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Alternative Formulae for Sum of Arithmetic and Geometric Progression

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***Abstract*—In this paper we generate an alternative formula for sums of Arithmetic and Geometric series by using Newton's Divided Difference Interpolating Polynomial operator. This alternate formula can be used to find sums of Arithmetic and Geometric series rather than using the two formulas of Arithmetic and Geometric series which is nth term formula and then sum formula. As we all know interpolation is constructing new data points within a given range. We have used the points which are in Arithmetic differences and Geometric ratios.**

***Index Terms*-- Arithmetic Progression (AP), Geometric Progression (GP), Interpolation, Newton's Divided Difference Interpolating Polynomialand Polynomial Interpolation.**

I. INTRODUCTION

A. Interpolation

It is the method to construct unknown data points within the range of a discrete set of known $(n+1)$ data points(x_i, y_i). This can be achieved by curve fitting and regression analysis. We have, $y_i = f(x_i)$ at $(n+1)$ points $x_1 \dots x_j, \dots x_{n+1}$: $x_j > x_{j-1}$. In general, we do not know the underlying function $f(x)$.

Following are some of the methods of interpolation:

- Linear interpolation
- Polynomial interpolation
- Spline interpolation, etc.

In this paper we are using polynomial interpolation (specifically, Newton's Divided Difference method) which is a generalization of linear interpolation (i.e. interpolant is a linear function) to construct an alternative formula for arithmetic and geometric progression.

Newton's Divided Difference Interpolating Polynomial for $n+1$ points:$y = f(x) = P_n(x) = f(x_1) + f(x_1,x_2)(x-x_1) + f(x_1,x_2,x_3)(x - x_1)(x - x_2) + \dots\dots. + f(x_1,x_2,x_3 \dots . x_{n+1})(x - x_1)(x - x_2) \dots . (x - xn)\ And, f x_1, x_2, \dots x_{n+1}] = \{f[x_2, x_3 \dots x_{n+1}] - f[x_1, x_2, \dots xn]\}/((x_{n+1} - (x_1)$.

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B. Arithmetic Progression(AP)

It is a sequence in which the difference between the two consecutive terms is constant. n^{th}term of an AP is given by :$a_n = a + (n-1)d$.

Where, a_n is the n^{th} term, a is the first term of the series, n is the total number of terms in the series, d is the common difference.Sum of an arithmetic sequence is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

C. Geometric Progression(GP)

It is a sequence in which ratio of any term to the previous term is same. n^{th}term of a GP is given by : $a_n = ar^{(n-1)}$

Where, a_n is the n^{th} term, a is the first term of the series, r is the common ratio, n is the number of terms in the series.

Sum of a geometric sequence is given by:$S_n = \frac{\{a(1-r^n)\}}{1-r}$.

II. MODEL DEVELOPMENT

A. Development for Arithmetic Progression

We have assumed xi to be in arithmetic progression and function $y_i = f(x_i)$to be the arithmetic sum of the i^{th} terms. We have considered a series with terms 1,3,5,7 with function, $f(x)$ equals to1, 4,9,16 respectively, which is the arithmetic sum of the terms. Here $a = 1$and $d = 2$.

To find Newton's Divided Difference Interpolating Polynomial, we have used the following table:

x_i	$f(x_i)$	$f(x_i,x_{i+1})$	$f(x_i,x_{i+1},x_{i+2})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3})$
1	1			
		3/2		
3	4		¼	
		5/2		0
5	9		¼	
		7/2		
7	16			

So, the polynomial we get is :$\frac{x^2}{4} + \frac{x}{2} + \frac{1}{4}$

: $\frac{[x\ (x+2)+1]}{4}$: $\frac{[x(x+2)+(2-1)1]}{2*2}$

Now, we have considered a series with terms 1, 5, 9, 13, 17, 21 with function, f(x) equals to 1, 6, 15, 28, 45, 66respectively, which is the arithmetic sum of the terms. Here $a = 1$and $d = 4$.

To find Newton's Divided Difference Interpolating Polynomial, we have used the following table:

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x_i	$f(x_i)$	$f(x_i, x_{i+1})$	$f(x_i,x_{i+1}, x_{i+2})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5})$
1	1					
		5/4				
5	6		1/8			
		9/4		0		
9	15		1/8		0	
		13/4		0		0
13	28		1/8		0	
		17/4		0		
17	45		1/8			
		21/4				
21	66					

So, the polynomial we get is : $\frac{x2}{8} + \frac{2x}{4} + \frac{3}{8}$

: $\frac{[x\ (x+4)+3]}{8}$: $\frac{[x(x+4)+(4-1)1]}{2*4}$

From the above discussed examples, we have come to a conclusion to an alternate Sum formula for AP using the Newton's Divided method which can be given by:

$$\frac{[x(x+d)+(d\ -\ a)a]}{2d}$$

Where, a is the first term, d is the common difference and x is any missing x_i value for which we have to calculate corresponding $f(x_i)$i.e. Sum up to that term.

Now, we have considered a series with terms 11, 17, 23, 29, 35, 41, 53, 59 and 47 is the missing term in the series with function, $f(x)$ equals to 11, 28, 51, 80, 115, 156, 256, 315 respectively, which is the arithmetic sum of the terms.

Here $a\ =\ 11$and $d\ =\ 6$,

To find Newton's Divided Difference Interpolating

x_i	$f(x_i)$	$f(x_i,x_{i+1})$	$f(x_i,x_{i+1}, x_{i+2})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5}, x_{i+6})$	$f(x_i,x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5}, x_{i+6}, x_{i+7})$
11	11							
		17/6						
17	28		1/12					
		23/6		0				
23	51		1/12		0			
		29/6		0		0		
29	80		1/12		0		0	
		35/6		0		0		0
35	115		1/12		0		0	
		41/6		0		0		
41	156		1/12		0			
		50/6		0				
53	256		1/12					
		59/6						
59	315							

Polynomial, we have used the following table:

So, the polynomial we get is:$\frac{x^2}{12} + \frac{6x}{12} - \frac{55}{12}$: $\frac{[x(x+6)+\ 11(6-11)]}{2*6}$

Here, for $x\ =\ 47$, the $y(x)\ =\ 203(1)$

Now, with Alternate Sum formula: $\frac{[x(x+d)+(d\ -a)a]}{2d}$

$y(x)$is: $\frac{1}{12}[47(47+6)\ +\ 11(6-11)] = 203$ (2)

From (1) and (2), we see that the value of $y(x)$ for x is same from both the formulae.

B. Comparison between Developed Formula and Original AP Formula

First, we calculate the arithmetic sum of nth term by using the original AP formula which is as follows:

$a_n\ =\ a\ +\ (n-1)d$ (1)

$S_n\ = n\frac{[2a+(n-1)d]}{2}$ (2)

For instance, we consider a long series 1, 3, 5, 7, 9, ... ,195, 197 and calculate the arithmetic sum till 197. Here $a\ = 1$ and $d\ = 2$.

To calculate n, we use (1):$197\ =\ 1\ +\ (n\ -\ 1)2$

$n\ =\ 99$

To calculate sum of nth term, we use (2):

$$S_n\ = 99\frac{[(2*1) +\ (99\ -\ 1)2]}{2}$$

$S_n\ = 9801$ (a)

Now, we compare this solution with the solution of our derived formula which is as follows:

$S_n\ = \frac{[x(x+d)+a(d-a)]}{2d}$ (3)

Considering the above mentioned series, we calculate the arithmetic sum of the same term i.e. 197.

To calculate, we use (3): $S_n\ = \frac{[197(197+2)+1(2-1)]}{2*2}$

$S_n\ =\ 9801$ (b)

From (a) and (b), we can conclude that the solution from both the formulae is same.
We can also justify that instead of using above two original formulae for first finding n and then calculating the arithmetic sum , we can use our derived formula to calculate the arithmetic sum in one go.

C. Development for Geometric Progression

We have considered a series with terms 1, 2, 4, 8 with function, $f(x)$ equals to 1, 3, 7, 15 respectively, which is the geometric sum of the terms. Here $a = 1$and $r = 2$.
To find Newton's Divided Difference Interpolating Polynomial, we have used the following table :

x_i	$f(x_i)$	$f(x_i,x_{i+1})$	$f(x_i,x_{i+1},x_{i+2})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3})$
1	1			
		2		
2	3		0	
		2		0
4	7		0	
		2		
8	15			

So, the polynomial we get is:$1 + 2(x - 1)$:

$$\frac{2x - 1}{2 - 1}$$

Now, we have considered a series with terms 4, 20, 100, 500, 2500, 12500 with function, $f(x)$ equals to 4, 24, 124, 624, 3124, 15624 respectively, which is the geometric sum of the terms. Here $a = 4$ and $r = 5$.
To find Newton's Divided Interpolating Polynomial, we have used the following table:

x_i	$f(x_i)$	$f(x_i,x_{i+1})$	$f(x_i,x_{i+1},x_{i+2})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4},x_{i+5})$
4	4					
		5/4				
20	24		0			
		5/4		0		
100	124		0		0	
		5/4		0		0
500	624		0		0	
		5/4		0		
2500	3124		0			
		5/4				
12500	15624					

So, the polynomial we get is: $4 + \frac{5x}{4} - \frac{20}{4}$

: $\frac{5x-4}{5-1}$

From the above discussed examples, we have come to a conclusion to an alternative Sum formula for GP using the Newton's Divided method which can be given by:

$$\frac{rx - a}{r - 1}$$

Where, a is the first term , r is the common ratio, and x is any missing x_i value for which we have to calculate corresponding $f(x_i)$i.e. Sum upto that term.

Now, we have considered a series with terms 3, 15, 75, 375, 1875, 46875 and 9375 is the missing term in the series with function, $f(x)$ equals to 3, 18, 93, 468, 2343, 58593 respectively, which is the geometric sum of the terms. Here $a = 3$ and $r = 5$.

To find Newton's Divided Difference Interpolating Polynomial, we have used the following table:

x_i	$f(x_i)$	$f(x_i,x_{i+1})$	$f(x_i,x_{i+1},x_{i+2})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4})$	$f(x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4},x_{i+5})$
3	3					
		5/4				
15	18		0			
		5/4		0		
75	93		0		0	
		5/4		0		0
375	468		0		0	
		5/4		0		
1875	2343		0			
		5/4				
46,875	58,593					

So, the polynomial we get is: $3 + \frac{5}{4}(x - 3)$: $\frac{5x-3}{4}$
Here, for $x = 9375$.

the $y(x) = \frac{[5(9375) - 3]}{4} = 11{,}718$ (1)

Now, with derived formula: $(rx - a)/(r - 1)$

$y(x)$is: $\frac{[5(9375) - 3]}{5-1} = 11{,}718$ (2)

From (1) and (2), we see that the value of $y(x)$ for x is same from both the formulae.

D. Comparison between Developed Formula and Original GP Formula

Similarly, we calculate the geometric sum of nth term by using the original GP formula which is as follows:

$a_n = ar^{(n-1)}$ (1)

$S_n = \frac{\{a(1-r^n)\}}{1-r}$(ii) (2)

For instance, we consider a long series 1, 2, 4, 8, 16, 32, 64...... ,262144 and calculate the geometric sum till 262144. Here $a = 1$and $r = 2$.

To calculate n, we use (1): $262144 = 1(2)^{(n-1)} n = 19$

To calculate sum of nth term, we use (2) :

$$S_n = \frac{\{1(1-2^{19})\}}{1-2}$$

$S_n = 524287$ (a)

Now, we compare this solution with the solution of our derived formula which is as follows:

$S_n = \frac{rx - a}{r-1}$ (3)

Considering the above mentioned series, we calculate the arithmetic sum of the same term i.e. 262144.

To calculate, we use (3):

$$S_n = \frac{\{2(262144) - 1\}}{2-1}$$

$S_n = 524287$ (b)

From (a) and (b), we can conclude that the solution from both the formulae is same.

We can also justify that instead of using above two original formulae for first finding n and then calculating the geometric sum , we can use our derived formula to calculate the geometric sum in one go.

III. CONCLUSION

In this study, the Newton's Divided Difference Interpolating Operator makes a great tool to find the general formula for sums of Arithmetic and Geometric Progression. We have used a combination between Mathematics and Computer Science. We conclude that we can use the alternate formula for fast calculation of the sums of Arithmetic and Geometric Progression instead of using the two formulas of Arithmetic and Geometric series which is nth term formula and then sum formula.

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