Structural Design Optimization using Basic T-Norm and T-Conorm based Intuitionistic Fuzzy Optimization Technique

Samir Dey, Tapan Kumar Roy

Abstract— **Real world engineering problems are usually designed by the presence of many conflicting objectives. In this paper we develop an approach to solve multi-objective structural design using basic t-norms and t- co-norms based intuitionistic fuzzy optimization technique. Here binary t-norms, t-co-norms are extended in the form of n-ary t-norms and t-co-norms and their basic properties are discussed with some special cases. In this paper we have considered a multi objective structural optimization model with weight and deflection as objectives and stress as constraint function. Here design variables are considered as cross sectional area of bars. This classical truss optimization example is presented here in to demonstrate the efficiency of our proposed optimization approach. Numerical example is given here to illustrate this structural model through this approximation method.**

Index Terms— **Intuitionistic Fuzzy Set, t-norms, t-conorms, Multi-Objective Intuitionistic Optimization, Structural Optimization.**

I. Introduction

Optimization is the process of minimizing or maximizing an objective function (e.g. cost, weight) of a structural system which has been frequently employed as the evaluation criterion in structural engineering applications. But in the practical optimization problems, usually more than one objective are required to be optimized, such as minimum mass or cost, maximum stiffness, minimum displacement at specific structural points, maximum natural frequency of free vibration, and maximum structural strain energy. This makes it necessary to formulate a multi-objective optimization problem. The first note on multi-objective optimization was given by Pareto; since then the determination of the compromise set of a multi-objective problem is called Pareto optimization. That is why the application of different optimization technique to structural problems has attracted the interest of many researchers. It has been seen that numerous engineering design problem need to deal with noisy data, manufacturing error or uncertainty of the environment during the design process. Fuzzy as well as intuitionistic fuzzy optimization in case of structural engineering not only helps the engineers in their design and analysis of systems but also leads to significant advances and new discoveries in fuzzy optimization theory and technique. This fuzzy set theory was first introduced by Zadeh [4]. As an extension Intuitionistic fuzzy set theory was first introduced by Atanassove [3].When an imprecise information cannot be expressed by means of conventional fuzzy set Intuitionistic Fuzzy set play an important role. In intuitionistic fuzzy (IF) set we usually consider degree of acceptance, degree of non-membership and a hesitancy function whereas we consider only membership function in fuzzy set. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. The concept of membership and non-membership was considered by Angelov[1] in optimization problem and gave intuitionistic fuzzy approach to solve optimization problems. Now intuitionistic fuzzy optimization (IFO) is an open field for research work. Very little research work has been carried out on IFO in application to structural optimization. Dey et al.[2] used intuitionistic fuzzy technique to optimize single objective two bar truss structural model. Dey et al.[9] used multi-objective intuitionistic optimization technique in their paper on three bar truss structural model.

An important concept in fuzzy as well as intuitionistic fuzzy set theory is triangular norms and conforms which are nothing but a generalized intersection and union of fuzzy sets. Alsina et al.[6] introduced the t-norm in fuzzy set theory and suggested that the t-norms could be used for the intersection of fuzzy sets. G..Deschrijver et al.[7] introduced the concept of intuitionistic fuzzy t-norm and t-conorm to investigate the theorems for similar representation of aggregated t-norm and t-conorm.

As per our best of knowledge this is the first time basic t-norms and t-conorm based intuitionistic fuzzy optimization programming technique is being used to solve multi-objective structural model in this paper. In the test problem we have considered three-bar planar truss subjected to a single load condition where the objective functions are weight of the truss and deflection of loaded joint and the design variables are the cross-sections of bars with the constraints as stresses in members.

The remainder of this paper is organized in the following way. In section II, structural optimization model is discussed. In section III, mathematics Prerequisites are discussed with. extended n-ary t-norms and t-co-norms and calculation of some of special cases. In section IV, we discuss about weighted fuzzy aggregation In section V, we proposed the technique to solve a multi-objective non-linear programming problem using extended basic t-norms and t-co-norm based

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intuitionistic fuzzy optimization. In section VI, multi-objective structural model is solved using extended basic t-norms and t-co-norm based intuitionistic fuzzy optimization. Numerical illustration of structural model of three bar truss and comparison of results by using different extended weighted t-norms and t-co-norm are discussed in section VII.Finally we draw conclusions in section VIII.

II. Multi-objective Structural Model

In the design problem of the structure i.e lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure .In truss structure system, the basic parameters (including allowable stress, etc) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions.

The multi-objective structural model can be expressed as

$$Minimize\ WT(A) \quad (1)$$

$$Minimize\ \delta(A)$$

$$subject\ to\ \sigma(A) \le [\sigma]$$

$$A^{\min} \le A \le A^{\max}$$

where $A = [A_1, A_2, ...A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar , $WT(A) = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure , $\delta(A)$ is the deflection of the loaded joint ,where L_i, A_i and ρ_i are the bar length ,cross section area and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is allowable stress of the group bars under various conditions, $A^{\min}$ and are the lower and upper bounds of cross section area A respectively.

III. Mathematical Preliminaries

A. Fuzzy Set

Let X denotes a universal set. Then the fuzzy subset A in X is a subset of order pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0,1]$ is called the membership function which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$ to each element $x \in X$. A is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set. It is clear that the range of membership function is a subset of non-negative real numbers.

B. $\alpha-$ Level set or $\alpha-$ cut of a Fuzzy Set

The $\alpha-$level set of a fuzzy set A of X is a crisp set A_α which contains all the elements of X that have membership values greater than or equal to α i.e $A = \{x : \mu_A(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

C. Intuitionistic Fuzzy Set

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) set A^i in the sense of Atanassove [3] is given by equation $\tilde{A}^i = \{< X, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x) > | x_i \in X\}$ where the function $\mu_{\tilde{A}^i}(x^i) : X \to [0,1]$; $x_i \in X \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$ and $\upsilon_{\tilde{A}^i}(x^i) : X \to [0,1]$; $x_i \in X \to \upsilon_{A^i}(x_i) \in [0,1]$ define the degree of membership and degree of non-membership of an element $x_i \in X$ to the set $\tilde{A}^i \subseteq X$,such that they satisfy the condition $0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1$, $\forall\ x_i \in X$. For each IFS $\tilde{A}^i$ in X the amount $\Pi_{\tilde{A}^i}(x_i) = 1 - (\mu_{\tilde{A}^i}(x^i) + \upsilon_{\tilde{A}^i}(x^i))$ is called the degree of uncertainty (or hesitation) associated with the membership of elements $x_i \in X$ in $\tilde{A}^i$ we call it intuitionistic fuzzy index of $\tilde{A}^i$ with respect of an element $x_i \in X$.

D. (α, β) cut of a Intuitionistic Fuzzy Set

A set of (α, β) - cut is generated by IFS $\tilde{A}^i$ where $\alpha, \beta \in [0,1]$ are fixed number such that $\alpha + \beta \le 1$ is denoted by $\tilde{A}^i_{\alpha,\beta} = \left\{ \begin{array}{l} < x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x) >: \\ x \in X, \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta, \alpha, \beta \in [0,1] \end{array} \right\}$ and defined as the crisp set of element x which belong to $\tilde{A}^i$ at least to the degree α and which belong to $\tilde{A}^i$ at most to the degree β .

E. Triangular Norm (T-Norm)

$T : [0,1] \times [0,1] \to [0,1]$ is said to be t-norm if it satisfies the following properties

i) $T(a,b) = T(b,a)\ \forall a,b \in [0,1]$ (commutativity)

ii) $T(T(a,b),c) = T(a,T(b,c))\ \forall a,b,c \in [0,1]$ (Associativity)

iii) $T(a,b) \le T(a,c)$ with $b \le c\ \ \forall a,b,c \in [0,1]$ (Monotonicity)

iv) $T(0,0) = 0, T(1,1) = 1;$

v) $T(a,1) = a\ \forall\ a \in [0,1]$ (Identity)

F. Extended n-ary Triangular Norm (T-Norm)

For the purpose of operations of multiple fuzzy sets ,it is useful to define the notation of multidimensional t-norms. Let $[0,1]^n$ be a n-dimensional cube and $(x_1, x_2, ..., x_n)$ and $(z_1, z_2, ..., z_n) \in [0,1]^n$ .A mapping $T : [0,1]^n \to [0,1]$ is called n-dimensional t-norm if it satisfies the following properties.

i) $T(x_1, x_2, ..., x_{i-1}, x_i, x_{i+1}, ..., x_n) = T(x_1, x_2, ..., x_{i-1}, x_j, x_{i+1}, ..., x_{j-1}, x_i, x_{j+1}, ..., x_n)$

ii) $T(T(x_1, x_2, ..., x_{n-1}, x_n), x_{n+1}, x_{n+2}, ..., x_{2n-1}) = T(x_1, x_2, ..., x_{n-1}, T(x_n, x_{n+1}, ..., x_{2n-1}))$

iii) For $(x_1, x_2, ..., x_{n-1}, x_n) \le (z_1, z_2, ..., z_{n-1}, z_n) \Rightarrow T(x_1, x_2, ..., x_{n-1}, x_n) \le T(z_1, z_2, ..., z_{n-1}, z_n)$ with $x_i = z_i$

for some i and $x_i \le z_i$ for some $i = 1, 2, ..., n$

iv) $T(0,0,..........,0) = 0,\ T(1,1,..........,1) = 1$

v) $T(1,1,....., x_i,.....,1) = x_i$

G. Properties of Extended n-ary Triangular Norm (T-Norm)

Due to associative law it is easy to extend a triangular norm T into n arguments the n-ary operation T_n on $[0,1]$ satisfies the following properties

i) $T_n(x_1, x_2, ..., x_n) = T_n(x_{\sigma_1}, x_{\sigma_2}, ..., x_{\sigma_n})$ where σ is a permutation of $\{1, 2, ..., n\}$ (Commutativity)

ii) $T_n(x_1, x_2, ..., x_n) = T_{i+1}(x_1, x_2, ..., x_i, T_{n-i}(x_{i+1}, .., x_j, ..., x_n))$
$= T_{n-j+1}(T(x_1, x_2, ..., x_j), x_{j+1}, ..., x_n)$

iii) $(\forall i \in N_n)(x_i \le x_i') \Rightarrow T_n(x_1, x_2, ..., x_n) \le T_n(x_1', x_2', ..., x_n')$ (monotonocity)

iv) $T_n(x_1, x_2, ..., x_{i-1}, 1, x_{i+1} ...., x_n) = T(x_1, x_2, ..., x_{i-1}, x_{i+1}, .., x_j, ..., x_n)$ (Identity Law)

A t-norm T_n is said to be continuous if T is continuous function on $[0,1]$ .From the above, we may call T_n an extension of triangular norm .In the sequel we omit number of argument n and simply write T of the class of mapping generated by triangular norm T .

H. Triangular Conorm (T-Conorm)

$S : [0,1] \times [0,1] \to [0,1]$ is said to be t-conorm if it satisfies the following properties

i) $S(a,b) = S(b,a)\ \forall a,b \in [0,1]$ (commutativity)

ii) $S(S(a,b),c) = S(a, S(b,c))\ \forall a,b,c \in [0,1]$ (Associativity)

iii) $S(a,b) \le S(a,c)$ with $b \le c\ \ \forall a,b,c \in [0,1]$ (Monotonocity)

iv) $S(0,0) = 0, S(1,1) = 1;$

v) $S(a,0) = a\ \forall\ a \in [0,1]$ (Identity)

I. Extended n-ary Triangular Conorm (T-Conorm)

For the purpose of operations of multiple fuzzy sets ,it is useful to define the notation of multidimensional t-norms. Let $[0,1]^n$ be a n-dimensional cube and $(x_1, x_2, ..., x_n)$ and $(z_1, z_2, ..., z_n) \in [0,1]^n$ .A mapping $S : [0,1]^n \to [0,1]$ is called n-dimensional t-norm if it satisfies the following properties.

i) $S(x_1, x_2, ..., x_{i-1}, x_i, x_{i+1}, ..., x_n) = S(x_1, x_2, ..., x_{i-1}, x_j, x_{i+1}, ..., x_{j-1}, x_i, x_{j+1}, ..., x_n)$

ii) $S(S(x_1, x_2, ..., x_{n-1}, x_n), x_{n+1}, x_{n+2}, ..., x_{2n-1}) = S(x_1, x_2, ..., x_{n-1}, S(x_n, x_{n+1}, ..., x_{2n-1}))$

iii) For $(x_1, x_2, ..., x_{n-1}, x_n) \le (z_1, z_2, ..., z_{n-1}, z_n) \Rightarrow$ $S(x_1, x_2, ..., x_{n-1}, x_n) \le S(z_1, z_2, ..., z_{n-1}, z_n)$ with $x_i = z_i$ for some i and $x_i \le z_i$ for some $i = 1, 2, ..., n$

iv) $S(0,0,..........,0) = 0,\ S(1,1,..........,1) = 1$

v) $S(0,0,....., x_i,.....,0) = x_i$

J. Properties of Extended n-ary Triangular Conorm (T-conorm)

Due to associative law it is easy to extend a triangular norm S into n arguments the n-ary operation S_n on $[0,1]$ satisfies the following properties

i) $S_n(x_1, x_2, ..., x_n) = S_n(x_{\sigma_1}, x_{\sigma_2}, ..., x_{\sigma_n})$ where σ is a permutation of $\{1, 2, ..., n\}$ (Commutativity)

ii) $S_n(x_1, x_2, ..., x_n) = S_{i+1}(x_1, x_2, ..., x_i, S_{n-i}(x_{i+1}, .., x_j, ..., x_n))$
$= S_{n-j+1}(S(x_1, x_2, ..., x_j), x_{j+1}, ..., x_n)$

iii) $(\forall i \in N_n)(x_i \le x_i') \Rightarrow S_n(x_1, x_2, ..., x_n) \le S_n(x_1', x_2', ..., x_n')$ (monotonocity)

iv) $S_n(x_1, x_2, ..., x_{i-1}, 0, x_{i+1} ...., x_n) = S(x_1, x_2, ..., x_{i-1}, x_{i+1}, .., x_j, ..., x_n)$ (Identity Law)

A t-norm S_n is said to be continuous if S is continuous function on $[0,1]$ .From the above, we may call S_n an extension of triangular norm .In the sequel we omit number of argument n and simply write S of the class of mapping generated by triangular norm S. .

K. Four Basic T-norm and T-conorm and their Generalization with Weight Factors

Let $\tilde{A}_j^i = \left\{ \begin{matrix} < x_j, \mu_{\tilde{A}_j^i}(x_j), \upsilon_{\tilde{A}_j^i}(x_j) >: \\ x_j \in X, \mu_{\tilde{A}_j^i}(x_j) \ge \alpha, \upsilon_{\tilde{A}_j^i}(x_j) \le \beta, \alpha, \beta \in [0,1] \end{matrix} \right\}$

be n intuitionistic fuzzy set for $j = 1, 2, ..., n$.

i)*Minimum t-norm and maximum t-conorm*

The intuitionistic fuzzy minimum t-norm and maximum t-co-norm can be defined as

$T_M(\mu_1(x_1), \mu_2(x_2), ..., \mu_n(x_n))$
$= \min\{\mu_1(x_1), \mu_2(x_2), ..., \mu_n(x_n)\}$

and $S_M(\upsilon_1(x_1), \upsilon_2(x_2), ..., \upsilon_n(x_n)) = \max\{\upsilon_1(x_1), \upsilon_2(x_2), ..., \upsilon_n(x_n)\}$

Similarly n-ary intuitionistic fuzzy minimum t-norm and maximum t-co-norm
with weight can be defined as

$T_M^w(w_1, \mu_1(x_1); w_2, \mu_2(x_2); .....; w_n, \mu_n(x_n)) = \min\{w_1\mu_1(x_1); w_2\mu_2(x_2); .....; w_n\mu_n(x_n)\}$ and

$$S_M^w\left(w_1,\upsilon_1(x_1);w_2,\upsilon_2(x_2);.....;w_n,\upsilon_n(x_n)\right)$$
$$=\max\left\{w_1\upsilon_1(x_1);w_2\upsilon_2(x_2);.....;w_n\upsilon_n(x_n)\right\}$$

ii) *Probabilistic t-norm and t-conorm*

The intuitionistic fuzzy probabilistic t-norm and t-co-norm can be defined as

$$T_P\left(\mu_1(x_1),\mu_2(x_2),.....,\mu_n(x_n)\right)=\prod_{i=1}^{n}\left(\mu_i(x_i)\right)$$ and

$$S_P\left(\upsilon_1(x_1),\upsilon_2(x_2),.....,\upsilon_n(x_n)\right)=1-\prod_{i=1}^{n}\left(1-\upsilon_i(x_i)\right)$$

Similarly n-ary intuitionistic fuzzy probabilistic t-norm and t-co-norm
with weight can be defined as

$$T_P^w\left(w_1,\mu_1(x_1);w_2,\mu_2(x_2);.....;w_n,\mu_n(x_n)\right)=\prod_{i=1}^{n}\left(\mu_i(x_i)\right)^{w_i}$$

and

$$S_P^w\left(w_1,\upsilon_1(x_1);w_2,\upsilon_2(x_2);.....;w_n,\upsilon_n(x_n)\right)$$
$$=1-\prod_{i=1}^{n}\left(1-\upsilon_i(x_i)\right)^{w_i}$$

iii)*Lukasewicz t-norm and t-conorm*

The intuitionistic fuzzy Lukasewicz t-norm and t-co-norm can be defined as

$$T_L\left(\mu_1(x_1),\mu_2(x_2),...,\mu_n(x_n)\right)=\max\left(\sum_{i=1}^{n}\mu_i(x_i)-(n-1),0\right)$$

and $S_L\left(\upsilon_1(x_1),\upsilon_2(x_2),.....,\upsilon_n(x_n)\right)=\min\left\{1,\sum_{i=1}^{n}\upsilon_i(x_i)\right\}$

Similarly n-ary intuitionistic fuzzy Lukasewicz t-norm and t-co-norm
with weight can be defined as

$$T_L^w\left(\mu_1(x_1),\mu_2(x_2),.....,\mu_n(x_n)\right)$$
$$=\max\left(\sum_{i=1}^{n}w_i\mu_i(x_i)-(n-1),0\right)$$ and

$$S_L^w\left(\upsilon_1(x_1),\upsilon_2(x_2),.....,\upsilon_n(x_n)\right)=\min\left\{1,\sum_{i=1}^{n}w_i\upsilon_i(x_i)\right\}$$

iv)*Weber (or Drastic Product) t-norm and t-conorm*

The intuitionistic fuzzy Weber (or Drastic Product) t-norm and t-co-norm can be defined as

$$T_D\left(\mu_1(x_1),\mu_2(x_2),.....,\mu_n(x_n)\right)$$
$$=\begin{cases}\min\left(\mu_i(x_i)\right) & if\ \max\left(\mu_i(x_i)\right)=1\\ 0 & otherwise\end{cases}$$

and $S_D\left(\upsilon_1(x_1),\upsilon_2(x_2),.....,\upsilon_n(x_n)\right)$

$$=\begin{cases}\max\left(\upsilon_i(x_i)\right) & if\ \min\left(\upsilon_i(x_i)\right)=0\\ 1 & if\ \min\left(\upsilon_i(x_i)\right)>0\end{cases}$$

Similarly n-ary intuitionistic fuzzy Lukasewicz t-norm and t-co-norm
with weight can be defined as

$$T_D^w\left(w_1,\mu_1(x_1);w_2,\mu_2(x_2);.....;w_n,\mu_n(x_n)\right)$$
$$=\begin{cases}\min\left(w_i\mu_i(x_i)\right) & if\ \max\left(w_i\mu_i(x_i)\right)=1\\ 0 & otherwise\end{cases}$$ and

$$S_D^w\left(w_1,\upsilon_1(x_1);w_2,\upsilon_2(x_2);.....,w_n,\upsilon_n(x_n)\right)$$
$$=\begin{cases}\max\left(w_i\upsilon_i(x_i)\right) & if\ \min\left(w_i\upsilon_i(x_i)\right)=0\\ 1 & if\ \min\left(w_i\upsilon_i(x_i)\right)>0\end{cases}$$

IV. Weighted Intuitionistic Fuzzy Aggregation

Weighted aggregation has been used quiet extensively especially in fuzzy decision making ,where the weight are used to represent the relative importance and the negligence the decision maker attaches to different decision criterion (goals or constraints).Weighted aggregation of fuzzy sets by using t-norm has been considered by Yagar [5] .He proposed to modify the membership function with the associated weight factors before the fuzzy aggregation. Xeshui Xu [8] presented intuitionistic fuzzy aggregation operator .The weighted aggregation is then the aggregation of the modified membership and non-membership functions and the general form of this idea is

$$D_1(x,w)=$$
$$T\left(I\left(\mu_1(x_1),w_1\right),I\left(\mu_2(x_2),w_2\right),..,I\left(\mu_k(x_k),w_k\right)\right);$$
$$D_2(x,w)$$
$$=S\left(I\left(\upsilon_1(x_1),w_1\right),I\left(\upsilon_2(x_2),w_2\right),......,I\left(\upsilon_k(x_k),w_k\right)\right)$$

Where w are vectors of weight factor $w_i\in[0,1]\ i=1,2,.....,k$ associated with the aggregated membership function $\mu_i(x_i)$ and non-membership function $\upsilon_i(x_i)$ .Here T is triangular norm and S is triangular conorm, I is a function of two variables that transforms the membership and non-membership with $\sum_{i=1}^{k}w_i=1,\ w_i\ge 0;$.

V. Mathematical Analysis

A. Intuitionistic Fuzzy Non-linear Programming (IFNLP) Optimization with different Weighted T-norm and T-conorm Operator to Solve Multi-Objective Non-linear Programming Problem (PMONLP)

A multi-objective non-linear parametric intuitionistic programming (MONLP) Problem can be formulated as

$$Minimize\ \left\{f_1(x),f_2(x),....,f_p(x)\right\}^T \tag{2}$$

Subject to $g_j(x)\le b_j;\quad j=1,2,.....,m$

$x>0$

Following Zimmermann [10],we have presented a solution algorithm to solve the MONLP Problem by fuzzy optimization technique.

Step-1: Solve the MONLP (2) as a single objective non-linear programming problem p th by taking one of the objective at a time and ignoring the others .These solutions are known as ideal solutions. Let x^i be the respective optimal solution for the i^{th} different objectives with same constraints and evaluate each objective values for all these i^{th} optimal solutions.

Step-2: From the result of step -1 determine the corresponding values for every objective for each derived solutions. With the values of all objectives at each ideal solutions ,pay-off matrix can be formulated as follows

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \vdots \\ x^p \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \dots\dots & f_p(x) \end{array} \\ \begin{bmatrix} f_1^*(x^1) & f_2^*(x^1) & \dots\dots & f_p^*(x^1) \\ f_1^*(x^2) & f_2^*(x^2) & \dots\dots & f_p^*(x^2) \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ f_1^*(x^p) & f_2^*(x^p) & \dots\dots & f_p^*(x^p) \end{bmatrix} \end{array}$$

Here $x^1, x^2, \dots\dots, x^p$ are the ideal solution of the objectives $f_1(x), f_2(x), \dots., f_p(x)$ respectively.

Step-3: From the result of step 2 now we find lower bound (minimum) L_i^{ACC}

and upper bound (maximum) U_i^{ACC} by using following rule $U_i^{ACC} = \max\{f_i(x^p)\}, L_i^{ACC} = \min\{f_i(x^p)\}$ where $1 \le i \le p$ .But in IFO The degree of non-membership (rejection) and the degree of membership (acceptance) are considered so that the sum of both value is less than one. To define the non -membership of NLP problem let $U_i^{\text{Re}\,j}$ and $L_i^{\text{Re}\,j}$ be the upper bound and lower bound of objective function $f_i(x)$ where $L_i^{ACC} \le L_i^{\text{Re}\,j} \le U_i^{\text{Re}\,j} \le U_i^{ACC}$ .For objective function of minimization problem ,the upper bound for non-membership function (rejection) is always equals to that the upper bound of membership function (acceptance).One can take lower bound for non-membership function as follows $L_i^{\text{Re}\,j} = L_i^{Acc} + \varepsilon_i$ where $0 < \varepsilon_i < (U_i^{Acc} - L_i^{Acc})$ based on the decision maker choice.

The initial intuitionistic fuzzy model with aspiration level of objectives becomes *Find* $\{x_i, i = 1, 2, \dots., p\}$

so as to satisfy $f_i(x) \le^i L_i^{Acc}$ with tolerance $P_i^{Acc} = (U_i^{Acc} - L_i^{Acc})$ for the degree of acceptance for $i = 1, 2, \dots.., p$. $f_i(x) \ge^i U_i^{\text{Re}\,j}$ with tolerance $P_i^{Acc} = (U_i^{Acc} - L_i^{Acc})$ for degree of rejection for $i = 1, 2, \dots.., p$ .Define the membership (acceptance) and non-membership (rejection) functions of above uncertain objectives as follows. For the $i^{th}, i = 1, 2, \dots., p$ objectives functions the linear membership function $\mu_i(f_i(x))$ and linear non-membership $\upsilon_i(f_i(x))$ is defined as follows

$\mu_i(f_i(x))$ and linear non-membership $\upsilon_i(f_i(x))$ is defined as follows

$$\mu_i(f_i(x)) = \begin{cases} 1 & if \quad f_i(x) \le L_i^{Acc} \\ \dfrac{e^{-T\left(\frac{f_i(x) - L_i^{Acc}}{U_i^{Acc} - L_i^{Acc}}\right)} - e^{-T}}{1 - e^{-T}} & if \quad L_i^{Acc} \le f_i(x) \le U_i^{Acc} \\ 0 & if \quad f_i(x) \ge U_i^{Acc} \end{cases}$$

$$\upsilon_i(f_i(x)) = \begin{cases} 0 & if \quad f_i(x) \le L_i^{\text{Re}\,j} \\ \left(\dfrac{f_i(x) - L_i^{\text{Re}\,j}}{U_i^{\text{Re}\,j} - L_i^{\text{Re}\,j}}\right)^2 & if \quad L_i^{\text{Re}\,j} \le f_i(x) \le U_i^{\text{Re}\,j} \\ 1 & if \quad f_i(x) \ge U_i^{\text{Re}\,j} \end{cases}$$

After determining the different membership functions for each of the objective functions, one can adopt following three type of decisions

i)Intuitionistic Min-Max Operator, ii) Probabilistic t-norm and t-conorm Operator ,iii) Lukasewicz t-norm and t-conorm Operator

i)According to the extension of the weighted intuitionistic min-max operator the MONLP (2)
can be formulated as

$$Maximize\ \mu_{\tilde{D}_1}(x;w) =$$
$$Max\left(Min\left\{w_1\mu_1(f_1(x)), w_2\mu_2(f_2(x)), \dots\dots, w_p\mu_p(f_p(x))\right\}\right)$$
$$Minimize\ \upsilon_{\tilde{D}_2}(x;w) =$$
$$Min\left(Max\left\{w_1\upsilon_1(f_1(x)), w_2\upsilon_2(f_2(x)), \dots\dots, w_p\upsilon_p(f_p(x))\right\}\right);$$
such that
$$0 \le \mu_i(f_i(x)) + \upsilon_i(f_i(x)) \le 1;\ for\ i = 1, 2, \dots, p.$$
$$\mu_i(f_i(x)) \ge \upsilon_i(f_i(x)),\ for\ i = 1, 2, \dots, p.$$
$$\mu_i(f_i(x)) \in [0,1], \upsilon_i(f_i(x)) \in [0,1],\ for\ i = 1, 2, \dots, p.$$
$$g_j(x) \le b_j;\ j = 1, 2, \dots., m.$$
$$x > 0; \sum_{i=1}^{p} w_i = 1; w_{ii} \in [0,1]\ for\ i = 1, 2, \dots, p.$$

According to Angelov [1] the above problem can be formulated as

$$Maximize\ (\alpha - \beta)$$
$$w_i\mu_i(f_i(x)) \ge \alpha; w_i\upsilon_i(f_i(x)) \le \beta;\ for\ i = 1, 2, \dots, p.$$
$$g_j(x) \le b_j;\ for\ j = 1, 2, \dots, m.$$
$$x > 0; 0 \le \alpha + \beta \le 1; \alpha \ge \beta; \alpha, \beta \in [0,1]$$
$$x > 0; \sum_{i=1}^{p} w_i = 1; w_i \in [0,1]$$

ii)According to the extension of the weighted intuitionistic Probabilistic operator the MONLP (2) can be formulated as

$$Maximize\ \mu_{\tilde{D}_1}(x;w) = Maximize \prod_{i=1}^{n} (\mu_i(x_i))^{w_i}$$
$$Minimize\ \upsilon_{\tilde{D}_2}(x;w) = Minimize \left\{1 - \prod_{i=1}^{n} (1 - \upsilon_i(x_i))^{w_i}\right\}$$

Subject to the same constraint as (i)

iii)According to the extension of the weighted intuitionistic Lukasewicz operator the MONLP (2) can be formulated as

$$Maximize\ \mu_{\tilde{D}_1}(x;w) = Maximize\left(\sum_{i=1}^{n}\mu_i(x_i)-(n-1),0\right)$$

$$Minimize\ \upsilon_{\tilde{D}_2}(x;w) = Minimize\left\{1,\sum_{i=1}^{n}w_i\upsilon_i(x_i)\right\}$$

Subject to the same constraint as (i)

Step-4: Solving any of the above two we will get the optimal solution of MONLP (2).

B. Pareto optimality Test

A numerical test of Pareto optimality for x^* can be performed by solving the following problem

$$Maximize\ R = \sum_{i=1}^{p}\xi_i \tag{3}$$

subject to

$$f_i(x)+\xi_i = f_i(x^*)\ \ i=1,2,...,p.$$

$$x\in X, \xi_i \ge 0\ \ i=1,2,...,p.$$

The optimal solution of (3),say x^{**} and $f_i(x^{**})$ are called strong Pareto optimal Solution provided ξ_i is very small, otherwise it is called weak Pareto-optimal Solution

VI. Solution of Multi-Objective structural optimization problem by Intuitionistic Fuzzy Optimization Technique

To solve the MOSOP (1) step 1 of V.A is used. After that according to step 2 pay-off matrix is formulated

$$\begin{array}{c} \\ A^1 \\ A^2 \end{array}\begin{array}{cc} WT(A) & \delta(A) \\ \end{array}$$
$$\begin{matrix} A^1 \\ A^2 \end{matrix}\begin{bmatrix} WT^*(A^1) & \delta^*(A^1) \\ WT^*(A^2) & \delta^*(A^2) \end{bmatrix}$$

In next step following step 2 we calculate the bound of the objective $U_{WT}^{Acc}, L_{WT}^{Acc}$ and $U_{WT}^{\mathrm{Re}j}, L_{WT}^{\mathrm{Re}j}$ for weight function $WT(A)$, such that $L_{WT}^{Acc} < WT(A) < U_{WT}^{Acc}$ and $L_{WT}^{\mathrm{Re}j} < WT(A) < U_{WT}^{\mathrm{Re}j}$ and $U_{\delta}^{Acc}, L_{\delta}^{Acc}$; $U_{\delta}^{\mathrm{Re}j}, L_{\delta}^{\mathrm{Re}j}$ for deflection $\delta(A)$, such that $L_{\delta}^{Acc} < \delta(A) < U_{\delta}^{Acc}$ and $L_{\delta}^{\mathrm{Re}j} < \delta(A) < U_{\delta}^{\mathrm{Re}j}$ with the condition $U_i^{Acc} = U_i^{\mathrm{Re}j}$; $L_i^{\mathrm{Re}j} = L_i^{Acc} + \varepsilon_i$ $\ for\ i = WT, \delta$ so as $0 < \varepsilon_i < \left(U_i^{Acc} - L_i^{Acc}\right)$ are identified.

According to IFO technique considering membership and non-membership function for MOSOP (1)

$$\mu_{WT(A)}(WT(A)) = \begin{cases} 1 & if\ \ WT(A) \le L_{WT}^{Acc} \\ \dfrac{e^{-T\left(\frac{WT(A)-L_{WT}^{Acc}}{U_{WT}^{Acc}-L_{WT}^{Acc}}\right)} - e^{-T}}{1-e^{-T}} & if\ \ L_{WT}^{Acc} \le WT(A) \le U_{WT}^{Acc} \\ 0 & if\ \ WT(A) \ge U_{WT}^{Acc} \end{cases}$$

$$\upsilon_{WT(A)}(WT(A)) = \begin{cases} 0 & if\ \ WT(A) \le L_{WT}^{\mathrm{Re}j} \\ \left(\dfrac{WT(A)-L_{WT}^{\mathrm{Re}j}}{U_{WT}^{\mathrm{Re}j}-L_{WT}^{\mathrm{Re}j}}\right)^2 & if\ \ L_{WT}^{\mathrm{Re}j} \le WT(A) \le U_{WT}^{\mathrm{Re}j} \\ 1 & if\ \ WT(A) \ge U_{WT}^{\mathrm{Re}j} \end{cases}$$

and

$$\mu_{\delta(A)}(\delta(A)) = \begin{cases} 1 & if\ \ \delta(A) \le L_{\delta}^{Acc} \\ \dfrac{e^{-T\left(\frac{\delta(A)-L_{\delta}^{Acc}}{U_{\delta}^{Acc}-L_{\delta}^{Acc}}\right)} - e^{-T}}{1-e^{-T}} & if\ \ L_{\delta}^{Acc} \le \delta(A) \le U_{\delta}^{Acc} \\ 0 & if\ \ \delta(A) \ge U_{\delta}^{Acc} \end{cases}$$

$$\upsilon_{\delta(A)}(\delta(A)) = \begin{cases} 0 & if\ \ \delta(A) \le L_{\delta}^{\mathrm{Re}j} \\ \left(\dfrac{\delta(A)-L_{\delta}^{\mathrm{Re}j}}{U_{\delta}^{\mathrm{Re}j}-L_{\delta}^{\mathrm{Re}j}}\right)^2 & if\ \ L_{\delta}^{\mathrm{Re}j} \le \delta(A) \le U_{\delta}^{\mathrm{Re}j} \\ 1 & if\ \ \delta(A) \ge U_{\delta}^{\mathrm{Re}j} \end{cases}$$

After determining the different membership functions for each of the objective functions, one can adopt following three types of decisions

i) According to the extension of the weighted intuitionistic min-max operator

$$Maximize\ (\alpha-\beta)$$
$$w_1\left(\mu_{WT}(WT(A))\right) \ge \alpha; w_2\left(\mu_{\delta}(\delta(A))\right) \ge \alpha;$$
$$w_1\left(\upsilon_{WT}(WT(A))\right) \le \beta; w_2\left(\upsilon_{\delta}(\delta(A))\right) \le \beta;$$
$$\sigma(A) \le [\sigma];$$
$$A^{\min} \le A \le A^{\max};$$
$$w_1 \ge 0, w_2 \ge 0,\ w_1 + w_2 = 1;\ w_1, w_2 \in [0,1]$$
$$0 \le \alpha+\beta \le 1; \alpha \ge \beta; \alpha, \beta \in [0,1];$$

ii)According to extension of weighted Probabilistic operator

$$Maximize\left\{\mu_{WT}(WT(A))^{w_1}\ \mu_{\delta}(\delta(A))^{w_2}\right\}$$
$$Minimize\left[1-\left(\left\{1-\upsilon_{WT}(WT(A))\right\}^{w_1}\left\{1-\upsilon_{\delta}(\delta(A))\right\}^{w_2}\right)\right]$$

such that

$$0 \le \mu_{WT}(WT(A)) + \upsilon_{WT}(WT(A)) \le 1;$$
$$0 \le \mu_{\delta}(\delta(A)) + \upsilon_{\delta}(\delta(A)) \le 1;$$
$$\mu_{WT}(WT(A)) \ge \upsilon_{WT}(WT(A));$$
$$\mu_{\delta}(\delta(A)) \ge \upsilon_{\delta}(\delta(A));$$
$$\sigma(A) \le [\sigma];$$
$$A^{\min} \le A \le A^{\max};$$
$$w_1 \ge 0, w_2 \ge 0,\ w_1 + w_2 = 1;\ w_1, w_2 \in [0,1]$$

iii) According to extension of weighted Lukasewicz operator

$$Maximize\left\{w_1\mu_{WT}(WT(A)) + w_2\mu_{\delta}(\delta(A)) - 1\right\}$$
$$Minimize\left\{w_1\upsilon_{WT}(WT(A)) + w_2\upsilon_{\delta}(\delta(A))\right\}$$

subject to the same constraint as (6.i)

Table.1 The input data for MOSOP (4)

Applied Load P (KN)	Material Density (ρ) (KN/m^3)	Length L (m)	Maximum allowable tensile stress $[\sigma_1^T],[\sigma_2^T]$ (KN/m^2)	Maximum allowable compressive stress $[\sigma_C]$(KN/m^2)	Young's Modulus E (KN/m^2)	$A_i^{\min}$ and $A_i^{\max}$ of cross section of bars $10^{-4}(m^2)$
20	100	1	20	15	2×10^8	$A_1^{\min}=0.1$, $A_1^{\max}=5$, $A_2^{\min}=0.1$, $A_2^{\min}=5$

Table 2.Optimal weight for equal importance on structural weight and deflection *i.e* $w_1 = w_2 = 1/2$ and for $\varepsilon_{WT} = 1.65$ $\varepsilon_\delta = 1.1$

Weighted paremeterized t-norm,t-conorm operator	$A_1^* \times 10^{-4} m^2$	$A_2^* \times 10^{-4} m^2$	$WT^* \times 10^2 KN$	$\delta^* \times 10^{-7} m$
Min-max operator	0.5513085	2.634761	4.194097	4.675712
Probabilistic	0.5723870.	3.475052	5.094007	3.645077
Lukasewicz	0.5723870	3.475052	5.094007	3.645077

For equal importance, the extension of weighted Min-max-t-norm t-co-norm operator gives minimum structural weights and probabilistic Lukaswicz give minimum deflection.

Table 3.Optimal weight for more importance on structural weight $w_1 = 0.8, w_2 = 0.2$ and for $\varepsilon_{WT} = 1.65$ $\varepsilon_\delta = 1.1$

Weighted paremeterized t-norm,t-conorm operator	$A_1^* \times 10^{-4} m^2$	$A_2^* \times 10^{-4} m^2$	$WT^* \times 10^2 KN$	$\delta^* \times 10^{-7} m$
Min-max operator	0.6007867	5.478828	7.178109	2.395491
Probabilistic	0.5650335	3.141327	4.739484	3.993976
Lukasewicz	0.5581935	1.428380	3.007189	7.757267

For more importance on structural weight ,the extension of weighted Lukasewicz-t-norm t-co-norm operator gives minimum structural weights and Max-min operator gives minimum deflection.

Table 4.Optimal weight for more importance on deflection $w_1 = 0.2, w_2 = 0.8,$ and for $\varepsilon_{WT} = 1.65$ $\varepsilon_\delta = 1.1$

Weighted paremeterized t-norm,t-conorm operator	$A_1^* \times 10^{-4} m^2$	$A_2^* \times 10^{-4} m^2$	$WT^* \times 10^2 KN$	$\delta^* \times 10^{-7} m$
Min-max operator	0.5720532	1.500028	3.118039	7.425522
Probabilistic	0.5723870	3.475051	5.094006	3.645077
Lukasewicz	0.5723870	3.645052	5.094007	3.645077

For more importance on deflection ,the extension of weighted Max-min t-norm t-co-norm operator gives minimum structural weights where as weighted Probabilistic and Lukasewicz give t-norm t-co-norm operator gives minimum deflection.

Table 5. Pareto Optimality test for $w_1 = w_2 = 1/2$

R	$A_1^* \times 10^{-4} m^2$	$A_2^* \times 10^{-4} m^2$	$WT^* \times 10^2 KN$	$\delta^* \times 10^{-7} m$
0.002084	0.1	3.809080	4.091923	3.645077

VII. Numerical Illustration

A well known three bar planer truss is considered is to minimize weight $WT(A_1, A_2)$ of the structure and minimize the deflection $\delta(A_1, A_2)$ at a loading point of a statistically loaded three bar planer truss subject to stress constraints on each of the truss members

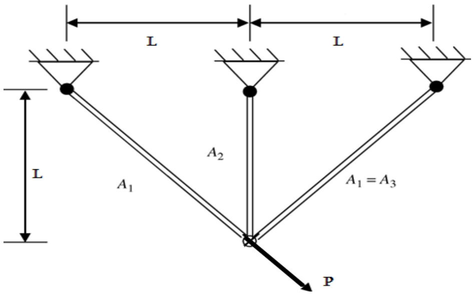

Fig.1. Design of Three-Bar Planer Truss

The multi-objective optimization problem can be stated as follows

$$\text{Minimize} \quad WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right) \qquad (4)$$

$$\text{Minimize} \quad \delta(A_1, A_2) = \frac{PL}{E\left(A_1 + \sqrt{2}A_2\right)}$$

$$\text{subject to} \quad \sigma_1(A_1, A_2) \equiv \frac{P\left(\sqrt{2}A_1 + A_2\right)}{\left(2A_1^2 + 2A_1A_2\right)} \le \left[\sigma_1^T\right];$$

$$\sigma_2(A_1, A_2) \equiv \frac{P}{\left(A_1 + \sqrt{2}A_2\right)} \le \left[\sigma_2^T\right];$$

$$\sigma_3(A_1, A_2) \equiv \frac{PA_2}{\left(2A_1^2 + 2A_1A_2\right)} \le \left[\sigma_3^C\right];$$

$$A_i^{\min} \le A_i \le A_i^{\max} \quad i = 1, 2$$

Where $P =$ applied load ; $\rho =$ material density ; $L =$ length ; $E =$ Young's modulus ; $A_1 =$ Cross section of bar-1 and bar-3; $A_2 =$ Cross section of bar-2; δ is deflection of loaded joint. $\left[\sigma_1^T\right]$ and $\left[\sigma_2^T\right]$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, $\left[\sigma_3^C\right]$ is maximum allowable compressive stress for bar 3.

Table.1 The input data for MOSOP (4)

Solution: According to step 2 pay-off matrix is formulated as follows

$$\begin{array}{c} \\ A^1 \\ A^2 \end{array} \begin{array}{cc} WT(A_1, A_2) & \delta(A_1, A_2) \\ \left[\begin{array}{c} 2.638958 \\ 19.14214 \end{array}\right. & \left.\begin{array}{c} 14.64102 \\ 1.656854 \end{array}\right] \end{array}$$

Here

$U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$

$L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_{WT} = 2.638958 + \varepsilon_{WT};$

Such that $0 < \varepsilon_{WT} < (19.14214 - 2.638958)$;

$U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 14.64102, L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_{\delta} = 1.656854 + \varepsilon_{\delta};$

such that $0 < \varepsilon_{\delta} < (14.64102 - 1.656854)$.

Here membership and non-membership function for objective functions $WT(A_1A_2), \delta(A_1A_2)$ for $T = 2$ are defined as follows

$$\mu_{WT}(WT(A_1, A_2)) = \begin{cases} 1 & \text{if } WT(A_1, A_2) \le 2.638958 \\ \dfrac{e^{-2\left(\frac{WT(A_1,A_2)-2.638958}{16.503182}\right)} - e^{-2}}{1 - e^{-2}} & \text{if } 2.638958 \le WT(A_1, A_2) \le 19.14214 \\ 0 & \text{if } WT(A_1, A_2) \ge 19.14214 \end{cases}$$

$$\upsilon_{WT}(WT(A_1, A_2)) = \begin{cases} 0 & \text{if } WT(A_1, A_2) \le 2.638958 + \varepsilon_{WT} \\ \left(\dfrac{WT(A_1, A_2) - (2.638958 + \varepsilon_{WT})}{16.503182 - \varepsilon_{WT}}\right)^2 & \text{if } 2.638958 + \varepsilon_{WT} \le WT(A_1, A_2) \le 19.14214 \\ 1 & \text{if } WT(A_1, A_2) \ge 19.14214 \end{cases}$$

$$\mu_{\delta}(\delta(A_1, A_2)) = \begin{cases} 1 & \text{if } \delta(A_1, A_2) \le 3.638958 \\ \dfrac{e^{-2\left(\frac{\delta(A_1,A_2)-3.638958}{11.002062}\right)} - e^{-2}}{1 - e^{-2}} & \text{if } 3.638958 \le \delta(A_1, A_2) \le 14.64102 \\ 0 & \text{if } \delta(A_1, A_2) \ge 14.64102 \end{cases}$$

$$\upsilon_{\delta}(\delta(A_1, A_2)) = \begin{cases} 0 & \text{if } \delta(A_1, A_2) \le 3.638958 + \varepsilon_{\delta} \\ \left(\dfrac{\delta(A_1, A_2) - (3.638958 + \varepsilon_{\delta})}{11.002062 - \varepsilon_{\delta}}\right)^2 & \text{if } 3.638958 + \varepsilon_{\delta} \le \delta(A_1, A_2) \le 14.64102 \\ 1 & \text{if } \delta(A_1, A_2) \ge 14.64102 \end{cases}$$

The optimal results of model (4) using t-norms and t-conorms are shown in table 2 to 4 and Pareto optimal solution is shown in table 5.

VIII. Conclusion

In this paper, we have proposed a method to solve multi-objective structural model in intuitionistic fuzzy environment. Here binary t-norms are expressed in extended n-ary t-norms and discussed their basic properties and some special cases. The said model is solved by using t-norms and t-conorm based on intuitionistic fuzzy optimization technique. A main advantage of the proposed method is that it allows the user to concentrate on the actual limitations in a problem during the specification of the flexible objectives. This approximation method can be applied to optimize different models in various fields of engineering and sciences.

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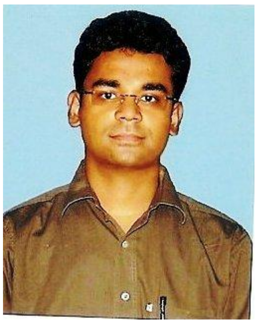

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