Exponential L1 control of nonlinear positive switched delay systems

Mingliang Ma, Yongzhen Wang, Yanchang Li

***Abstract*—The exponential L_1 control for a class of nonlinear switched positive delayed systems is discussed in this paper. First, by choosing a nonlinear Lyapunov-Krasovskii functional, a sufficient condition for exponential L_1 stability of the systems is established in terms of linear programming. Then, a feedback control law containing nonlinear functions and a state feedback control law are designed, respectively. Finally, an illustrative example is provided to illustrate the effectiveness of the results.**

***Index Terms*—Exponential stability, L_1 performance, nonlinear positive switched systems, linear programming.**

I. Introduction

A switched system is a special class of hybrid systems which is composed of discrete-time or continuous-time dynamic subsystems and a law determining when and how the switching events occur. Such system has gained wide concerns from variety researchers for extensive applications in manufacturing systems [1], chemical processing [2], air traffic control [3], etc. A system with positive constraint is called a positive system, whose state trajectories always remain non-negative as long as the initial condition is non-negative. There have been a large amount of excellent papers on the stability analysis, controller design and so on[4-7].

Delays are universal in real engineering processes and have very complex impacts on system dynamics. Although many results have been reported for time-delay systems [8-9], only recently have positive systems with time delays become a topic of major interest. To list a few, a necessary and sufficient stability criterion for positive systems with constant delays is obtained in [10] by means of a linear co-positive Lyapunov functional.

The results mentioned above are mainly concerned with switched linear positive systems. Up to now, few efforts are devoted to switched nonlinear positive systems. As we all know, main reasons lie in two aspects: (i) the nonlinearity problem has been a hard topic in the science and engineering fields, (ii) how to define the positivity of a nonlinear system is not an easy work. However, there is no result on the control of exponential L1 control for a class of nonlinear switched positive delayed systems, which motivates our present work.

The reminder of the paper is organized as follows: In section 2, some preliminaries and problem formulation are presented. By average dwell time method, the problem of exponential L_1 control for a class of nonlinear switched positive delayed systems is investigated in section 3, then an effective algorithm are given to obtain the gain matrices. Section 4 gives a numerical example to show that the obtained results are effective. Conclusion is contained in section 5.

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II. Problem Formulation And Preliminaries

Consider the nonlinear switched positive system:

$$\begin{cases} \dot{x}(t) = A_{0\sigma(t)} f(x(t)) + A_{1\sigma(t)} f(x(t-\tau(t))) + B_{\sigma(t)} u(t) + E_{\sigma(t)} w(t), \\ y(t) = C_{\sigma(t)} g(x(t)). \end{cases} \tag{1}$$

where $x(t) = (x_1(t),...,x_n(t))^T \in \Re^n$, $y(t) \in \Re^n$, $u(t) \in \Re^m$, and $w(t) \in \Re_+^r$ are system state, system output, control input, and exogenous disturbance input, respectively, the differentiable function $\tau(t)$ represents the time delay, $f(x) = (f_1(x_1),..., f_n(x_n))^T \in \Re^n$ and, $g(x) = (g_1(x_1),..., g_n(x_n))^T \in \Re^n$. The function $\varphi(\theta)$ is an initial vector-value function on $\theta \in [-\tau, 0]$ and $x_t = x(t+\theta)$ denotes a solution of system (1).The switching law $\sigma(t)$ is a mapping defining from $[0,\infty)$ to a finite set $S = \{1,2,...,N\}$, $N \in \mathbb{N}_+$, and it is continuous from the right everywhere for a switching sequence $0 \le t_0 < t_1 < ...$. For each $p \in S$, $A_{0p} \in \Re^{n\times n}$, $A_{1p} \in \Re^{n\times n}$, $B_p \in \Re^{n\times m}$, $E_p \in \Re^{n\times r}$, $C_p \in \Re^{n\times n}$.

For the convenience of later development, the following assumptions are given:

Assumption 1. For each $p \in S$, A_{0p} is a Metzler matrix and $A_{1p} \succeq 0$, $B_p \succeq 0$, $E_p \succeq 0$, $C_p \succeq 0$.

Assumption 2. The time delay function $\tau(t)$ satisfies $0 \le \tau(t) \le \tau$ and $\dot{\tau}(t) \le d \le 1$ for a known positive constant τ and an unknown constant d.

Assumption 3. The nonlinear functions $f(x)$ and $g(x)$

lie in sector fields satisfying

$$\eta_1 x_i^2 \le f_i(x_i)x_i \le \eta_2 x_i^2, \tag{2}$$

$$\eta_3 x_i^2 \le g_i(x_i)x_i \le \eta_4 x_i^2, \tag{3}$$

for $x_i \in \Re$ and $i = 1,2,...,n$, where $0 < \eta_1 \le \eta_2$, $0 < \eta_3 \le \eta_4$, and $f_i(0) = 0$.

Definition 1. A system is said to be positive if the initial condition $x(t_0) \succeq 0$ implies the state $x(t) \succeq 0$ and the output $y(t) \succeq 0$ for all the inputs $u(t) \succeq 0$ and $w(t) \succeq 0$.

Definition 2. System (1) is said to be exponentially L_1 state if

(i) for $w(t) = 0$ the origin of the system is exponentially stable with any switching signal.

(ii) There exists positive constants δ, ς, γ and nonnegative real-valued function $W(x(0))$ such that

$$\delta \int_0^\infty e^{-\varsigma t}\|y(t)\|_1 dt \le W(x(0)) + \gamma \int_0^\infty \|w(t)\|_1 dt \tag{4}$$

Definition 3. Let $\sigma(t)$ be a switching signal and $N_\sigma(t_1,t_2)$ be the switching number of $\sigma(t)$ in time interval $[t_1,t_2]$. If there exist two constants $N_0 \ge 0$ and $\tau^* > 0$ such that

$$N_\sigma(t_1,t_2) \le N_0 + (t_2 - t_1)/\tau^* \tag{5}$$

then τ^* is an average dwell time of the switching signal $\sigma(t)$ and N_0 is the chatter bound.

Lemma 1. Let $A \in \Re^{n\times n}$ be a Metzler matrix. Then the following statements are equivalent:

(i) The matrix A is Hurwitz;

(ii) There exists a vector $v \succ 0$ in $\Re^n$ such that $Av \prec 0$.

III. MAIN RESULTS

In this section, we consider the exponential L_1 stability analysis and controller design of nonlinear positive switched systems.

A. Exponential L_1 stability

The first subsection gives absolute exponential L_1 stability of system (1) with $u(t) = 0$.

Theorem 1. Under Assumptions 1-3, if there exit constants $\mu > 0$, $\lambda > 1$, $\gamma > 0$ and vectors $v^{(p)} \succ 0$ with $v^{(p)} \in \Re^n$, $\rho^{(p)} \succ 0$ with $\rho^{(p)} \in \Re^n$, $\vartheta^{(p)} \succ 0$ with $\vartheta^{(p)} \in \Re^n$ such that

$$A_{0p}^T v^{(p)} + \eta_4 C_p^T 1_n + \frac{1}{\eta_1}\mu v^{(p)} + \rho^{(p)} + \tau\vartheta^{(p)} \prec 0, \tag{6}$$

$$A_{1p}^T v^{(p)} - (1-d)e^{\mu\tau}\rho^{(p)} \prec 0, \tag{7}$$

$$E_p^T v^{(p)} - \gamma 1_n \prec 0, \tag{8}$$

$$v^{(p)} \preceq \lambda v^{(q)}, \tag{9}$$

$$\rho^{(p)} \preceq \lambda \rho^{(q)}, \tag{10}$$

$$\vartheta^{(p)} \preceq \lambda \vartheta^{(q)}, \tag{11}$$

Hold $\forall (p,q) \in S \times S$, $p \ne q$, where $1_n = (\underbrace{1,...,1}_{n})^T$, then, under the average dwell time switching satisfying

$$\tau^* \ge \frac{\ln\lambda}{\mu_0}, \tag{12}$$

system (1) is positive and absolutely exponentially L_1 stable, where $\mu_0 \in (0,\mu)$.

Proof. The positivity of system (1) can be obtained by Lemma 2. Thus, $x_t \succeq 0$ for $t \in [-\tau, +\infty)$. Suppose a switching sequence $0 \le t_0 \le t_1 \le \cdots$ and $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$ and the $\sigma(t_m)$ subsystem is active in the interval $[t_m, t_{m+1})$, $k \in \mathbb{N}$. We prove the absolute exponential L_1 stability via a two-step strategy. In the paper, we construct a nonlinear Lyapunov functional:

$$\begin{aligned} &V(t,x_t) \\ &= x^T(t)v^{(\sigma(t))} \\ &\quad + \int_{t-\tau(t)}^{t} e^{\mu(-t+s)} f^T(x(s))\rho^{(\sigma(t))}ds \\ &\quad + \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\mu(-t|+s)} f^T(x(s))\vartheta^{(\sigma(t))}dsd\theta \end{aligned} \tag{13}$$

where $v^{(\sigma(t))} \succ 0$ with $v^{(\sigma(t))} \in \Re^n$, $\rho^{(\sigma(t))} \succ 0$ with $\rho^{(\sigma(t))} \in \Re^n$, $\vartheta^{(\sigma(t))} \succ 0$ with $\vartheta^{(\sigma(t))} \in \Re^n$.

First, consider $w(t) = 0$. Choose the Lyapunov functional candidate (13), then

$$\begin{aligned} &\dot{V}(t,x_t) \\ &= f^T(x(t))A_{0\sigma(t_k)}^T v^{(\sigma(t_k))} \\ &\quad + f^T(x(t-\tau(t)))A_{1\sigma(t_k)}^T v^{(\sigma(t_k))} \\ &\quad - \mu\int_{t-\tau(t)}^{t} e^{\mu(-t+s)} f^T(x(s))\rho^{(\sigma(t_k))}ds \\ &\quad + f^T(x(t))\rho^{(\sigma(t_k))} \\ &\quad - (1-\dot{\tau}(t))e^{-\mu\tau(t)} f^T(x(t-\tau(t)))A_{1\sigma(t_k)}^T \rho^{(\sigma(t_k))} \\ &\quad - \mu\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\mu(-t+s)} f^T(x(s))\vartheta^{(\sigma(t))}dsd\theta \\ &\quad + \tau f^T(x(t))\vartheta^{(\sigma(t_k))} \\ &\quad - \int_{-\tau}^{0} e^{\mu\theta} f^T(x(s))\vartheta^{(\sigma(t_k))}d\theta \end{aligned} \tag{14}$$

for $t \in [t_k, t_{k+1})$. Together $x_t \succeq 0$ with Assumption 3 gives $\eta_1 x_t \preceq f(x_t) \preceq \eta_2 x_t$. Using this and Assumption 2, (14) is transformed into

$$\begin{aligned}\dot{V}(t,x_t) &= -\mu V(t,x_t) + f^T(x(t))\left(A_{0\sigma(t_k)}v^{(\sigma(t_k))}\right.\\ &\left.+\rho^{(\sigma(t_k))} + \tau\vartheta^{(\sigma(t))}\right) + \mu x^T(t)v^{(\sigma(t_k))}\\ &+ f^T(x(t-\tau(t)))\left(A_{1\sigma(t_k)}v^{(\sigma(t_k))}\right.\\ &\left.-(1-d)e^{\mu T}\rho^{(\sigma(t_k))}\right)\\ &-\int_{-\tau}^{0} e^{u\theta} f^T(x(s))\vartheta^{(\sigma(t_k))}d\theta\\ &\le -\mu V(t,x_t) + f^T(x(t))\\ &\times\left(A_{0\sigma(t_k)}v^{(\sigma(t_k))} + \frac{1}{\eta_1}uv^{(\sigma(t_k))}\right.\\ &\left.+\rho^{(\sigma(t_k))} + \tau\vartheta^{(\sigma(t))}\right) + f^T(x(t-\tau(t)))\\ &\times\left(A_{1\sigma(t_k)}v^{(\sigma(t_k))} - (1-d)e^{\mu T}\rho^{(\sigma(t_k))}\right)\end{aligned} \tag{15}$$

By (6), (7), and $\eta_4 C_P^T 1n \succ 0$, it follows that $\dot{V}(t,x_t) \le -\mu V(t,x_t)$ for $t \in [t_k, t_{k+1})$. Thus,

$$V(t,x_t) \le e^{-\mu(t-t_k)} V_{\sigma(t_k)}(t_k, x_{t_k}) \tag{16}$$

By (6), (7), we get

$$V(t,x_t) \le e^{-\mu(t-t_k)}\lambda V_{\sigma(t_k^-)}(t_k^-, x_{t_k^-}) \tag{17}$$

Using an iterative reduction leads to

$$\begin{aligned}V(t,x_t) &\le e^{-\mu(t-t_{k-1})}\lambda V_{\sigma(t_{k-1})}(t_{k-1}, x_{t_{k-1}})\\ &\le e^{-\mu(t-t_{k-1})}\lambda^2 V_{\sigma(t_{k-1}^-)}(t_{k-1}^-, x_{t_{k-1}}^-)\\ &\le e^{-\mu(t-t_{k-2})}\lambda^2 V_{\sigma(t_{k-2})}(t_{k-2}, x_{t_{k-2}})\\ &\le \cdots \le e^{-\mu(t-t_0)}\lambda^{N_{\sigma(t_0,t)}} V_{\sigma(t_0)}(t_0, x_{t_0})\end{aligned} \tag{18}$$

With Definition 3 and $\lambda > 1$ in mind, we have

$$V(t,x_t) \le e^{N_0 In\lambda} e^{(\frac{In\lambda}{\tau^*}-\mu_0)(t-t_0)} V_{\sigma(t_0)}(t_0, x_{t_0}) \tag{19}$$

Recalling the Lyapunov functional (13), we can get

$$\begin{aligned}\varepsilon_1\|x(t)\|_1 &\le V(t,x_t) \le \varepsilon_2\|x\|_1\\ &+ (\varepsilon_3 + \tau\varepsilon_4)\int_{t-\tau}^{t}\|x(s)\|_1 ds\end{aligned} \tag{20}$$

where , $\varepsilon_1 = \min_{P\in S}\{V_i^{(P)}\}$ $\varepsilon_2 = \max_{p\in S}\{V_i^{(p)}\}$, $\varepsilon_3 = \max_{p\in S}\{P_i^{(p)}\}$ $\varepsilon_4 = \max_{p\in S}\{\vartheta_i^{(p)}\}$, $i = 1,\cdots n$, with $v_i^{(p)}$, $\rho_i^{(p)}$, and $\vartheta_i^{(p)}$ are $v^{(p)}$, $\rho^{(p)}$, and $\vartheta^{(p)}$, respectively. Combining (19) and (20) gives

$$\begin{aligned}\|x(t)\|_1 &\le e^{N_0 In\lambda} e^{(\frac{In\lambda}{\tau^*}-\mu_0)(t-t_0)}\\ &\times\frac{\varepsilon_2 + \tau\varepsilon_2 + \tau^2\varepsilon_4}{\varepsilon_1}\sup_{-\tau\le s\le 0}\|x(t)\|_1\end{aligned} \tag{21}$$

Next, consider $w(t) \ne 0$. Noting (2) and Assumption 1, then

$$\begin{aligned}&\dot{V}(t,x_t)\\ &= -\mu V(t,x_t) + f^T(x(t))\left(A_{0\sigma(t_k)}v^{(\sigma(t_k))}\right.\\ &\left.+\rho^{(\sigma(t_k))} + \tau\vartheta^{(\sigma(t))}\right) + \mu x^T(t)v^{(\sigma(t_k))}\\ &+ f^T(x(t-\tau(t)))\left(A_{1\sigma(t_k)}v^{(\sigma(t_k))}\right.\\ &\left.-(1-d)e^{\mu T}\rho^{(\sigma(t_k))}\right) + \mathrm{g}^T(x(t))C_{\sigma(t_k)}^T 1_n\\ &-\int_{-\tau}^{0} e^{u\theta} f^T(x(s))\vartheta^{(\sigma(t_k))}d\theta\\ &+ w^T(t)\left(E_{\sigma(t_k)}^T v^{(\sigma(t_k))} - \gamma 1_n\right) + +\Gamma(t)\\ &\le \mu V(t,x_t) + f^T(x(t))\left(A_{0\sigma(t_k)}v^{(\sigma(t_k))}\right.\\ &+\eta_4 C_{\sigma(t_k)}^T 1_n + \frac{1}{\eta_1}\mu v^{(\sigma(t_k))} + \rho^{\sigma(t_k)} + \Gamma(t)\\ &\left.+\tau\vartheta^{\sigma(t_k)}\right) + f^T(x(t-\tau(t)))\left(A_{1\sigma(t_k)}v^{(\sigma(t_k))} -\right.\\ &\left.(1-d)e^{\mu T}\rho^{(\sigma(t_k))}\right) + w^T(t)\left(E_{\sigma(t_k)}^T v^{(\sigma(t_k))} - \gamma 1_n\right)\end{aligned} \tag{22}$$

Where $\Gamma(t) = \gamma\|w(t)\|_1 - \|y(t)\|_1$. Recalling $x(t) \succeq 0$, $w(t) \succeq 0$, and $f(x) \succeq 0$, we have

$$\dot{V}(t,x_t) \le -\mu V(t,x_t) + \Gamma(t) \tag{23}$$

By (6-8). Thus

$$\begin{aligned}V(t,x_t) &\le \lambda V_{\sigma(t_k^-)}(t_k^-, x_{t_k^-})e^{-u(t-t_k)} + \int_{t_k}^{t} e^{-u(t-s)}\Gamma(s)ds\\ &\le \cdots \le \lambda^{N_\sigma(t_0,t)} e^{-u(t-t_0)} V_{\sigma(t_0)}(t_0, x_{t_0})\\ &+ \lambda^{N_\sigma(t_0,t)}\int_{t_0}^{t_1} e^{-u(t-s)}\Gamma(s)ds\\ &+ \lambda^{N_\sigma(t_0,t)-1}\int_{t_1}^{t_2} e^{-u(t-s)}\Gamma(s)ds - \cdots\\ &+ \int_{t_m}^{t} e^{-u(t-s)}\Gamma(s)ds\\ &= e^{-u(t-t_0)+N_\sigma(t_0,t)In\lambda} V_{\sigma(t_0)}(t_0, x_{t_0})\\ &+ \int_{t_0}^{t} e^{-u(t-s)+N_\sigma(s,t)In\lambda}\Gamma(s)ds\end{aligned} \tag{24}$$

Multiplying both sides of (24) by $e^{-N_\sigma(t_0,t)In\lambda}$ yields

$$\begin{aligned}&e^{-N_\sigma(t_0,t)In\lambda} V_{\sigma(t_m)}(x(t))\\ &\le e^{-u(t-t_0)} V_{\sigma(t_0)}(t_0, x_{t_0})\\ &+ e^{-N_\sigma(t_0,t)In\lambda}\int_{t_0}^{t} e^{-u(t-s)+N_\sigma(s,t)In\lambda}\Gamma(s)ds\\ &= e^{-u(t-t_0)} V_{\sigma(t_0)}(t_0, x_{t_0})\\ &- \int_{t_0}^{t} e^{-u(t-s)-N_\sigma(t_0,s)In\lambda}\Gamma(s)ds\end{aligned} \tag{25}$$

By definition 3 and (12), we have

$$N_\sigma(t_0,s) \le N_0 + \frac{\mu_0(s-t_0)}{In\lambda} \tag{26}$$

Noting the fact and $e^{-N_\sigma(t_0,t)In\lambda}V_{\sigma(t_m)}(t,x_t)>0$ and Substituting (26) into (25) then

$$\int_{t_0}^{t} e^{-u(t-s)-\mu_0(s-t_0)-N_0 In\lambda}\|y(s)\|_1 ds \le e^{-u(t-t_0)}V_{\sigma(t_0)}(t_0,x_{t_0}) + \gamma\int_{t_0}^{t} e^{-u(t-s)}\|w(s)\|_1 ds \quad (27)$$

Taking integrator both sides of (27) from 0 to ∞, we get

$$\int_0^\infty e^{-u_0 s-N_0 In\lambda}\|y(s)\|_1 ds \le V_{\sigma(t_0)}(x_0)+\gamma\int_0^\infty \|w(s)\|_1 ds \quad (28)$$

that is

$$\delta\int_0^\infty e^{-\varsigma t}\|y(t)\|_1 dt \le V_{\sigma(t_0)}(x_0)+\gamma\int_0^\infty \|w(t)\|_1 dt \quad (29)$$

Where $\delta = e^{-N_0 In\lambda}$ and $\varsigma=\mu_0$. By Definition 2, system (1) is exponentially L_1 state under average dwell time.

B. Exponential L_1 stabilization

This section is devoted to exponential L_1 stabilization of system (1).

Theorem 2. Under Assumptions 1-3, if there exist constants $\mu>0$, $\lambda>0$, $\varpi_p>0$, $\gamma>0$, and vectors $v^{(p)}>0$, $\rho^{(p)}\succ 0$ and $\vartheta^{(p)}\succ 0$, such that

$$A_{0p}^T v^{(p)} + z^{(p)} + \eta_4 C_P^T 1_n + \frac{1}{\eta_1}uv^{(p)} + \rho^{(p)} + \tau\vartheta^{(p)} \prec 0, \quad (30)$$

$$A_{0p}\hat{v}^{(p)T}B_p^T v^{(p)} + B_p\hat{v}^{(p)}z^{(p)T} + \varpi_p I_n \succeq 0, \quad (31)$$

$$A_{1p}^T v^{(p)} - (1-d)e^{u\tau}\rho^{(p)} \prec 0, \quad (32)$$

$$E_p^T v^{(p)} - \gamma 1_n \prec 0, \quad (33)$$

$$v^{(p)} \preceq \lambda v^{(q)}, \quad (34)$$

$$\rho^{(p)} \preceq \lambda\rho^{(q)}, \quad (35)$$

$$\vartheta^{(p)} \preceq \lambda\vartheta^{(q)}, \quad (36)$$

hold $\forall (p,q)\in S\times S$, $p\neq q$, where $\hat{v}^{(p)}\succ 0$ with $\hat{v}^{(p)}\in\Re^m$ is given and $1_n=\underbrace{(1,\cdots,1)}_{n}^T$. Then, under the feedback control law

$$u(t)=k_{\sigma(t)}f(x(t))=\frac{1}{\hat{v}^{(p)T}B_T^p v^{(p)}}\hat{v}^{(p)}z^{(p)T}f(x(t)). \quad (37)$$

and the average dwell time switching satisfying (14), the resulting closed-loop system (1) is positive and absolutely exponentially L_1 state

Proof. we first prove the positivity of the resulting closed-loop (1). Since $B_p\succeq 0$, $v^{(p)}\succ 0$ and $\hat{v}^{(p)}\succ 0$, $\hat{v}^{(p)T}B_p^T v^{(p)}\succ 0$ $A_{0p}+B_p k_p+\frac{\varpi_p}{\hat{v}^{(p)T}B_p^T v^{(p)}}I_n\succeq 0$. By (30)-(32), which implies $A_{0p}+B_p k_p$ is a Metzler matrix for each $p\in S$. Together this with Assumption 1, the resulting closed-loop system. Nothing the fact $k_p^T B_p v^{(p)}=z^{(p)}$, we have

$$(A_{0p}+B_p k_p)^T v^{(p)} + \frac{1}{\eta_4}C_p^T 1_n + \frac{1}{\eta_1}uv^{(p)} + p^{(p)} + \tau\vartheta^{(p)} \prec 0$$

which implies under the feedback control law (37). So, the resulting closed-loop system (1) is exponentially L_1 state by Theorem 1.

IV. SIMULATION EXAMPLE

In this section, we provide an example to illustrate the effectiveness of the proposed design. Consider system (1) with

$$A_{01}=\begin{bmatrix}-3.5 & 0.1\\ 0.3 & -3\end{bmatrix}, A_{11}=\begin{bmatrix}0.2 & 0.5\\ 0.8 & 0.6\end{bmatrix},$$

$$A_{02}=\begin{bmatrix}-5 & 0.1\\ 0.1 & -2.5\end{bmatrix}, A_{12}=\begin{bmatrix}0.6 & 0.3\\ 0.8 & 0.5\end{bmatrix},$$

$$B_1=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.3\end{bmatrix}, G_1=\begin{bmatrix}1\\ 2\end{bmatrix}, C_1=\begin{bmatrix}0.1 & 0.2\\ 0.4 & 0.1\end{bmatrix},$$

$$B_2=\begin{bmatrix}0.3 & 0.1\\ 0.2 & 0.1\end{bmatrix}, G_2=\begin{bmatrix}2\\ 1\end{bmatrix}, C_2=\begin{bmatrix}0.2 & 0.5\\ 0.3 & 0.1\end{bmatrix},$$

$$R_1=\begin{bmatrix}0.2\\ 0.3\end{bmatrix},\ R_2=\begin{bmatrix}0.3\\ 0.2\end{bmatrix},\ \varepsilon=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},\ \delta=\begin{bmatrix}1\\ 2\end{bmatrix}.$$

$$E_1=\begin{bmatrix}0.05 & 0.01\\ 0.03 & 0.03\end{bmatrix}, E_2=\begin{bmatrix}0.02 & 0.05\\ 0.08 & 0.06\end{bmatrix},$$

Assuming that $\tau(t)=0.1+0.2\sin(t)$, we can get $\tau=0.3$ and $d=0.2$. $f_i(x_i(t))=x_i(t)+\frac{x_i(t)}{x_i^2(t)+1}$, $g_i(x_i(t))=x_i(t)$, then $\eta_1=1, \eta_2=2, \eta_3=1, \eta_4=1$. Choose μ=0.5, λ=1.5 . From Theorem 2, we have

$$v^{(1)}=\begin{bmatrix}1.1329\\ 2.0026\end{bmatrix}, \rho^{(1)}=\begin{bmatrix}1.7454\\ 2.1851\end{bmatrix}, \vartheta^{(3)}=\begin{bmatrix}1.5327\\ 1.1585\end{bmatrix},$$

$$v^{(2)}=\begin{bmatrix}1.0258\\ 1.5684\end{bmatrix}, \rho^{(2)}=\begin{bmatrix}1.2111\\ 2.0162\end{bmatrix}, \vartheta^{(3)}=\begin{bmatrix}1.3981\\ 1.5641\end{bmatrix},$$

$$z^{(1)}=\begin{bmatrix}2.8144\\ 3.6684\end{bmatrix}, z^{(2)}=\begin{bmatrix}2.9847\\ 3.04147\end{bmatrix},$$

and $\varsigma_1=23.1507, \varsigma_2=22.5642, \gamma=0.5684$, Thus

$$K_1=\begin{bmatrix}0.7915 & 0.4987\end{bmatrix}, K_2=\begin{bmatrix}06659 & 0.8021\end{bmatrix}$$

Fig. 1 shows the simulation results of the states $x_1(t), x_2(t)$. Fig. 2 shows the switching signal. Fig. 3, shows the simulations of the output $y(t)$.

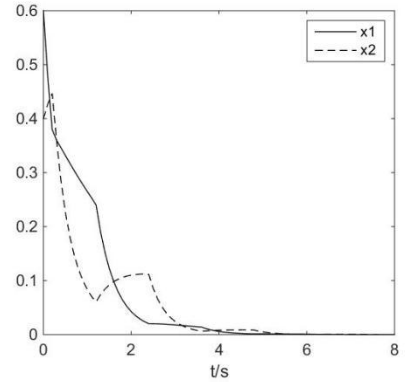

Fig. 1. State trajectories of the closed-loop system (1).

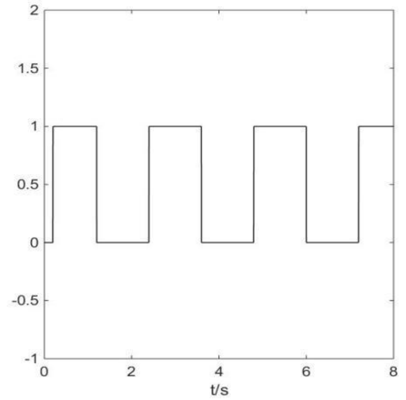

Fig. 2. Switching signal of system (1).

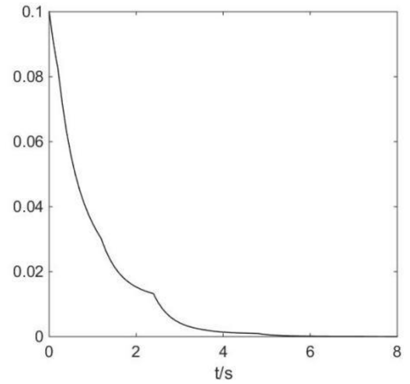

Fig. 3. The simulation of the output $y(t)$.

V. CONSLUSION

This paper has investigated exponential L_1 stability and stabilization of nonlinear positive switched systems with ADT. Based on Lyapunov-Krasovskii functional, some sufficient conditions and a controller are obtained to guarantee that the closed-loop system is exponential L_1 stability. Finally, a numerical example is provided to show the effectiveness of the proposed method.

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