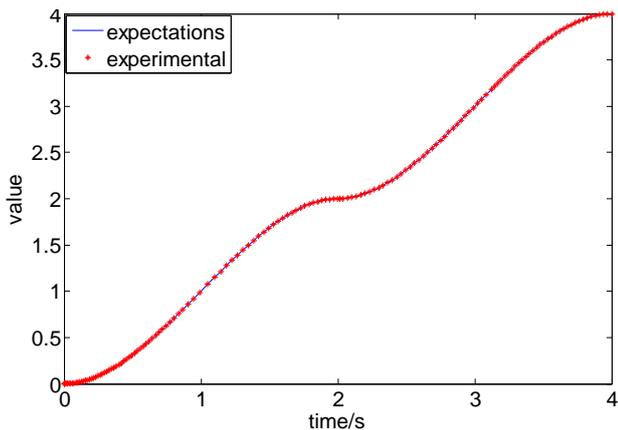


Fig.5. Joint 2 trajectory tracking curve

As shown in Figure 6 to Figure 7, the simulation results show that the algorithm can significantly improve the tracking accuracy and ensure the stability and controllability of the controller.

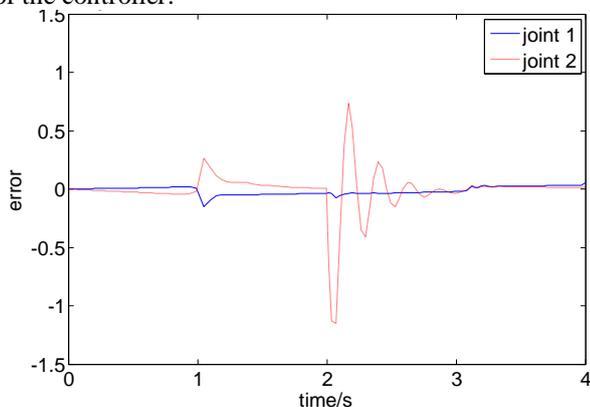


Fig.6. Error curve

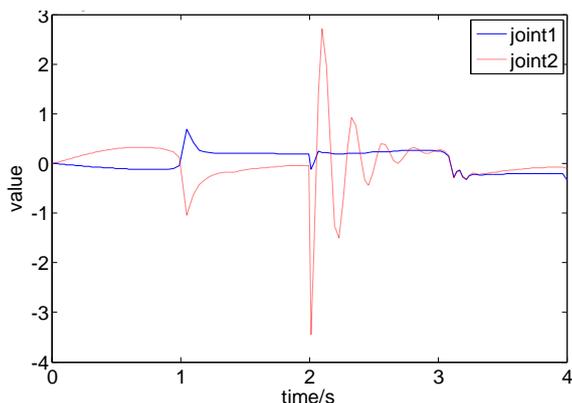


Fig.7. Error derivative curve

Figure 8 shows the simulation results of the control signal (torque). From the simulation results, the optimized PID controller control signal has a good degree of smoothness. The controller has strong controllability.

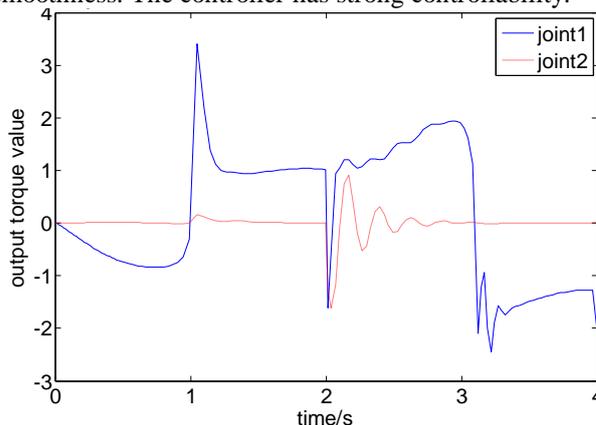


Fig.8. Output torque curve

Figure 9 to Figure 10 show the velocity response curves for joint 1 and joint 2. From the simulation results, the controller can quickly track the target trajectory.

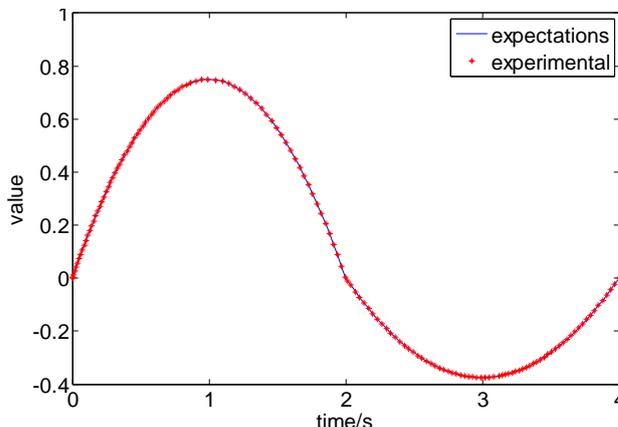


Fig.9. Joint 1 speed response curve

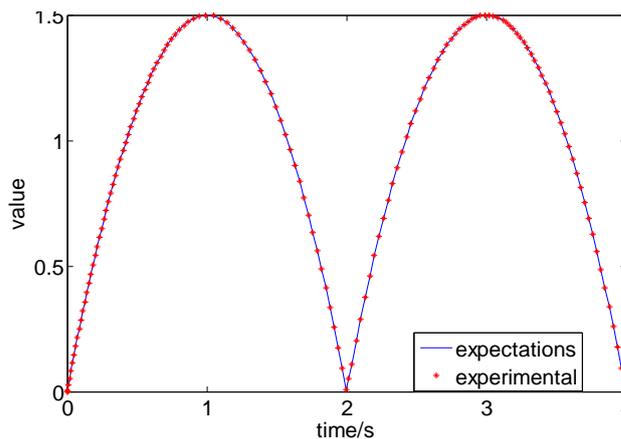


Fig.10. Joint 2 speed response curve

### III. CONCLUSION

This paper designs a nonlinear dynamic model to track and control the robotic arm system. The main contents are as follows: Before the design, the cubic polynomial of the initial joint angle position to the desired end position is obtained by giving the trajectory function, and the torque values of the two joints are obtained according to the dynamic model of the

nonlinear two-degree-of-freedom robotic arm. Then use the S function module to design the simulation model. From the results of simulation experiments, the expected tracking effect was achieved.

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