Robust output feedback control of positive switched systems with time-varying delays

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Abstract—**This paper investigates the robust output feedback control of positive switched systems with time-varying delays. Firstly, by using the Lyapunov-Krasovskii functional and ADT approach, sufficient conditions are obtained to guarantee that the corresponding system is robustly exponentially stable. Then a static output feedback controller is designed and some sufficient conditions are obtained to ensure that the closed-loop system is robustly exponentially stable. Such conditions can be easily solved by linear programming. Finally, a numerical example is provided to show the effectiveness of the proposed method.**

Index Terms—**Positive switched delay systems, Robustly exponentially stable, Average dwell time, Linear programming.**

I. INTRODUCTION

Positive switched systems, which consist of a family of positive subsystems and a switching signal governing the switching among them, have also been highlighted by many researchers due to their broad applications in communication systems [1], formation flying [2], and systems theory [3-5].. Due to the existence of the positive constraint, many achievements on general switched systems may be not applicable to switched positive systems any more. Therefore, although switched systems and positive systems have been studied well in control theory in the past few decades [6-8], it is necessary to investigate the switched positive linear systems.

Delays are universal in real engineering processes and have very complex impacts on system dynamics. Although many results have been reported for time-delay systems [9-12], only recently have positive systems with time delays become a topic of major interest, especially the time-varying delay. [13] considered the problem of finite-time L_1 control for a class of positive switched linear systems with time-varying delay. In [14], the problem of static output-feedback L_1 finite-time control for switched positive systems with time-varying delay was investigated. [15] studied the absolute exponential stability and stabilization with L_1-gain performance of switched nonlinear positive systems with time-varying delay.

On the other hand, The static output feedback problem is one of the most important open questions in control engineering. A survey of static output-feedback control is given by [16]. Conditions often involve two Riccati equations coupled by a spectral radius condition[17]. [18] presented an algorithm for computing the optimal H_2 static output-feedback gain that is in standard use, along with a convergence proof. Up to now, the exist results related to positive switched systems main focus on the state feedback controller design, the robust output feedback control of positive switched systems with time-varying delays was not fully investigated, which motivates our present study.

In this paper, we are interested in investigating the problem of robust output feedback control of positive switched systems with time-varying delays. Firstly, the definition of exponential stability is given. Secondly, by using the ADT approach and copositive Lyapunov-Krasovskii functional method, a static output feedback law is designed and sufficient conditions are obtained to guarantee that the closed-loop system is exponentially stable, such conditions can be easily solved by linear programming.

The rest of the paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. Robust exponential stability analysis and controller design are developed in Section 3. An example is provided in Section 4. Finally, Section 5 concludes this paper.

Notations. In this paper, $A \succ 0$ ($\succeq 0$) means that all elements of matrix A are positive(non-negative). The superscript T denotes the transpose. R_+^n, R^n and $R^{n\times n}$ denote the n-dimensional non-negative (positive) vector space, the n-dimensional Euclidean space and the set of real $n\times n$ matrices. 1-norm $\|x\|$ is defined by $\|x\| = \sum_{k=1}^{n} |x_k|$, where x_k is the k th element of $x \in R^n$, if not explicitly stated, matrices are assumed to have compatible dimensions.

II. PROBLEM STATEMENTS AND PRELIMINARIES

Consider the following positive switched delay systems:

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$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + G_{\sigma(t)}u(t), \\ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0], \\ y(t) = Cx(t). \end{cases} \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ represent the system state and control input. $\sigma(t):[0,\infty) \to \underline{S} = \{1, 2, \cdots, S\}$ is the switching signal, where S is the number of subsystems. $\forall p \in \underline{S}$, A_p, A_{dp} and G_p are constant matrices with appropriate dimensions, p denotes p th subsystem and t_q denotes the q th switching instant. $\varphi(\theta)$ is the initial condition on $[-\tau, 0]$, $\tau > 0$, $d(t)$ denotes the time-varying delay satisfying $0 \le d(t) \le \tau$, $\dot{d}(t) \le h < 1$, where τ and h are known positive constants.

Then we will present some definitions and lemmas for the positive switched systems (1).

Definition 1 [1]. System (1) is said to be positive if for any initial conditions $\varphi(\theta) \succeq 0$, $\theta \in [-\tau, 0]$, and any switching signals $\sigma(t)$, the corresponding trajectory $x(t) \succeq 0$ holds for all $t \ge 0$.

Definition 2 [1]. A is called a Metzler matrix if the off-diagonal entries of matrix A are non-negative.

Lemma 1 [1]. A matrix A is a Metzler matrix if and only if there exists a positive constant η such that $A + \eta I_n \succeq 0$.

Lemma 2 [3]. System (1) is positive if and only if A_p, $\forall p \in \underline{S}$ are Metzler matrices and $\forall p \in \underline{S}$, $A_{dp} \succeq 0$ and $G_p \succeq 0$.

Definition 3 [22]. For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denotes the switching number of $\sigma(t)$ over the interval $[T_1, T_2)$. For given $T_\alpha > 0$ and $N_0 > 0$, if the inequality

$$N_\sigma(T_1, T_2) \le N_0 + \frac{T_2 - T_1}{T_\alpha} \tag{2}$$

holds, then the positive constant T_α is called an average dwell time, and N_0 is called a chattering bound. Generally speaking, we choose $N_0 = 0$.

Definition 4 [1]. If there exist positive constants $\xi_1 > 0$ and $\xi_2 > 0$ such that the state response satisfies $\|x(t)\|_1 \le \xi_1 e^{-\xi_2(t-t_0)} \|x(t_0)\|_1$, $\forall t \ge t_0$ with arbitrary non-negative initial conditions, then system (1) is exponentially stable under a proper switching signal.

The aim of this paper is to design a static output feedback controller $u(t)$ and find a class of switching signals $\sigma(t)$ for positive switched system (1) such that the corresponding closed-loop system is exponentially stable.

III. Main Results

A. exponential stability analysis

In this subsection, we will focus on the problem of exponential stability analysis for positive switched system (1) with $u(t) = 0$. The following theorem gives sufficient conditions of exponential stability for system (1) with ADT.

Theorem 1. Consider the positive switched systems (1) with $u(t) = 0$, for given constant $\lambda_p > 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{S}$, such that the following inequalities hold:

$$\Psi_p = diag\{\psi_{p1}, \psi_{p2} \cdots \psi_{pn}, \psi'_{p1}, \psi'_{p2} \cdots \psi'_{pn}\} \le 0 \tag{3}$$

where

$$\psi_{pr} = a_{pr}^T + \upsilon_{pr} + \tau\vartheta_{pr} + \lambda_p \nu_{pr}$$

$$\psi'_{pr} = a_{dpr}^T \nu_p - (1-h)\nu_{pr}, \ \lambda = \min_{p \in \underline{S}}\{\lambda_p\}$$

$a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, $\nu_p = [\nu_{p1}, \nu_{p2}, \cdots, \nu_{pn}]^T$, $\upsilon_p = [\nu_{p1}, \upsilon_{p2}, \cdots, \upsilon_{pn}]^T$ and $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \cdots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr} and ϑ_{pr} represent the rth elements of the vectors ν_p, υ_p and ϑ_p, respectively, then under the following ADT scheme

$$T_a > T_a^* = \frac{\ln \mu}{\lambda} \tag{4}$$

the system (1) is exponentially stable,where $\mu \ge 1$ satisfies

$$\nu_p \preceq \mu \nu_q, \ \upsilon_p \preceq \mu \upsilon_q, \ \vartheta_p \preceq \mu \vartheta_q, \ \forall p, q \in \underline{S} \tag{5}$$

Proof. Construct the following co-positive Lyapunov-Krasovskii functional candidate for system (3):

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &= x^T(t)\nu_p + \int_{t-d(t)}^{t} e^{\lambda_p(-t+s)} x^T(s)\upsilon_p ds \\ &+ \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(-t+s)} x^T(s)\vartheta_p ds d\theta \end{aligned} \tag{6}$$

where ν_p, υ_p and $\vartheta_p \in R_+^n$, $\forall p \in \underline{S}$.

For the sake of simplicity, $V_{\sigma(t)}(t, x(t))$ is written as $V_{\sigma(t)}(t)$ in this paper.

Along the trajectory of system (1), we have

$$\begin{aligned}
\dot{V}_{\sigma(t)}(t) &= x^T(t)A_p^T\nu_p + x^T(t-d(t))A_{dp}^T\nu_p \\
&\quad -\lambda_p\int_{t-d(t)}^{t} e^{\lambda_p(-t+s)}x^T(s)\upsilon_p ds \\
&\quad + x^T(t)\upsilon_p - (1-\dot{d}(t))e^{-\lambda_p d(t)}x^T(t-d(t))\upsilon_p \\
&\quad -\lambda_p\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(-t+s)}x^T(s)\vartheta_p ds d\theta + \tau x^T(t)\vartheta_p \\
&\quad -\int_{-\tau}^{0} e^{\lambda_p\theta}x^T(t+\theta)\vartheta_p d\theta \\
&\le x^T(t)A_p^T\nu_p + x^T(t-d(t))A_{dp}^T\nu_p \\
&\quad -\lambda_p\int_{t-d(t)}^{t} e^{\lambda_p(-t+s)}x^T(s)\upsilon_p ds + x^T(t)\upsilon_p \\
&\quad -(1-h)x^T(t-d(t))\upsilon_p \\
&\quad -\lambda_p\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(-t+s)}x^T(s)\vartheta_p ds d\theta + \tau x^T(t)\vartheta_p \\
&\quad -\int_{t-d(t)}^{t} x^T(s)\vartheta_p ds
\end{aligned} \tag{7}$$

From (6) and (7) leads to

$$\begin{aligned}
\dot{V}_{\sigma(t)}(t) &\le -\lambda_p V_{\sigma(t)} + x^T(t)\left(A_p^T\nu_p + \upsilon_p + \tau\vartheta_p + \lambda_p\nu_p\right) \\
&\quad + x^T(t-d(t))\left(A_{dp}^T\nu_p - (1-h)\upsilon_p\right)
\end{aligned} \tag{8}$$

Substituting (3) into (8) yields

$$\dot{V}_{\sigma(t)}(t) \le -\lambda_p V_{\sigma(t)}(t) \tag{9}$$

Integrating both sides of (9) during the period $[t_k, t)$ for $t \in [t_k, t_{k+1})$ leads to

$$V_{\sigma(t)}(t) \le e^{-\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k) \tag{10}$$

For any $T_f > 0$, let N be the switching number of $\sigma(t)$ over $[t_0, t)$, and denote $t_1, t_2, \cdots, t_k, \cdots, t_N$ as the switching instants over the $[t_0, t)$. Then, for $t \in [t\ , t)$, $V_{\sigma(t_k)}(t_k) \le \mu V_{\sigma(t_k^-)}(t_k^-)$ is easily obtained from (6). Let $\lambda = \min_{p\in\underline{S}}\{\lambda_p\}$, From (10) , we have

$$\begin{aligned}
& V_{\sigma(t)}(t) \\
&\le e^{-\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k) \\
&\le \mu e^{-\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k^-)}(t_k^-) \\
&\le \mu e^{-\lambda_{\sigma(t_k)}(t-t_k)}e^{-\lambda_{\sigma(t_{k-1})}(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1}) \\
&\le \mu^2 e^{-\lambda_{\sigma(t_k)}(t-t_k)}e^{-\lambda_{\sigma(t_{k-1})}(t_k-t_{k-1})}V_{\sigma(t_{k-}^-)}(t_{k-1}^-) \\
&\le \cdots \le \mu^{N_{\sigma(0,t)}}e^{-\lambda(t-t_0)}V_{\sigma(t_0)}(t_0)
\end{aligned} \tag{11}$$

With Definition 3 and $\mu \ge 1$ in mind, we have

$$V_{\sigma(t)}(t) \le e^{(\frac{\ln\mu}{\tau_\alpha}-\lambda)(t-t_0)}V_{\sigma(t_0)}(t_0) \tag{12}$$

Noting the definition of $V_{\sigma(t)}(t)$ and (6), we have

$$V_{\sigma(t)}(t) \ge \|x(t)\|\varepsilon_1. \tag{13}$$

$$\begin{aligned}
V_{\sigma(0)}(0) &\le \varepsilon_2\|x(t_0)\|_1 + \tau\varepsilon_3\|x(t_0)\|_1 + \tau^2\varepsilon_4\|x(t_0)\|_1 \\
&\le (\varepsilon_2 + \tau\varepsilon_3 + \tau^2\varepsilon_4)\|x(t_0)\|_1
\end{aligned} \tag{14}$$

From (12)-(14), we obtain

$$\|x(t)\|_1 \le \frac{(\varepsilon_2 + \tau\varepsilon_3 + \tau^2\varepsilon_4)}{\varepsilon_1}e^{(\frac{\ln\mu}{T_\alpha}-\lambda)(t-t_0)}\|x(t_0)\|_1 \tag{15}$$

where $\varepsilon_1 = \min_{p\in\underline{S}}\{\nu_{pi}\}, \varepsilon_2 = \max_{p\in\underline{S}}\{\nu_{pi}\}, \varepsilon_3 = \max_{p\in\underline{S}}\{\upsilon_{pi}\},$ $\varepsilon_4 = \max_{p\in\underline{S}}\{\vartheta_{pi}\}, i = 1, 2, \cdots, n,$ respectively.

Then, (15) can be written as

$$\|x(t)\|_1 \le \xi_1 e^{\xi_2(t-t_0)}\|x(t_0)\|_1 \tag{16}$$

where $\xi_1 = \dfrac{(\varepsilon_2 + \tau\varepsilon_3 + \tau^2\varepsilon_4)}{\varepsilon_1}, \xi_2 = \dfrac{\ln\mu}{T_\alpha} - \lambda.$

According to Definition 4, the positive switched system (1) is exponentially stable. This completes the proof.

B. Static output feedback controller design

In this section, we concern with the static output feedback controller design of positive switched delay system (1). Under the controller $u(t) = K_{\sigma(t)}x(t)$, the corresponding closed-loop system is given by

$$\begin{cases}
\dot{x}(t) = (A_{\sigma(t)} + G_{\sigma(t)}K_{\sigma(t)})x(t) + A_{d\sigma(t)}x(t-d(t)), \\
x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0], \\
y(t) = Cx(t).
\end{cases} \tag{17}$$

By Lemma 1, to guarantee the positivity of system (17), $A_p + G_pK_p$ should be Metzler matrices, $\forall p \in \underline{S}$. The following Theorem 2 gives some sufficient conditions to guarantee that the closed-loop system (17) is exponentially stable.

Theorem 2. Consider the positive switched system (17). For given constant $\lambda_p > 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{S}$, such that (5) and the following conditions hold:

$$A_p + G_pK_p \text{ are Metzler matrices.} \tag{18}$$

$$\bar{\Psi}_p = diag\{\bar{\psi}_{p1}, \bar{\psi}_{p2}\cdots\bar{\psi}_{pn}, \bar{\psi}'_{p1}, \bar{\psi}'_{p2}, \cdots\bar{\psi}'_{pn}\} \le 0 \tag{19}$$

where

$$\bar{\psi}_{pr} = a_{pr}^T + \upsilon_{pr} + \tau\vartheta_{pr} + g_{pr} + \lambda_p\nu_{pr}$$

$$\psi'_{pr} = a_{dpr}^T\nu_p - (1-h)\nu_{pr}$$

$\lambda = \min_{p\in\underline{S}}\{\lambda_p\}$, $g_p = K_p^TG_p^T\nu_p$, g_{pr} represents the rth elements of vector g_p, $a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, and $\nu_p =$ $[\nu_{p1}, \nu_{p2}, \cdots, \ \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \cdots, \upsilon_{pn}]^T$ and $\vartheta_p =$ $[\vartheta_{p1}, \vartheta_{p2}, \cdots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr} and ϑ_{pr} represents the rth elements of the vectors ν_p, υ_p and ϑ_p, respectively, $\mu \ge 1$ satisfies (5), then under the ADT scheme (4), the resulting closed-loop system (17) is exponentially stable

Proof. By Lemma 1, we know that $p \in \underline{S}$, $A_p + G_p K_p$ is a Metzler matrix for each $p \in \underline{S}$. According to Lemma 2, the system (33) is positive if A_{dp} and G_p are all nonnegative. Replacing A_p in (3) with $A_p + G_p K_p$ and letting $g_p = K_p^T G_p^T \nu_p$, similar to Theorem 1, we easily obtain that the resulting closed-loop system (17) is exponentially stable.

The proof is completed.

Next, an procedure is presented to obtain the feedback gain matrices K_p, $p \in \underline{S}$.

Step1. By adjusting the parameters λ_p and solving (5) and (19) via linear programming, positive vectors $\nu_p, \upsilon_p, \vartheta_p$ and g_p can be obtained.

Step2. Substituting ν_p and g_p into $g_p = K_p^T G_p^T \nu_p$, K_p can be obtained.

Step 3. The gain K_p is substituted into $A_p + G_p K_p$. If $A_p + G_p K_p$ are Metzler matrices, then K_p are admissible. Otherwise, return to Step 1.

IV. Example

In this section, a numerical example is provided to show the effectiveness of the proposed approach.

Consider price dynamic model described by switched positive system (1) with the parameters:

$$A_1 = \begin{bmatrix} -1.2 & 0.2 \\ 0.4 & -1 \end{bmatrix},\ A_{d1} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.2 \end{bmatrix},\ G_1 = \begin{bmatrix} 1.5 \\ 1.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & 0.1 \\ 0.2 & -1.1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix},\ G_2 = \begin{bmatrix} 1.3 \\ 1 \end{bmatrix},$$

Assuming that $d(t) = 0.2 + 0.1\cos(t)$, we can get $\tau = 0.3$ and $h = 0.1$. Choosing the parameters $\lambda_1 = \lambda_2 = 0.15$, $\mu = 1.4$ and solving the inequalities in Theorem 2 by linear programming, we get

$$\nu_1 = \begin{bmatrix} 2.1162 \\ 1.3568 \end{bmatrix},\ \upsilon_1 = \begin{bmatrix} 2.6422 \\ 2.2451 \end{bmatrix},\ \vartheta_1 = \begin{bmatrix} 1.8724 \\ 2.0746 \end{bmatrix},$$

$$\nu_2 = \begin{bmatrix} 2.2657 \\ 2.0242 \end{bmatrix},\ \upsilon_2 = \begin{bmatrix} 1.9987 \\ 2.7814 \end{bmatrix},\ \vartheta_2 = \begin{bmatrix} 1.8951 \\ 2.0956 \end{bmatrix},$$

$$g_1 = \begin{bmatrix} 0.8265 \\ 0.7894 \end{bmatrix},\ g_2 = \begin{bmatrix} 0.9021 \\ 0.8533 \end{bmatrix}$$

By $g_p = K_p^T G_p^T \nu_p$, we have

$$K_1 = \begin{bmatrix} 0.6921 & 0.7007 \end{bmatrix}, K_2 = \begin{bmatrix} 0.6235 & 0.6875 \end{bmatrix}$$

It is easy to verify that (18) is satisfied. Then, according to (4), we get $T_\alpha^* = 2.2431$.

Choosing $T_\alpha = 2.3$, the simulation results are shown in Figs. 1-3, where the initial conditions of system (1) are $x(0) = [0.4 \quad 0.3]^T$ .The state trajectories of the open-loop system are shown in Fig 1. According to the ADT scheme, the switching signals $\sigma(t)$ is depicted in Fig 2. The state trajectories of the closed-loop system with ADT are shown in Fig 3, which implies that the corresponding closed-loop system is exponentially stable.

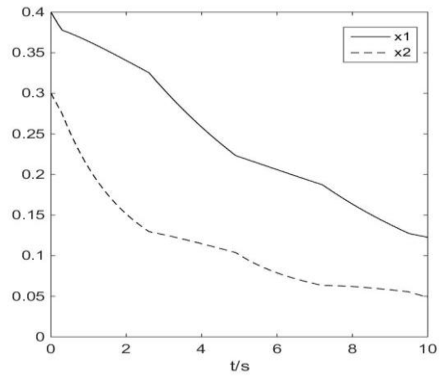

Fig. 1. State trajectories of open-loop system (1)

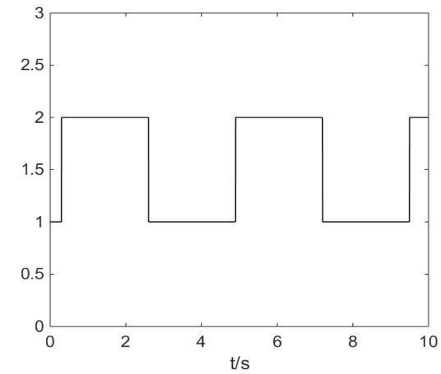

Fig.2. Switching signal of system (1)

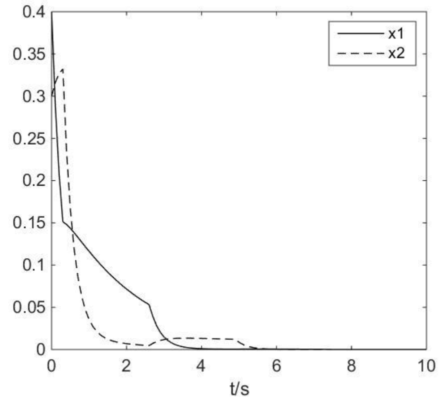

Fig. 3. State trajectories of closed-loop system (1).

V. Conclusion

In this paper, we have studied the problem of robust output feedback control of positive switched delayed systems with ADT. Based on the ADT approach, a static output feedback controller is constructed to guarantee that the closed-loop system is exponentially stable, the obtained sufficient conditions can be solved by linear programming. Finally, an example is given to show the effectiveness of the proposed method.

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