Optical analysis of two-axis scanning mirror and non-linearity correction of 2D-PSD

Mingliang Ma, Yanchang Li ,Yongzhen Wang

***Abstract*—This paper investigates the optical analysis of two-axis scanning mirror and non-linearity correction of 2D-PSD. Firstly, we consider the scan mirror about the pivot range of the ratio and the scanning locus of swinging different coordinate systems. The formula of the scan trace of the scan mirror with different angle of pivot shaft are also obtained. In the actual measurement, the position sensitive detectors (PSD) are used to measure the scan mirror angle. In order to reduce the nonlinearity of scan trace, a bilateral linear interpolation method was given to reduce the nonlinear characteristic of PSD.**

***Index Terms*—Scan mirror, Position sensitive detector, non-linear compensation, bilateral linear interpolation.**

I. INTRODUCTION

In space remote sensing instrument, two-dimensional scanning pendulum is used to swing the two axes to achieve large scanning scene, and the observation of different targets on the ground. The rotating 45 degree plane mirror is the most widely used scanning method in optics and mechanics [1-4]. 45 degrees mirror has the advantages of small scanning mirror, cold space observation, radiometric calibration standard, and more applications in the three axis stabilized satellite platform. In this paper, the method of calculating the swing angle of scanning mirror under shafting is given, and the scanning line on PSD is analyzed. However, because of the structural characteristics of PSD, the nonlinearity is always existence. This nonlinearity leads to a decrease in the confidence of the data measured by the whole device. In order to make up for this defect of PSD and make the output of PSD all have good linearity on the whole photosensitive surface [6-8], it is necessary to compensate the PSD linearly to improve the linearity of PSD.

The aim of this paper is to analysis the optical character of two-axis scanning mirror and proposed a bilateral linear interpolation method to correct the nonlinearity of 2D-PSD. Finally, a simulation result is provided to show the effective of the proposed method.

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II. OPTICAL THEORY ANALYSIS

Fig. 1 is the scan mirror. XYZ is the reference coordinate, $\vec{N}$ refers to the normal direction of two-dimensional scanning mirror, and the shaft angle between $\vec{N}$, and X is 135 degrees; The scanning mirror short axis along the Z axial direction; The scanning mirror long axis and the Q axis is parallel. $\vec{P}$ is a unit vector along the optical axis direction of the optical road. $\vec{P'}$ is the image vector of $\vec{P}$. Assume that the scanning mirror coordinates the axis Z, swings the α angle (pitch angle) to realize the north and south scan of the object, and swings β around the X axis angle (azimuth) for step by step motion, to achieve another one-dimensional image acquisition.

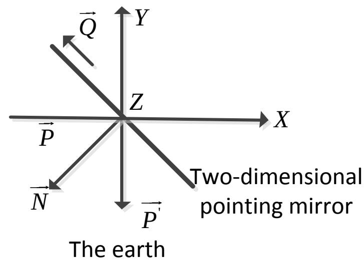

Fig.1 Sketch map of 45° mirror imaging

Next, we analyses the swing angle and the scanning trace of the 45 degree scanning mirror.

According to the conjugate principle of the image of the mirror and the rotation theorem of the mirror. In the selected reference coordinate, when the scanning mirror rotates, the relation between the image vector $\vec{A'}$ and the object vector $\vec{A}$ is expressed as:

$$\overrightarrow{A_0'} = G10 \cdot R \cdot G10^{-1} \overrightarrow{A_0} \tag{1}$$

From Fig.2, $\vec{N}$ represents the mirror normal vector, $\vec{A}$ represents the incident vector, and $\vec{A'}$ represents the reflection vector.

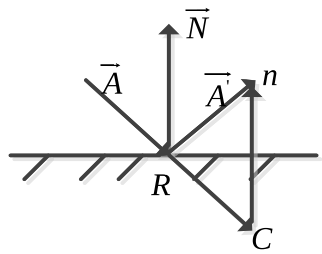

Fig. 2. Sketch a mirror reflection

R is the reflection matrix of the scan mirror, which can be represented as

$$R=\begin{bmatrix}1-2{N_x}^2 & -2N_xN_y & -2N_xN_z \\ -2N_xN_y & 1-2{N_y}^2 & -2N_yN_z \\ -2N_xN_z & -2N_yN_z & 1-2{N_z}^2\end{bmatrix} \quad (2)$$

where, N_x, N_y, N_z are projections of the scan mirror normal vector in the selected reference coordinate system, respectively.

The vector $\vec{A}$ rotates around the unit vector $\vec{P}$, whose rotation angle is θ, and the rotation matrix $S_{p,\theta}$ of vector $\vec{A'}$ is $G10$

$$S_{P,\theta}=\begin{bmatrix}\cos\theta+2P_x^2\sin^2\theta/2 & -P_z\sin\theta+2P_xP_y\sin^2\theta/2 & P_y\sin\theta+2P_xP_z\sin^2\theta/2 \\ P_z\sin\theta+2P_xP_y\sin^2\theta/2 & \cos\theta+2P_y^2\sin^2\theta/2 & -P_x\sin\theta+2P_yP_z\sin^2\theta/2 \\ -P_y\sin\theta+2P_xP_z\sin^2\theta/2 & P_x\sin\theta+2P_yP_z\sin^2\theta/2 & \cos\theta+2P_z^2\sin^2\theta/2\end{bmatrix} \quad (3)$$

P_x, P_y, P_z is the projection of the rotation axis in the selected reference coordinate system. If it rotates of the two shafts, the rotation matrix is the product of the two rotation matrix.

From Fig.1, The scanning mirror short axis along the Z axial direction; The scanning mirror long axis and the $\vec{Q}$ axis is parallel. The unit normal vector of a scanning mirror is $\vec{N}$:

$$\vec{N}=-\frac{\sqrt{2}}{2}\vec{i}-\frac{\sqrt{2}}{2}\vec{j}+0\vec{k} \quad (4)$$

$$R45^{\circ}=\begin{bmatrix}0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1\end{bmatrix} \quad (5)$$

$$\begin{aligned} G10_{X,Z} &= G45^{\circ}{}_{X,Z} \\ &= S45^{\circ}{}_{X,\beta}\cdot S45^{\circ}{}_{Z,\alpha} \\ &= \begin{bmatrix}1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta\end{bmatrix}\times\begin{bmatrix}\cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1\end{bmatrix} \end{aligned} \quad (6)$$

From the equation, the R is the reflection matrix of the scanning mirror, $S_{Z,\alpha}$ is the rotation matrix around the short axis Z of the scanning mirror, and $S_{X,B}$ is the rotation matrix around the X axis. The α, β angle at which the scanning mirror is required to achieve a scanning field of view. It is necessary to note that when calculating the rotation matrix $G10$ of two axes, it is necessary to pay attention to the rotation of the two axes. When the matrix is multiplied, the order of the rotation matrix should be fixed, the rotation matrix is prior, and the rotation matrix of the motion axis is behind.

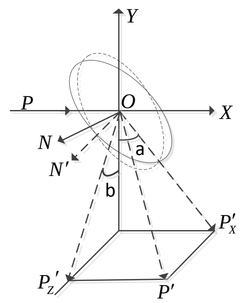

Fig. 3. Sketch of a scanner field

As shown in Figure 3, by studying the optical axis unit vector $\vec{P'}$ in the XOZ plane to calculate swing range of a, b with the scanning mirror(north and south direction of the scanning field angle, the east and west direction scanning field width). The 45 degree mirror axis vector $\vec{P}$, scanning mirror and the axis of rotation matrix $G10_{X,Z}$ into equation (1) can be obtained by reference coordinates after scanning mirror and two-dimensional rotation axis when the corresponding image vector $\vec{P'}$:

$$\vec{P'}=G45^{\circ}\cdot \mathrm{R}45^{\circ}\cdot G45^{\circ-1}\vec{P}=\begin{bmatrix}P'_X \\ P'_Y \\ P'_Z\end{bmatrix} \quad (7)$$

and

$$\tan(a)=\frac{P'_X}{P'_Y} \quad (8)$$

$$\tan(b)=\frac{P'_Z}{P'_Y} \quad (9)$$

From (7)-(9), the relation between the scanning mirror's swing angle α, β and the scanning field angle a, b can be obtained.

III. NON-LINEARITY CORRECTION OF 2D-PSD

In this section, we considered the non-linearity correction of 2D-PSD. Consider the common used 2D-PSD. Because of the distance between the four electrodes is different, the current received by the 4 electrodes is different, so that the current received by each electrode is in one-to-one correspondence with the location of the spot [9]. If the geometric center of the PSD photosensitive surface is the origin, we have the following formula for calculating the coordinate of the incident light spot [10]:

$$\begin{cases} x=\dfrac{(X_2+Y_2)-(X_1+Y_1)}{X_1+X_2+Y_1+Y_2}\cdot\dfrac{L_x}{2} \\ y=\dfrac{(X_2+Y_1)-(X_1+Y_2)}{X_1+X_2+Y_1+Y_2}\cdot\dfrac{L_y}{2} \end{cases} \quad (10)$$

where, X_1, X_2, Y_1 and Y_2 represent the output signal of each electrode, respectively; x and y are the position coordinates of the incident light spot; L_x and L_y are the length of the photosensitive surface of the 2D-PSD in the x and y directions respectively.

Because of the design and manufacture of PSD, the output of PSD in the photosensitive surface is nonlinear. Among the influence factors of measurement accuracy in the scan mirror around the rotating axis of the scanning line, the nonlinear PSD occupies the main factors. In order to improve the accuracy of measurement, it is necessary to compensate the nonlinear error of PSD.

A. Discretization of error function

Since the output of PSD on the whole photosensitive surface is continuously changed, and this change is nonlinear, in order to compensate for the nonlinearity of PSD, PSD needs to be discretized. The PSD and laser are fixed on the high precision numerical control platform, and the laser beam is irradiated perpendicularly to the sensitive surface of PSD. Adjust the PSD, such that the laser in the direction of the numerical control platform driven along the x axis moving, output y remains unchanged, repeat the above steps, make the output x remain unchanged. Then the laser beam will be adjusted to the maximum negative direction of the x axis and the y axis of photosensitive surface, then along the x direction and y direction with a certain step of numerical control platform, record the physical coordinates of CNC platform (x_k, y_J) and PSD output coordinates, so that the entire PSD is separated into a series of grid points. As shown in Fig. 4, the output point of PSD should be at the grid point in theory, and the physical coordinates of the numerical control platform correspond to the output coordinates of the PSD.

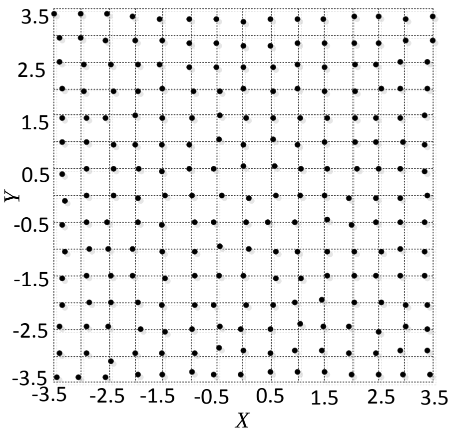

Fig. 4. PSD output(before corrected)

B. Bilateral linear interpolation compensation method

Fig. 5 shows a cell in the network point of the discretized PSD. The points $1, 2, 3, 4$ are the physical coordinates of the measured numerical control platform, and the four point $1', 2', 3', 4'$ are the measured PSD output coordinate points. Point A'_x and point A''_x are the projection of x coordinates of $A(x,y)$ on $3'$-$4'$ and $1'$-$2'$; A'_y and y''_y are the projections of the y coordinates of $A(x,y)$ on $1'$-$4'$ and $2'$-$3'$, respectively. As shown in Figure 5, the coordinate of the points 1, 2, 3, 4 are $1(x_k, y_J)$, $2(x_{k+1}, y_J)$, $3(x_{k+1}, y_{J+1})$, $4(x_k, y_{J+1})$, and the coordinate of the points $1'$, $2'$, $3'$, $4'$ are $1'(x_{kJ}, y_{KJ})$, $2'(x_{k+1}, y_{k+1J})$, $3'(x_{k+1J+1}, y_{k+1J+1})$, $4'(x_{kJ+1}, y_{kJ+1})$, respectively. Taking the direction of x axis as an example, the bilateral linear compensation will be introduced.

Because the PSD output $x_{k+1J} - x_{kJ}$ changes in the x direction of $1' - 2'$ points, is related to the displacement variation $x_{k+1} - x_k$ of numerical control platform in the x direction. So we can get the physical displacement Δx_1 of A''_x on the 1-2.

$$\Delta x_1 = \frac{x - x_{kJ}}{x_{k+1J} - x_{kJ}}(x_{k+1} - x_k) \tag{11}$$

In the same way, we can get the physical displacement Δx_2 corresponding to the x coordinates of the A'_x points on the 3-4.

$$\Delta x_2 = \frac{x - x_{kJ+1}}{x_{k+1J+1} - x_{kJ+1}}(x_{k+1} - x_k) \tag{12}$$

Thus, the physical displacement Δx corresponding to the x coordinates of the points $A(x,y)$ can be obtained by formula (11).

$$\Delta x = \frac{y_{kJ+1} - y}{y_{k+1J} - y_{kJ}}\Delta x_1 + \frac{y - y_{kJ}}{y_{kJ+1} - y_{Jk}}\Delta x_2 \tag{13}$$

Similarly, the physical displacement Δy of the point $A(x,y)$ in the direction y can be obtained.

$$\Delta y_1 = \frac{y - y_{kJ}}{y_{kJ+1} - y_{kJ}}(y_{J+1} - y_J) \tag{14}$$

$$\Delta y_2 = \frac{y - y_{k+1J}}{y_{k+1J+1} - y_{k+1J}}(y_{J+1} - y_J) \tag{15}$$

$$\Delta y = \frac{x_{k+1J} - x}{x_{k+1J} - x_{kJ}}\Delta y_1 + \frac{x - x_{kJ}}{x_{k+1J} - x_{kJ}}\Delta y_2 \tag{16}$$

By using the obtained Δx and Δy, the coordinates (X,Y) of the points $A(x,y)$ after compensation can be obtained.

$$\begin{cases} X = x_k + \Delta x \\ Y = y_k + \Delta y \end{cases} \tag{17}$$

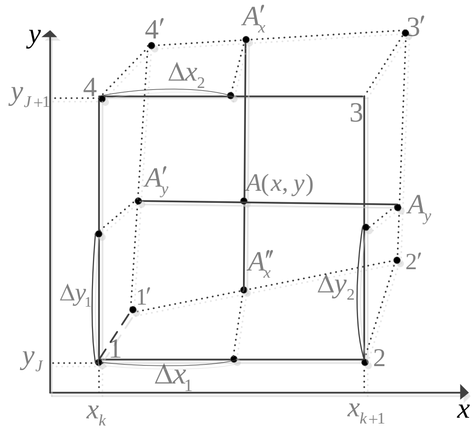

Fig. 5. Discrete grid points of PSD

C. Experimental verification of the compensation algorithm

This experiment uses the two-dimensional pillow shaped PSD developed by Zhejiang University. Its photosensitive area is $L \times L$=8x8mm, the spectral range is 380nm, the position measurement error is ±20-±100um, and the resolution is 5um. Through the optical theory above, the Fig. 6 is the rotation of the 45 degree mirror through the bilateral linear interpolation algorithm. The track map of the north and south sides of the field is scanned at 3.5 degrees, and the scanning field of view of the east west side of the object is 3.5. After comparison of Fig. 6 with Fig. 4, it can be observed that the compensated optical path is essentially straight line, and the linearity of PSD is greatly improved, especially the PSD edge.

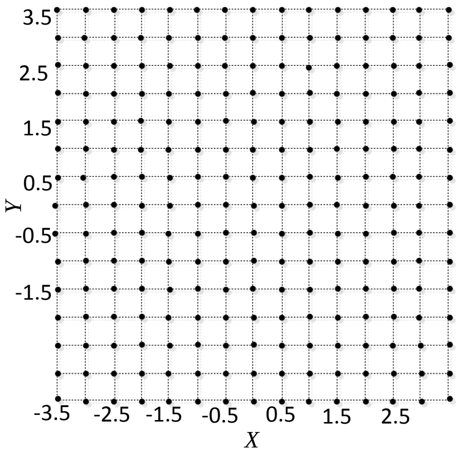

Fig. 6. Scan trace which rotates around X/Z axis (after corrected)

IV. CONCLUSION

By bilateral linear interpolation method, nonlinear compensation of rotating scanning line of shafting 45 degrees mirror is carried out, and the effective area of PSD is enlarged. The compensation accuracy is related to the discrete step distance. In order to improve the precision further, the step distance is reduced and the discrete workload is increased. This method has great application value in practice.

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