Guaranteed cost finite-time control for positive switched delay systems with ADT

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***Abstract*—This paper investigates the guaranteed cost finite-time boundedness for positive switched delay systems with average dwell time(ADT). The notion of guaranteed cost finite-time boundedness is first introduced. First, by using the Lyapunov-Krasovskii functional and ADT approach, sufficient conditions are obtained to guarantee that the corresponding system is finite-time boundedness. Then a state feedback controller is designed to ensure that the closed-loop system is guaranteed cost finite-time boundedness(GCFTB). The obtained conditions can be easily solved by linear programming. Finally, a practical example is provided to illustrate the effectiveness of the proposed method.**

***Index Terms*—Positive switched delay systems, Guaranteed cost control, Finite-time boundedness, Average dwell time, Linear programming.**

I. INTRODUCTION

Positive switched systems, which consist of a number of positive subsystems and a switching signal governing the switching among this subsystems, have been paid much attention in recent years, see [1-5] and references therein. The problems of stability analysis and controller synthesis of the systems have been investigated by many researchers due to their broad applications in communication networks [6], network employing TCP[7], systems theory[8-11], and so on.

So far, many existing results related to stability analysis focus on the asymptotic stability or exponential stability in the area of positive switched systems, which reflects the behavior of the system in an infinite time interval. But in many practical conditions, one is more interested in what happens over a finite-time interval. The concept of finite-time stability(FTS) was firstly defined in [12]. Then F. Amato et.al [13] extended this definition to finite time boundedness (FTB) when they dealt with the behavior of the state in the presence of external disturbances. Some related results can be found in [14-19]. Recently, [20] firstly extends the concept of FTS to positive switched systems and gives some FTS conditions of positive switched systems. In [21], the problem of finite-time stability and stabilization of fractional-order positive switched systems is considered via mode-dependent average dwell time approach. The problem of finite-time L_1 control for a class of positive switched linear systems with time-varying delays has been investigated in [22]. On the other hand, guaranteed cost control has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound[23]. So it is necessary to study the design problem of guaranteed cost finite-time controller. There are some results about this problem, see [24-27] and references therein. But there are few results available on guaranteed cost finite-time control for positive switched systems with time-varying delays, which motivates our present study.

In this paper, we are interested in investigating the problem of GCFTB for positive switched delay systems with ADT. The main contributions of this paper can be summarized as follows: (1) The definition of guaranteed cost finite-time boundedness is given, it is different from the general one, it takes full advantage of the characteristics of nonnegative states of positive switched systems. (2) By using the ADT approach and copositive Lyapunov- Krasovskii functional method, a state feedback law is designed and sufficient conditions are obtained to guarantee that the closed-loop system is GCFTB, such conditions can be easily solved by linear programming.

The paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. GCFTB analysis and controller design are developed in Section 3. A practical example is provided in Section 4. Finally, Section 5 concludes this paper.

Notations. In this paper, $A \succ 0$ ($\succeq 0$) means that all elements of matrix A are positive(non-negative). The superscript T denotes the transpose. R_+^n, R^n and $R^{n\times n}$ denote the n-dimensional non-negative (positive) vector space, the n-dimensional Euclidean space and the set of real $n\times n$ matrices. 1-norm $\|x\|$ is defined by $\|x\| = \sum_{k=1}^{n} |x_k|$, where x_k is the k th element of $x \in R^n$, if not explicitly stated, matrices are assumed to have compatible dimensions.

II. PROBLEM STATEMENTS AND PRELIMINARIES

Consider the following positive switched delay systems:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + G_{\sigma(t)}u(t) + B_{\sigma(t)}w(t), \\ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0]. \end{cases} \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ represent the system state and control input. $w(t) \in R^l$ is the disturbance input, which satisfies

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$$\exists d>0: \quad \int_0^T \|w(t)\| dt < d. \qquad (2)$$

$\sigma(t):[0,\infty)\rightarrow \underline{S}=\{1,2,\cdots,S\}$ is the switching signal, where S is the number of subsystems. $\forall p\in\underline{S}$, A_p, A_{dp}, B_p and G_p are constant matrices with appropriate dimensions, p denotes p th subsystem and t_q denotes the q th switching instant. $\varphi(\theta)$ is the initial condition on $[-\tau,0]$, $\tau>0$, $d(t)$ denotes the time-varying delay satisfying $0\le d(t)\le\tau$, $\dot{d}(t)\le h<1$, where τ and h are known positive constants.

Then we will present some definitions and lemmas for the following positive switched systems.

$$\begin{cases}\dot{x}(t)=A_{\sigma(t)}x(t)+A_{d\sigma(t)}x(t-d(t))+B_{\sigma(t)}w(t),\\ x(\theta)=\varphi(\theta), \quad \theta\in[-\tau,0].\end{cases} \qquad (3)$$

Definition 1 [1]. System (3) is said to be positive if for any initial conditions $\varphi(\theta)\succeq 0$, $\theta\in[-\tau,0]$, disturbance input $w(t)\succeq 0$ and any switching signals $\sigma(t)$, the corresponding trajectory $x(t)\succeq 0$ holds for all $t\ge 0$.

Definition 2 [1]. A is called a Metzler matrix if the off-diagonal entries of matrix A are non-negative.

Lemma 1 [1]. A matrix A is a Metzler matrix if and only if there exists a positive constant η such that $A+\eta I_n\succeq 0$.

Lemma 2 [3]. System (3) is positive if and only if A_p, $\forall p\in\underline{S}$ are Metzler matrices and $\forall p\in\underline{S}$, $A_{dp}\succeq 0$, $B_p\succeq 0$ and $G_p\succeq 0$.

Definition 3 [22]. For any $T_2>T_1\ge 0$, let $N_\sigma(T_1,T_2)$ denotes the switching number of $\sigma(t)$ over the interval $[T_1,T_2)$. For given $T_\alpha>0$ and $N_0>0$, if the inequality

$$N_\sigma(T_1,T_2)\le N_0+\frac{T_2-T_1}{T_\alpha} \qquad (4)$$

holds, then the positive constant T_α is called an average dwell time, and N_0 is called a chattering bound. Generally speaking, we choose $N_0=0$.

Definition 4 (FTS). For a given time constant T_f and vectors $\delta\succ\varepsilon\succ 0$, positive switched system (3) with $w(t)\equiv 0$ is said to be FTS with respect to $(\delta,\varepsilon,T_f,\sigma(t))$, if

$$\sup_{-\tau\le t\le 0}\{x^T(t)\delta\}\le 1\Rightarrow x^T(t)\varepsilon<1,\ \forall t\in[0,T_f].$$

if the above condition is satisfied for any switching signals $\sigma(t)$, system (3) is said to be uniformly FTS with respect to (δ,ε,T_f).

Definition 5 (FTB). For a given time constant T_f and vectors $\delta\succ\varepsilon\succ 0$, positive switched system (3) is said to be FTB with respect to $(\delta,\varepsilon,T_f,d,\sigma(t))$, where $w(t)$ satisfies (2), if

$$\sup_{-\tau\le t\le 0}\{x^T(t)\delta\}\le 1\Rightarrow x^T(t)\varepsilon<1,\ \forall t\in[0,T_f].$$

Now we give some definitions about GCFTB for the positive switched delay systems (1).

Definition 6. Define the cost function of positive switched systems (1) as follows:

$$J=\int_0^{T_f}[x^T(t)R_1+u^T(t)R_2]dt \qquad (5)$$

where $R_1\succ 0$ and $R_2\succ 0$ are two given vectors.

Remark 1. It should be noted that the proposed cost function is different from the general one[23-26], this definition provides a more useful description, because it takes full advantage of the characteristics of nonnegative states of positive switched systems.

Definition 7. (GCFTB) For a given time constant T_f and vectors $\delta\succ\varepsilon\succ 0$, consider the positive switched system (1) and cost function (5), if there exist a control law $u(t)$ and a positive scalar J^* such that the closed-loop system is FTB with respect to $(\delta,\varepsilon,T_f,d,\sigma(t))$ and the cost function satisfies $J\le J^*$, then the corresponding closed- loop system is called GCFTB, where J^* is a guaranteed cost value and $u(t)$ is a guaranteed cost finite-time controller.

The aim of this paper is to design a state feedback controller $u(t)$ and find a class of switching signals $\sigma(t)$ for positive switched system (1) such that the corresponding closed-loop system is GCFTB.

III. MAIN RESULTS

A. *Guaranteed cost finite-time stability analysis*

In this subsection, we will focus on the problem of GCFTB for positive switched system (3). The following theorem gives sufficient conditions of GCFTB for system (3) with ADT.

Theorem 1. Consider the positive switched systems (3), for given constants T_f, $\lambda_p>0$, γ and vectors $\delta\succ\varepsilon\succ 0$ and $R_1\succ 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p\in\underline{S}$, and positive constants ξ_1, ξ_2, ξ_3, ξ_4, such that the following inequalities hold:

$$\Psi_p=diag\{\psi_{p1},\psi_{p2}\cdots\psi_{pn},\psi'_{p1},\psi'_{p2}\cdots\psi'_{pn}\}\le 0 \qquad (6)$$

$$\xi_1\varepsilon\prec\nu_p\prec\xi_2\delta,\ \upsilon_p\prec\xi_3\delta,\ \vartheta_p\prec\xi_3\delta \qquad (7)$$

$$\xi_2 b_{pr}^T\delta<\gamma \qquad (8)$$

$$\xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4+\gamma d<\xi_1 e^{-\lambda T_f} \qquad (9)$$

where

$$\psi_{pr}=a_{pr}^T-\lambda_p\nu_{pr}+\upsilon_{pr}+\tau\vartheta_{pr}+R_1$$

$$\psi'_{pr}=a_{dpr}^T\nu_p-(1-h)\nu_{pr},\ \lambda=\max_{p\in\underline{S}}\{\lambda_p\}$$

$a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, $\nu_p=[\nu_{p1},\nu_{p2},\cdots,\nu_{pn}]^T$, $\upsilon_p=[\nu_{p1}$,

$\upsilon_{p2},\cdots,\upsilon_{pn}]^T$ and $\vartheta_p=[\vartheta_{p1},\vartheta_{p2},\cdots,\vartheta_{pn}]^T$, ν_{pr} , υ_{pr} and ϑ_{pr} represent the rth elements of the vectors ν_p, υ_p and ϑ_p, respectively, then under the following ADT scheme

$$T_a > T_a^* = \max\left\{\frac{T_f \ln\mu}{\ln(\xi_1 e^{-\lambda T_f}) - \ln(\xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4+\gamma d)}, \frac{\ln\mu}{\lambda}\right\} \quad (10)$$

The system (3) is GCFTB with respect to $(\delta,\varepsilon,T_f,d,\sigma(t))$, where $\mu\ge 1$ satisfies

$$\nu_p \preceq \mu\nu_q,\ \upsilon_p \preceq \mu\upsilon_q,\ \vartheta_p \preceq \mu\vartheta_q,\ \forall p,q\in \underline{S} \quad (11)$$

and the guaranteed cost value of system (3) is given by

$$\begin{aligned} J &= \int_0^{T_f} x^T(s)R_1 ds \le J^* \\ &= e^{2\lambda T_f}(\xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4+\gamma d) \end{aligned} \quad (12)$$

Proof. Construct the following co-positive Lyapunov-Krasovskii functional candidate for system (3):

$$\begin{aligned} V_{\sigma(t)}(t,x(t)) &= x^T(t)\nu_p + \int_{t-d(t)}^{t} e^{\lambda_p(t-s)}x^T(s)\upsilon_p ds \\ &\quad + \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)}x^T(s)\vartheta_p ds d\theta \end{aligned} \quad (13)$$

where ν_p, υ_p and $\vartheta_p \in R_+^n$, $\forall p\in\underline{S}$.

For the sake of simplicity, $V_{\sigma(t)}(t,x(t))$ is written as $V_{\sigma(t)}(t)$ in this paper.

Along the trajectory of system (3), we have

$$\begin{aligned} \dot V_{\sigma(t)}(t) &= x^T(t)A_p^T\nu_p + x^T(t-d(t))A_{dp}^T\nu_p \\ &\quad + w^T(t)B_p^T\nu_p + \lambda_p\int_{t-d(t)}^{t} e^{\lambda_p(t-s)}x^T(s)\upsilon_p ds \\ &\quad + x^T(t)\upsilon_p - (1-\dot d(t))e^{\lambda_p d(t)}x^T(t-d(t))\upsilon_p \\ &\quad + \lambda_p\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)}x^T(s)\vartheta_p ds d\theta + \tau x^T(t)\vartheta_p \\ &\quad - \int_{-\tau}^{0} e^{-\lambda_p\theta}x^T(t+\theta)\vartheta_p d\theta \\ &\le x^T(t)A_p^T\nu_p + x^T(t-d(t))A_{dp}^T\nu_p + w^T(t)B_p^T\nu_p \\ &\quad + \lambda_p\int_{t-d(t)}^{t} e^{\lambda_p(t-s)}x^T(s)\upsilon_p ds + x^T(t)\upsilon_p \\ &\quad - (1-h)x^T(t-d(t))\upsilon_p \\ &\quad + \lambda_p\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)}x^T(s)\vartheta_p ds d\theta + \tau x^T(t)\vartheta_p \\ &\quad - \int_{t-d(t)}^{t} x^T(s)\vartheta_p ds \end{aligned} \quad (14)$$

From (7), (8), (13) and (14) leads to

$$\begin{aligned} &\dot V_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) + x^T(t)R_1 \\ &\le x^T(t)\left(A_p^T\nu_p - \lambda_p\nu_p + \upsilon_p + \tau\vartheta_p + R_1\right) \\ &\quad + x^T(t-d(t))\left(A_{dp}^T\nu_p - (1-h)\upsilon_p\right) + \gamma\|w(t)\| \end{aligned} \quad (15)$$

Substituting (6) into (15) yields

$$\dot V_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) + x^T(t)R_1 - \gamma\|w(t)\| \le 0 \quad (16)$$

It implies

$$\dot V_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) - \gamma\|w(t)\| \le 0 \quad (17)$$

Integrating both sides of (17) during the period $[t_k,t)$ for $t\in[t_k,t_{k+1})$ leads to

$$V_{\sigma(t)}(t) \le e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k) + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds. \quad (18)$$

For any $T_f>0$, let N be the switching number of $\sigma(t)$ over $[0,T_f)$, and denote $t_1,t_2,\cdots,t_k,\cdots,t_N$ as the switching instants over the interval $[0,T_f)$. Then, for $t\in[0,T_f)$, $V_{\sigma(t_k)}(t_k)\le\mu V_{\sigma(t_k^-)}(t_k^-)$ is easily obtained from (11). From (17) , we have

$$\begin{aligned} &V_{\sigma(t)}(t) \\ &\le e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k) + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds \\ &\le \mu e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k^-)}(t_k^-) \\ &\quad + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds \\ &\le \mu e^{\lambda_{\sigma(t_k)}(t-t_k)}\Big[e^{\lambda_{\sigma(t_{k-1})}(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1}) \\ &\quad + \gamma\int_{t_{k-1}}^{t_k} e^{\lambda_{\sigma(t_{k-1})}(t_k-s)}w^T(s)ds\Big] \\ &\quad + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds \le \cdots \\ &\le \mu^N e^{\lambda t}V_{\sigma(0)}(0) + \mu^N\gamma\int_0^{t_1} e^{\lambda(t-s)}w^T(s)ds \\ &\quad + \mu^{N-1}\gamma\int_{t_1}^{t_2} e^{\lambda(t-s)}w^T(s)ds + \cdots \\ &\quad + \gamma\int_{t_k}^{t} e^{\lambda(t-s)}w^T(s)ds \\ &= \mu^N e^{\lambda T_f}V_{\sigma(0)}(0) \\ &\quad + \gamma\int_0^{t} \mu^{N_{\sigma(t)}(s,t)}e^{\lambda(t-s)}w^T(s)ds \\ &\le \mu^N e^{\lambda T_f}V_{\sigma(0)}(0) + \mu^N\gamma\int_0^{t} e^{\lambda T_f}w^T(s)ds \\ &\le \mu^N e^{\lambda T_f}\left(V_{\sigma(0)}(0)+\gamma d\right) \end{aligned} \quad (19)$$

Noting the definition of $V_{\sigma(t)}(t)$ and (7), we have

$$V_{\sigma(t)}(t) \ge \xi_1 x^T(t)\varepsilon. \quad (20)$$

$$\begin{aligned} &V_{\sigma(0)}(0) \le \xi_2 x^T(0)\delta + \tau e^{\lambda\tau}\xi_3 \sup_{-\tau\le\theta\le 0}\{x^T(\theta)\delta\} \\ &+\tau^2 e^{\lambda\tau}\xi_4 \sup_{-\tau\le\theta\le 0}\{x^T(\theta)\delta\} \\ &\le (\xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4)\sup_{-\tau\le\theta\le 0}\{x^T(\theta)\delta\} \\ &\le \xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4. \end{aligned} \quad (21)$$

From (19)-(21), we obtain

$$x^T(t)\varepsilon \le \frac{1}{\xi_1}e^{(\lambda+\frac{\ln\mu}{T_a})T_f}\left(\xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4+\gamma d\right).$$

(22)

Substituting (10) into (22), one has

$$x^T(t)\varepsilon < 1. \quad (23)$$

According to Definition 5, we conclude that the system (3) is FTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$.

Next, we will give the guaranteed cost value of system (3). According to (15), denoting $\nabla(t) = \gamma\|w(t)\| - x^T(t)R_1$ and integrating both sides of (16) from t_k to t for $t \in [t_k, t_{k+1}]$, it gives rise to

$$V_{\sigma(t)}(t) \le e^{\lambda_p(t-t_k)}V_{\sigma(t_k)}(t_k) + \int_{t_k}^{t} e^{\lambda_p(t-s)}\nabla(s)ds \quad (24)$$

Similar to the proof process of (19), for any $t \in [0, T_f]$, we can obtain

$$V_{\sigma(t)}(t) \le \mu^{N_{\sigma(t)}(0,t)}e^{\lambda t}V_{\sigma(0)}(0) + \int_0^t e^{\lambda(t-s)}\mu^{N_{\sigma(t)}(s,t)}\nabla(s)ds \quad (25)$$

From (25), we can get

$$\int_0^t \mu^{N_{\sigma(t)}(s,t)}e^{\lambda(t-s)}x^T(s)R_1 ds \le \mu^{N_{\sigma(t)}(0,t)}e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma\int_0^t \mu^{N_{\sigma(t)}(s,t)}e^{\lambda(t-s)}\|w(s)\|ds \quad (26)$$

Multiplying both sides of (26) by $\mu^{-N_{\sigma(t)}(0,t)}$ leads to

$$\int_0^t \mu^{-N_{\sigma(t)}(0,s)}e^{\lambda(t-s)}x^T(s)R_1 ds \le e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma\int_0^t \mu^{-N_{\sigma(t)}(0,s)}e^{\lambda(t-s)}\|w(s)\|ds \quad (27)$$

Noting that $N_{\sigma(t)}(0,s) \le \frac{s}{T_\alpha}$ and $T_\alpha > \frac{\ln_\mu}{\lambda}$, we obtain that $0 < N_{\sigma(t)}(0,s) \le \frac{s}{T_\alpha} \le \frac{\lambda s}{\ln\mu}$, that is, $e^{-\lambda s} \le \mu^{-N_{\sigma(t)}(0,s)} < 1$.

Then (27) can be turned into

$$\int_0^t e^{-\lambda s}e^{\lambda(t-s)}x^T(s)R_1 ds \le \int_0^t \mu^{-N_{\sigma(t)}(0,s)}e^{\lambda(t-s)}x^T(s)R_1 ds \le e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma\int_0^t e^{\lambda(t-s)}\|w(s)\|ds \quad (28)$$

Let $t = T_f$, then multiplying both sides of (28) by $e^{-\lambda T_f}$ leads to

$$\int_0^{T_f} e^{-2\lambda s}x^T(s)R_1 ds \le V_{\sigma(0)}(0) + \gamma\int_0^{T_f} e^{-\lambda s}\|w(s)\|ds \le V_{\sigma(0)}(0) + \gamma\int_0^{T_f}\|w(s)\|ds \quad (29)$$

Substituting (2) into (29) yields

$$e^{-2\lambda T_f}\int_0^{T_f} x^T(s)R_1 ds \le V_{\sigma(0)}(0) + \gamma d \quad (30)$$

which can be rewritten as

$$\int_0^{T_f} x^T(s)R_1 ds \le e^{2\lambda T_f}\left(V_{\sigma(0)}(0) + \gamma d\right) \quad (31)$$

Substituting (21) into (31), the guaranteed cost value of system (3) is given by

$$J = \int_0^{T_f} x^T(s)R_1 ds \le J^* = e^{2\lambda T_f}(\xi_2 + \tau e^{\lambda\tau}\xi_3 + \tau^2 e^{\lambda\tau}\xi_4 + \gamma d) \quad (32)$$

Therefore, according to Definition 7, we can conclude that the claim of the theorem is true. Thus, the proof is completed.

Remark 2. Generally, when we consider the asymptotic stability, $\dot{V}_{\sigma(t)}(t)$ is required to be negative. As the difference between asymptotic stability and FTB, the restriction is relaxed in the proof process of Theorem 1. Moreover, if $\mu = 1$ in (10), then one can obtain $T_\alpha^* = 0$, which means that the switching signal can be arbitrary.

B. Controller design

In this section, we concern with the GCFTB controller design of positive switched delay system (1). Under the controller $u(t) = K_{\sigma(t)}x(t)$, the corresponding closed-loop system is given by

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} + G_{\sigma(t)}K_{\sigma(t)})x(t) + A_{d\sigma(t)}x(t-d(t)) + B_{\sigma(t)}w(t), \\ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0]. \end{cases} \quad (33)$$

By Lemma 1, to guarantee the positivity of system (33), $A_p + G_pK_p$ should be Metzler matrices, $\forall p \in \underline{S}$. The following Theorem 2 gives some sufficient conditions to guarantee that the closed-loop system (1) is GCFTB.

Theorem 2. Consider the positive switched system (1). For given constants T_f, $\lambda_p > 0$, γ, vectors $\delta \succ \varepsilon \succ 0$, $R_1 \succ 0$ and $R_2 \succ 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{S}$, and positive constants ξ_1, ξ_2, ξ_3, ξ_4, such that (7)-(9) and the following conditions hold:

$$A_p + G_pK_p \text{ are Metzler matrices.} \quad (34)$$

$$\bar{\Psi}_p = diag\{\bar{\psi}_{p1}, \bar{\psi}_{p2} \cdots \bar{\psi}_{pn}, \bar{\psi}'_{p1}, \bar{\psi}'_{p2}, \cdots \bar{\psi}'_{pn}\} \le 0 \quad (35)$$

where

$$\bar{\psi}_{pr} = a_{pr}^T - \lambda_p\nu_{pr} + \upsilon_{pr} + \tau\vartheta_{pr} + R_1 + g_{pr}$$

$$\psi'_{pr} = a_{dpr}^T\nu_p - (1-h)\nu_{pr}$$

$\lambda = \max_{p\in\underline{S}}\{\lambda_p\}$, $g_p = K_p^T\left(G_p^T\nu_p + R_2\right)$, g_{pr} represents the rth elements of vector g_p, $a_{pr}(a_{dpr})$ represents the rth column vector of the matrix $A_p(A_{dp})$, and $\nu_p = [\nu_{p1}, \nu_{p2}, \cdots, \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \cdots, \upsilon_{pn}]^T$ and $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \cdots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr} and ϑ_{pr} represents the rth elements of the vectors ν_p, υ_p and ϑ_p, respectively, $\mu \ge 1$ satisfies (11), then under the ADT scheme (10), the resulting closed-loop system (33) is GCFTB with respect to $(\delta, \varepsilon, T_f, d, \sigma(t))$ and the guaranteed cost value of system (33) is given by

$$J=\int_0^{T_f}\left(x^T(s)R_1+x^T(s)K_{\sigma(t)}^T R_2\right)ds \le J^*=e^{2\lambda T_f}(\xi_2+\tau e^{\lambda\tau}\xi_3+\tau^2 e^{\lambda\tau}\xi_4+\gamma d) \quad (36)$$

Proof. By Lemma 1, we know that $p\in\underline{S}$, $A_p+G_pK_p$ is a Metzler matrix for each $p\in\underline{S}$. According to Lemma 2, the system (33) is positive if A_{dp}, B_p and G_p are all nonnegative. Replacing A_p in (6) with $A_p+G_pK_p$ and letting $g_p=K_p^T\left(G_p^T\nu_p+R_2\right)$, similar to Theorem 1, we easily obtain that the resulting closed-loop system (33) is GCFTB with respect to $(\delta,\varepsilon,T_f,d,\sigma(t))$ and the guaranteed cost value is given by (36).

The proof is completed.

Next, an procedure is presented to obtain the feedback gain matrices K_p, $p\in\underline{S}$.

Step1. By adjusting the parameters λ_p and solving (7)-(9), (11) and (35) via linear programming, positive vectors $\nu_p,\upsilon_p,\vartheta_p$ and g_p can be obtained.

Step2. Substituting ν_p and g_p into $g_p=K_p^T\left(G_p^T\nu_p+R_2\right)$, K_p can be obtained.

Step 3. The gain K_p is substituted into $A_p+G_pK_p$. If $A_p+G_pK_p$ are Metzler matrices, then K_p are admissible. Otherwise, return to Step 1.

IV. Example

In this section, a practical price dynamics example is provided to show the effectiveness of the proposed approach.

In an economic system composed of n goods, $\hat{x}_p(t)$ denotes the prices of the p-th good at time t and suppose that the demand f_{ip} and the supply g_{ip} of good p depend on the current prices $\hat{x}(t)$ and the previous prices $\hat{x}(t-d(t))$ of all goods, where $\hat{x}(t)=[\hat{x}_1(t),\ldots,\hat{x}_n(t)]^T$. We have

$$\dot{\hat{x}}(t)=f_{ip}(\hat{x}(t),\hat{x}(t-d(t)))-g_{ip}(\hat{x}(t),\hat{x}(t-d(t))),$$

it is significant to learn whether the prices can tend to constant values $\overline{x}_p$. Here, it is assumed that demand and supply functions are linear and the influence coefficients of $\overline{x}_p(t)$ on $f_{ip}(g_{ip})$ are negative (nonnegative), while the influence coefficients of $\hat{x}_p(t)$ on $f_{iq}(g_{iq})$, for $q\neq p$ are nonnegative (negative); the influence coefficients of $\hat{x}_p(t-d(t))$ on $f_{ip}(g_{ip})$ are positive (negative) for all q.

Then, by means of an appropriate transformation, price dynamics can be described by the state equations

$$\dot{\hat{x}}(t)=A_i(\hat{x}(t)-\overline{x}(t))+A_{di}(\hat{x}(t-d(t)-\overline{x}).$$

Letting $x(t)=\hat{x}(t)-\overline{x}$, the above system is described as

$$\dot{x}(t)=A_ix(t)+A_{di}x(t-d(t).$$

which can be seen as positive switched system (1) with time-varying delay.

Consider price dynamic model described by switched positive system (1) with the parameters:

$$A_1=\begin{bmatrix}-3.5&0.1\\0.3&-3\end{bmatrix},\ A_{d1}=\begin{bmatrix}0.2&0.5\\0.8&0.6\end{bmatrix},\ B_1=\begin{bmatrix}0.2&0.1\\0.1&0.3\end{bmatrix},\ G_1=\begin{bmatrix}1\\2\end{bmatrix},$$

$$A_2=\begin{bmatrix}-5&0.1\\0.1&-2.5\end{bmatrix},\ A_{d2}=\begin{bmatrix}0.6&0.3\\0.8&0.5\end{bmatrix},\ B_2=\begin{bmatrix}0.3&0.1\\0.2&0.1\end{bmatrix},\ G_2=\begin{bmatrix}2\\1\end{bmatrix},$$

$$R_1=\begin{bmatrix}0.2\\0.3\end{bmatrix},\ R_2=\begin{bmatrix}0.3\\0.2\end{bmatrix},\ \varepsilon=\begin{bmatrix}0.1\\0.1\end{bmatrix},\ \delta=\begin{bmatrix}1\\2\end{bmatrix}.$$

Assuming that $d(t)=0.2+0.2\sin(t)$, we can get $\tau=0.4$ and $h=0.2$. Choosing the parameters $T_f=10$, $\lambda_1=\lambda_2=0.1$, $\gamma=1$, $\mu=1.4$ and solving the inequalities in Theorem 2 by linear programming, we get

$$\nu_1=\begin{bmatrix}1.3134\\2.2270\end{bmatrix},\ \upsilon_1=\begin{bmatrix}2.6612\\3.3618\end{bmatrix},\ \vartheta_1=\begin{bmatrix}2.0759\\2.2647\end{bmatrix},$$

$$\nu_2=\begin{bmatrix}1.2785\\2.3077\end{bmatrix},\ \upsilon_2=\begin{bmatrix}2.7685\\3.2799\end{bmatrix},\ \vartheta_2=\begin{bmatrix}2.1055\\2.2593\end{bmatrix},$$

$$g_1=\begin{bmatrix}0.5749\\0.7909\end{bmatrix}\quad g_2=\begin{bmatrix}0.8084\\0.7548\end{bmatrix}$$

By $g_p=K_p^T\left(G_p^T\nu_p+R_2\right)$, we obtain

$$K_1=\begin{bmatrix}0.0948 & 0.1304\end{bmatrix}, K_2=\begin{bmatrix}0.1596 & 0.1490\end{bmatrix}$$

It is easy to verify that (34) is satisfied. Then, according to (10), we get $T_\alpha^*=3.3647$.

Let

$$w(t)=\begin{bmatrix}f_1(t)\\f_2(t)\end{bmatrix}, f_1(t)=f_2(t)=\begin{cases}\dfrac{1+\sin(t)}{t} & 3\le t\le 10,\\ 0 & others.\end{cases}$$

Choosing $T_\alpha=3.5$, the simulation results are shown in Figs. 1-3, where the initial conditions of system (1) are $x(0)=[0.5\quad 0.2]^T$ and $x(\theta)=0$, $\theta\in[-\tau\quad 0)$, which meet the condition $\sup_{-\tau\le t\le 0}\{x^T(t)\delta\}\le 1$. The state trajectory of the closed-loop system is shown in Fig. 1. The switching signal $\sigma(t)$ is depicted in Fig. 2. Fig. 3 plots the evolution of $x(t)\varepsilon$, which implies that the corresponding closed-loop system is GCFTB with respect to $(\delta,\varepsilon,T_f,d,\sigma(t))$, and the cost value $J^*=37.9791$, which can be obtained by (36).

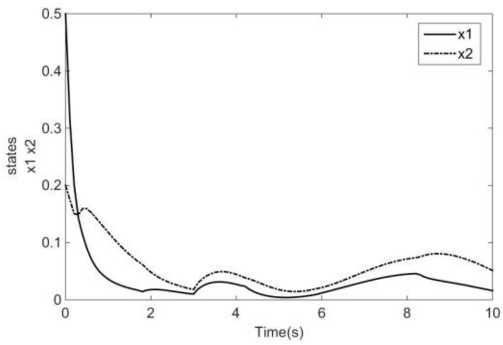

Fig. 1. State trajectories of closed-loop system (1)

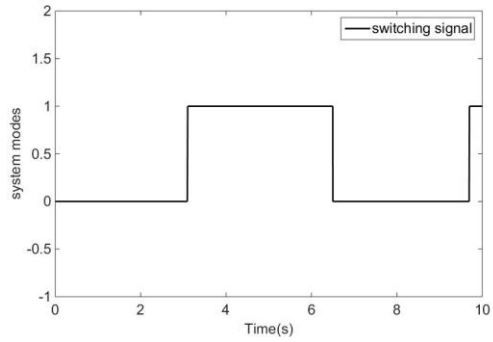

Fig.2. Switching signal of system (1)

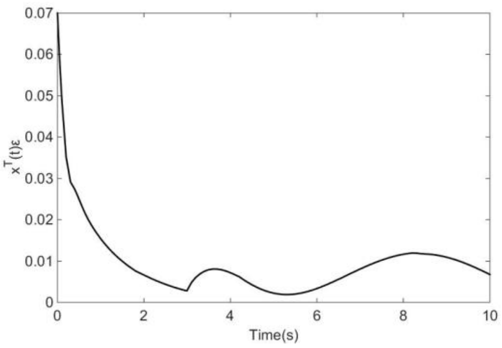

Fig. 3. The evolution of $x^T(t)\varepsilon$ of system (1)

V. Conclusion

In this paper, we have studied the problem of guaranteed cost finite-time control for positive switched delay systems with ADT. A novel guaranteed cost performance index is introduced. Based on the ADT approach, a state feedback controller is constructed to guarantee that the closed-loop system is GCFTB, the obtained sufficient conditions can be solved by linear programming. Finally, an example is given to illustrate the effectiveness of the proposed method.

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