

A MRC Based RNS to Binary Converter Using the Moduli Set $\{2^{2n+1}-1, 2^{n-1}, 2^{2n}-1\}$

H. K Bello, K.A Gbolagade

Abstract— This paper presents a new reverse converter for the moduli set $2^{2n+1}-1, 2^{n-1}, 2^{2n}-1$. We used Mixed Radix conversion to convert the residues of the moduli to binary number. It is a 5-bit dynamic range and when compare to the literature state of the art reverse converter of the same dynamic range, it is considered to be slightly faster.

Index Terms— Dynamic Range, Reverse converter, Mixed Radix conversion, Moduli set, Residues, Binary number, RNS

I. INTRODUCTION

For many decades in computer arithmetic, Residue Number System (RNS) has been an interesting research field. It usually uses positional bases that are relatively prime to each other e.g 7,11,13,15 etc. It is used in Digital Signal Processing (DSP), Cryptography system and other systems where we need high speed and low-power hardware implementation [1]. This is possible because of the following advantages of RNS properties: carry free and borrow free of addition and subtraction respectively and error correcting properties [2,3]. These properties provide RNS with a high performance computing architecture [4]. RNS is capable of performing high speed of arithmetic, though, not popular in general arithmetic and computing due to its difficulties in division, magnitude detection and most especially error correction [5].

In this paper, a novel and efficient reverse converter for the $\{2^{2n+1}-1, 2^{n-1}, 2^{2n}-1\}$ moduli set is proposed using a Mixed Radix Conversion method for the reverse conversion. The rest of this paper is organized as follows: Section II presented literature review; In section III, we discussed the proposed architecture of the system; In Section IV, we presented the hardware realization; In section V, performance evaluation was discussed and lastly section VI we presented the conclusion of the paper.

II. LITERATURE REVIEW

A. The Residue Number System (RNS)

RNS is an integer that capable of supporting high speed

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concurrent automatic [6]. It may be defined as a set of relatively prime integer called moduli which can be denoted as $m_1, m_2, m_3 \dots m_n$, where m_i is the i^{th} modulus [7] and $\text{gcd}(m_i, m_j) = 1$ for $i \neq j$. Each X can be represented as a set of smaller integers called $x_1, x_2, x_3 \dots x_n$

where $X \bmod m_i = x_i; x_i = |X|_{m_i}$

RNS has the capabilities to support parallel carry free addition, borrow free subtraction and single step multiplication without partial fraction [8][9].

Many moduli sets are in used in RNS with different Dynamic Range (DR). The most popular $3n$ -bit DR moduli set known as traditional moduli set is $\{2^n-1, 2^n, 2^{n+1}\}$ [10]. Others are $\{2^{n-1}, 2^n-1, 2^{n+1}\}$, $\{2^{n-1}-1, 2^n-1, 2^n\}$ [11]. The dynamic range of $3n$ -bits produced by this moduli set will not be able to be used by any application with higher DR. $4n$ -bit DR 4 moduli set such as $\{2^n-1, 2^n, 2^{n+1}, 2^{n+1}+1\}$, $\{2^n-1, 2^n, 2^{n+1}, 2^{n+1}-1\}$, $\{2^n-1, 2^n, 2^{n+1}, 2^{n-1}-1\}$ [12] were suggested. In order to raise parallelism in RNS arithmetic, $5n$ -bit moduli set were suggested $\{2^n, 2^{2n}-1, 2^{2n}+1\}$, $\{2^n-1, 2^n, 2^{n+1}, 2^{2n}+1\}$ etc. were also proposed.

1) Selection of Moduli in RNS

In RNS system, the Dynamic Range (DR) is the product of all the moduli such that the interval can be uniquely represented in RNS [13], this is an important aspect to be considered in the choice of moduli in RNS. Proper choice of moduli when designing RNS system is very essential; this is because the moduli choice affects the complexity of forward conversion, reverse conversion and RNS arithmetic circuits. Also, the speed of resulting conversion depends on selected moduli [14], therefore, moduli selection and data conversion are very critical in RNS to binary conversion [15]. However, it is a common fact that as the number of moduli increases, the speed of residue arithmetic units increases and the conversion from residue-binary becomes slower and complex [16]. Abdyullahi and Skavantzios stated that the moduli set m_i and m_j satisfy the following criteria:

They must be pairwise prime (i.e. $\text{gcd}(m_i, m_j) = 1$)
Each moduli m_i should be as small as possible so that operation modulo m_i require minimum computation time

The moduli m_i, m_j should imply simple binary to Residue Number system and Residue Number System to binary conversion.

The moduli product should be large enough in order to implement the decision dynamic range.

However, in a case where a large dynamic range (DR) is required, large moduli might be in a better performance.

In addition, another fundamental area where attention should be taken is the hardware selection, which is also a determining factor for RNS performance. The simplified conversion equation of the system is computed by using adders like Carry

Propagation Adder (CPA), Carry Save Adder (CSA) [17] etc.

2) Data Conversion method

The three main parts of RNS systems are binary to residue (forward) conversion, arithmetic operation and residue to binary (reverse) conversion [10]. Data conversion in RNS is either through Chinese Remainder Theorem (CRT) or Mixed Radix Conversion (MRC) which is also either forward or reverse conversion.

Forward Conversion: This is the conversion of a binary or decimal number to its RNS equivalent

Reverse Conversion: This is the conversion of RNS into its binary or decimal number.

Reverse conversion is somehow difficult, many algorithms have used various moduli sets to perform reverse conversion e.g. $\{2^n, 2^n - 1, \dots, 2^n + 1\}$ [18], $\{2^n, 2^{2n} - 1, 2^{2n} + 1\}$ [19], $\{2^n, 2^n - 1, 2^{2n} - 1\}$ [20]. The Traditional methods used in reverse conversion are the Chinese Remainder Theorem (CRT) and the Mixed Radix Conversion (MRC) [21].

a) Chinese Remainder Theorem

The Chinese Remainder Theorem (CRT) is one of the methods used in reverse conversion, i.e conversion from RNS to binary or decimal numbers [8]. Each number in the dynamic range (DR) will have a unique representation in RNS if the moduli of RNS are correctly chosen [22].

Given a set of pairwise relatively prime moduli $\{m_1, m_2, m_3, \dots, m_n\}$ with residues $\{x_1, x_2, x_3, \dots, x_n\}$ in the system of X i.e. $x_i = |X|_{m_i}$ then that number and its residue are related as follows

$$|X|_M = |\sum_{i=1}^n x_i |M_i^{-1}|_{m_i} M_i|_M$$

Where M is the product of m_i and $M_i = \frac{M}{m_i}$ and $\gcd(m_i, m_j) = 1$ (for $i \neq j$)

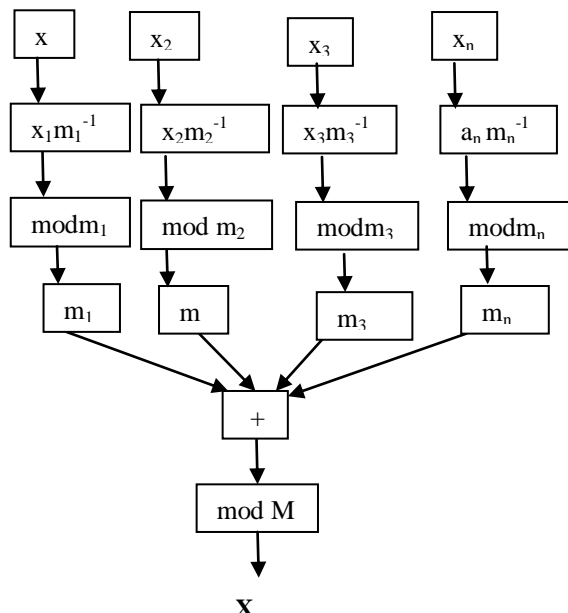


Fig1. A schematic diagram of CRT

b) Mixed Radix Conversion (MRC) Algorithm

Let m_1, m_2, m_3 be a three moduli set with residues x_1, x_2, x_3 respectively, the mixed radix algorithm is derived as follows:

	m_1	m_2	m_3
	x_1	x_2	x_3
$x_1 = a_1$		$-a_1$	$-a_1$
		$(x_2 - a_1)m_1^{-1} = a_2$	$(x_3 - a_1)m_1^{-1}$
		$-a_2$	$-a_2$
			$(x_3 - a_1)m_1^{-1} - a_2m_2^{-1} = a_3$

From the above diagram,,

$$X = a_1 + a_2m_1 + a_3 m_1m_2 \quad (1)$$

where $a_1 = x_1$

$$a_2 = |(x_2 - a_1)m_1^{-1}|_{m_2} \quad (2)$$

$$a_3 = |(x_3 - a_1)m_1^{-1} - a_2m_2^{-1}|_{m_3} \dots$$

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$$a_k = |(((x_k - a_1)|_{m_1})^{-1}|_{m_k} -$$

Mixed Radix numbers are very significant because they are weighted number system and facilitate implementation of operations such as magnitude comparison [23] which posed a lot of problems in RNS.

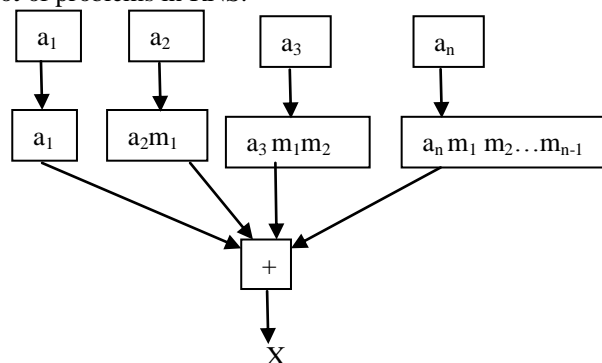


Fig. 2 A schematic diagram of MRC

III. THE PROPOSED ALGORITHM

In this paper, we assumed $m_1 = 2^{2n+1} - 1, m_2 = 2^{n-1}$ and $m_3 = 2^{2n} - 1$

Theorem 1: The moduli set $\{2^{2n+1} - 1, 2^{n-1}, 2^{2n} - 1\}$ are pairwise relatively prime numbers

Proof: Using Euclid's theorem $\gcd(m_1, m_2) = \gcd(m_2, |m_1|_{m_2}) = 1$

$$\gcd(2^{2n+1} - 1, 2^{n-1}) = \gcd(2^{n-1}, |2^{2n+1} - 1|_{2^{n-1}}) = \gcd(2^{n-1}, -1) = 1$$

$$\text{Then, } \gcd(2^{2n+1} - 1, 2^{2n} - 1) = \gcd(2^{2n} - 1, |2^{2n+1} - 1|_{2^{2n} - 1})$$

$$\begin{aligned} \text{To evaluate } |2^{2n+1} - 1|_{2^{2n} - 1} &= |2^{2n+1}|_{2^{2n} - 1} - |1|_{2^{2n} - 1} \\ |2^{2n+1} - 1|_{2^{2n} - 1} &= |2^{2n+1}|_{2^{2n} - 1} - |1|_{2^{2n} - 1} \\ &= |2 \times 2^{2n}|_{2^{2n} - 1} - |1|_{2^{2n} - 1} \\ &= |2|_{2^{2n} - 1} \times |2^{2n}|_{2^{2n} - 1} - 1 \\ &= 2 \times |2^{2n} - 1| + 1|_{2^{2n} - 1} = 2 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

Implies $\gcd(2^{2n} - 1, 1) = 1$

Also

$$\gcd(2^{2n} - 1, 2^{n-1}) = \gcd(2^{n-1}, |2^{2n} - 1|_2^{n-1})$$

$$\text{if } |2^{2n} - 1|_2^{n-1} = -1$$

$$\text{then } \gcd(2^{n-1}, |2^{2n} - 1|_2^{n-1}) = \gcd(2^{n-1}, -1) = 1$$

By the above proof, it is confirmed that $2^{2n+1} - 1$, 2^{n-1} , $2^{2n} - 1$ are relatively prime since the greatest common divisor (gcd) are 1. Therefore, our proposed moduli set can be used in RNS and we can proceed to determine its reverse converter.

Theorem 2: Given the moduli set $2^{2n+1} - 1$, 2^{n-1} , $2^{2n} - 1$ where $m_1 = 2^{2n+1} - 1$, $m_2 = 2^{n-1}$, $m_3 = 2^{2n} - 1$, the following hold to be true:

$$|m_2^{-1}|_{m_3} = 2^{n+1} \quad (3)$$

$$|m_1^{-1}|_{m_2} = 2^{n-1} - 1 \quad (4)$$

$$|m_1^{-1}|_{m_3} = 1 \quad (5)$$

Proof:

From (3), if it can be demonstrated that 2^{n+1} is the multiplicative inverse of $2^{2n} - 1$

$$\begin{aligned} |2^{n-1}|_2^{2n-1} &= 1 \\ |2^{n-1} \times 2^{n+1}|_2^{2n-1} &= |2^{2n}|_2^{2n-1} \\ &= |2^{2n} - 1 + 1|_2^{2n-1} \\ &= 1 \end{aligned}$$

Implies 2^{n+1} is the multiplicative inverse of $2^{2n} - 1$ with respect to $2^{2n} - 1$

$$\begin{aligned} \text{From (4), } |(2^{2n+1} - 1)(2^{n-1} - 1)|_2^{n-1} &= |2^{3n} - 2^{n-1} + 1|_2^{n-1} \\ &= |2^{3n}|_2^{n-1} - |2^{n-1}|_2^{n-1} + |1|_2^{n-1} \\ &= |2^{3n}|_2^{n-1} + 1 \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

Implies $2^{n-1} - 1$ is the multiplicative inverse of $2^{2n+1} - 1$ with respect to $2^{2n} - 1$

$$\begin{aligned} \text{From (5), } |(2^{2n+1} - 1) \times 1|_2^{2n-1} &= |(2^{2n+1} - 1)|_2^{2n-1} \\ &= |2 \times 2^{2n} - 1|_2^{2n-1} \\ &= |2|_2^{2n-1} \times |2^{2n}|_2^{2n-1} - |1|_2^{2n-1} \\ &= 2 \times |2^{2n} - 1 + 1|_2^{2n-1} - |1|_2^{2n-1} \\ &= 2 \times 1 - 1 \\ &= 1 \end{aligned}$$

Implies that 1 is the multiplicative inverse of $2^{2n+1} - 1$ with respect to $2^{2n} - 1$

Property 1: Modulus 2^s of a number is equivalent to s least significant figure of the number.

Property 2: modulo $2^s - 1$ multiplication of a residue number by 2^t where s and t are positive integer numbers is equivalent to t-bit circular left shifting.

Property 3: modulo $2^s - 1$ of a negative number is equivalent to the one's complement of the number which is obtained by subtracting the number $2^{2n+1} - 1$, $2^{n-1} 2^{2n} - 1$ will have residues x_1 , x_2 , x_3 respectively.

Using mixed radix conversion, let the binary representation of the residue be

$$x_1 = \underbrace{x_{1,2n} x_{1,2n-1} x_{1,2n-2} \dots x_{1,1} x_{1,0}}_{2n+1} \quad (6)$$

$$x_2 = \underbrace{x_{2,n-2} x_{2,n-3} x_{2,n-4} \dots x_{2,1} x_{2,0}}_{2n} \quad (7)$$

(n-1) bit

$$x_3 = \underbrace{x_{3,2n-1} x_{3,2n-2} x_{3,2n-3} \dots x_{3,1} x_{3,0}}_{2n\text{-bit}} \quad (8)$$

Given the residues $a_n, a_{n-1}, a_{n-2}, \dots, a_1$, any number X can be uniquely expressed in a mixed radix as [3] $X \cong (z_n, z_{n-1}, z_{n-2}, \dots, z_1)$ which can be written as

$$X = z_n a_{n-1} a_{n-2} \dots a_1 + z_3 a_2 a_1 + z_2 a_1 + z_1 \quad (9)$$

$$0 \leq z_i < a_i$$

Given the above association between a mixed radix representation and let a_i 's corresponds to m_i 's in equation (9) with residue representation $x_n x_{n-1} x_{n-2} \dots x_1$

$$X = z_n m_{n-1} m_{n-2} \dots m_1 + z_3 m_2 m_1 + z_2 m_1 + z_1 \quad (10)$$

$$|X|_{m_1} = z_1 = x_1$$

Thus, if we consider the moduli set $2^{2n+1} - 1$ and 2^{n-1}

$z_1 = (x, y)$ and by applying the mixed radix algorithm in (1) for the two moduli set and also apply equation (6).

$$X = a_1 + a_2(2^{2n+1} - 1) + a_3(2^{n-1})(2^{2n} - 1) \quad (11)$$

$$a_1 = x_1 = \underbrace{x_{1,2n} x_{1,2n-1} x_{1,2n-2} \dots x_{1,1} x_{1,0}}_{2n+1} \quad (12a)$$

$$\begin{aligned} a_2 &= |(x_2 - x_1) m_1^{-1}|_{m_2} \\ &= |(x_2 - x_1) 2^{n-1} - 1|_2^{n-1} \\ &= |2^{n-1}(x_2 - x_1) - (x_2 - x_1)|_2^{n-1} \\ &= |(x_1 - x_2)|_2^{n-1} \\ &= |x_1|_2^{n-1} - |x_2|_2^{n-1} \\ &= |x_1|_2^{n-1} + |x_2'|_2^{n-1} \\ &= y_1 + y_2 \end{aligned} \quad (12b)$$

where $y_1 = |x_1|_2^{n-1}$ and $y_2 = |x_2'|_2^{n-1}$

$$y_1 = \underbrace{x_{1,n-2} x_{1,n-3} x_{1,n-4} \dots x_{1,1} x_{1,0}}_{n-1} \quad (12c)$$

$$y_2 = \underbrace{x'_{1,n-2} x'_{1,n-3} x'_{1,n-4} \dots x'_{1,1} x'_{1,0}}_{n-1} \quad (12c)$$

$$\begin{aligned} a_3 &= |(x_3 - a_1) m_1^{-1}|_{m_3} - a_2 |m_2^{-1}|_{m_3} \\ &= ((x_3 - x_1) \times 1 - a_2) \times 2^{n+1} |2^{2n} - 1| \\ &= |2^{n+1}(x_3 - x_1) - a_2|_2^{2n-1} \\ &= 2^{n+1} N |2^{2n} - 1| \end{aligned} \quad (13)$$

$$\text{where } N = x_3 + y_3 + y_4 + 1 \quad (14)$$

$$\text{or } N = x_3 + y_3 + y_4 + 0 \quad (15)$$

$$\begin{aligned} \text{and } y_3 &= |-x_1|_2^{2n-1} = |x_1'|_2^{2n-1} \\ &= | -x_{1,2n-1} x_{1,2n-2} x_{1,2n-3} \dots x_{1,1} x_{1,0} |_2^{2n-1} \\ &= \underbrace{x'_{1,2n-1} x'_{1,2n-2} x'_{1,2n-3} \dots x'_{1,1} x'_{1,0}}_{2n} \\ &= \underbrace{x'_{1,2n-1} x'_{1,2n-2} x'_{1,2n-3} \dots x'_{1,1} x'_{1,0}}_{2n} \\ y_4 &= |-a_2|_2^{2n-1} = |a_2'|_2^{2n-1} \\ &= | -a_{1,2n-1} a_{1,2n-2} a_{1,2n-3} \dots a_{1,0} |_2^{2n-1} \\ &= \underbrace{a'_{1,2n-1} a'_{1,2n-2} a'_{1,2n-3} \dots a'_{1,1} a'_{1,0}}_{2n} \end{aligned}$$

From equation (1)

$$\begin{aligned} X &= a_1 + a_2(2^{n+1} - 1) + a_3(2^{2n+1} - 1) \times 2^{n-1} \\ &= a_1 + 2^{2n+1}a_2 - a_2 + 2^{3n}a_3 - 2^{n-1}a_3 \end{aligned} \quad (16)$$

$$\text{Let } X = \beta_3 + \beta_4 \quad (17)$$

$$\begin{aligned} \text{where, } \beta_4 &= 2^{3n}a_3 \\ &= \underbrace{00\dots0}_{3n} \underbrace{a_{3,2n-1}a_{3,2n-2}a_{3,2n-3} \dots a_{3,0}}_{2n} \end{aligned} \quad (18)$$

$$\beta_3 = \beta_1 + \beta_2 \quad (19)$$

$$\beta_1 = a_1 + 2^{2n+1}a_2 \quad (20)$$

$$\begin{aligned} &= 2^{2n+1} \underbrace{a_{2,n-2}a_{2,n-3}a_{2,n-4} \dots a_{2,0}}_{n-1} + \underbrace{a_{1,2n}a_{1,2n-1}a_{1,2n-2} \dots a_{1,0}}_{2n+1} \\ &= \underbrace{00\dots0}_{2n+1} \underbrace{a_{2,n-2}a_{2,n-3}a_{2,n-4} \dots a_{2,0}}_{n-1} + \underbrace{a_{1,2n}a_{1,2n-1}a_{1,2n-2} \dots a_{1,0}}_{2n+1} \end{aligned}$$

$$\begin{aligned} \beta_2 &= -a_2 - 2^{n-1}a_3 \\ &= -a_{2,n-2}a_{2,n-3}a_{2,n-4} \dots a_{2,0} \underbrace{00\dots0}_{n-1} - 2^{n-1} \underbrace{a_{3,2n-1}a_{3,2n-2}a_{3,2n-3} \dots a_{3,0}}_{2n-1} \underbrace{00\dots00}_{2n-1} \\ &= \underbrace{a'_{2,n-2}a'_{2,n-3}a'_{2,n-4} \dots a'_{2,0}}_{2n-1} \underbrace{00\dots0}_{n-1} + \underbrace{a'_{3,2n-1}a'_{3,2n-2}a'_{3,2n-3} \dots a'_{3,0}}_{2n-1} \underbrace{11\dots11}_{2n-1} \\ &= \underbrace{a'_{2,n-2}a'_{2,n-3}a'_{2,n-4} \dots a'_{2,0}}_{3n-1} + \underbrace{a'_{3,2n-1}a'_{3,2n-2}a'_{3,2n-3} \dots a'_{3,0}}_{3n-1} \end{aligned} \quad (21)$$

We add 1 to β_2 so as to make it of the same bit with β_1

$$\beta_2 = 1 \underbrace{a'_{2,n-2}a'_{2,n-3}a'_{2,n-4} \dots a'_{2,0}}_{3n} + \underbrace{a'_{3,2n-1}a'_{3,2n-2}a'_{3,2n-3} \dots a'_{3,0}}_{3n} \quad (22)$$

From equation (14) and (15)

$$\begin{aligned} X &= \underbrace{a'_{3,2n-1}a'_{3,2n-2}a'_{3,2n-3} \dots a'_{3,0}}_{2n} \underbrace{00\dots00}_{3n} + \underbrace{\beta_{3,3n-1} \beta_{3,3n-2} \dots \beta_{3,0}}_{3n} \\ &= \underbrace{a'_{3,2n-1}a'_{3,2n-2}a'_{3,2n-3} \dots a'_{3,0}}_{2n} \underbrace{\beta_{3,3n-1} \beta_{3,3n-2} \dots \beta_{3,0}}_{3n} \end{aligned} \quad (23)$$

IV. HARDWARE REALIZATION

The hardware realization of the proposed system is based on (13), (14) or (15). This can be seen on table1. The carry propagation Adder1 (CPA1) is used to add up x_1 and x_2 . The carry save adder 1 (CSA1) is used to compute a_3 . Carry Propagation adder 2 (CPA2) and Carry propagation adder 3 (CPA3) are in parallel where either of them can be used to compute (14) or (15). Carry propagation adder 4 (CPA 4) is used to add β_1 and β_2 to obtain β_3 . 1 is appended to the concatenated result in equation (22) as to make β_2 a 3n-bit number.

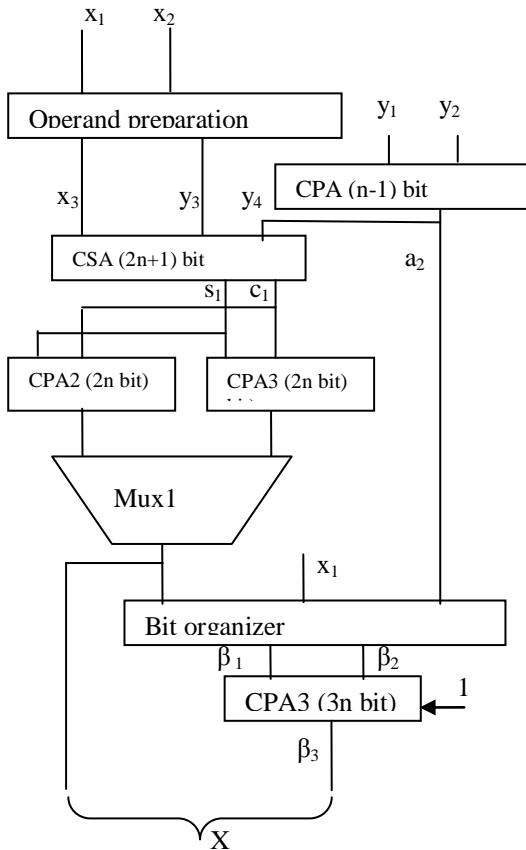


Fig. 3: Hardware implementation of

V. PERFORMANCE EVALUATION

The performance of the proposed system is evaluated by comparing it with [8], [9] and [4] of the same DR. the author [8] compared the performance evaluation of his work with one of the best converter in literature $\{2^n-1, 2^n+1, 2^{2n}-1\}$ [16] and found that the performance [8] is better than [16]. The performance evaluation of our proposed system is better than [8], [9] and [24].

TABLE 1: DELAY COMPARISON

Converters	DR bits	Delay
[9]	5n bits	(13n+4)
[24]	5n bits	(9n +6)
[8]	5n bits	(6n+3)
Proposed	5n bits	(6n+1)

VI. CONCLUSION

The author of [8] compared his work with one of the best converter in literature [16]. In this paper the proposed converter is compared with [8], [9] & [24] and the proposed system is slightly found faster.

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