

Influence of Chemical Reaction on Unsteady MHD Free Convective Flow Past a Vertical Plate with Uniform Heat and Mass Flux through Porous Medium

R. Mohana Ramana and J. Girish Kumar

Abstract:

An attempt has been made to study unsteady MHD free convective flow combined with heat and mass transfer of electrically conducting, viscous incompressible fluid past an infinite vertical plate immerses in a porous medium considering into the account of viscous dissipation and subject to uniform heat and mass flux in the presence of chemical reaction analytically. The governing non-linear coupled partial differential equations are reduced to ordinary differential equations by using perturbation technique. The effects of governing physical parameters on velocity, temperature and concentration are analysed graphically.

Keywords: MHD, chemical reaction, heat and mass flux, porous medium, viscous dissipation etc.

I. INTRODUCTION

MHD is related with the study of the interaction with magnetic fields and electrically conducting fluids in motion. There are number of applications in MHD principles including MHD pumps, MHD generators and MHD flow meters etc.

*R.Mohana Ramana, Department of GEBH(Mathematics),
Sree vidyanikethan Engineering college
A.Rangampet, India*

*J.Girish Kumar, Department of Mathematics, Government
Degree College, India*

MHD flows of convection have many applications in the field of stellar and planetary magnetospheres, chemical engineering, aeronautical plasma flows and electronics. The phenomenon of combined heat and mass transfer is observed in the dispersion and formation of fog, distribution of moisture and temperature over groves of fruit trees and agricultural fields, damage of crop due to freezing and environmental pollution. MHD convective flows with combined heat and mass transfer with chemical reaction arise in many transport processes in many branches of science and engineering. Sparrow et al. [1] investigated the effect of magnetic field on free convective heat transfer. Soundalgekar et al. [2] studied the effect of heat and mass transfer on unsteady natural convective flow along vertical porous plate with constant suction. Raptis [3] discussed unsteady free convective flow through a porous medium. Lahurakir et al. [4] presented an exact solution of unsteady forced and free convection flow embedded in a porous medium past an infinite vertical plate. Anjalidevi et al. [5] examined the effect of

chemical reaction on the flow in the presence of heat transfer and magnetic field. Sreekanth et al. [6] studied transient MHD free convective flow of an incompressible viscous dissipative fluid. Soundalgekar et al. [7] analysed the transient free convection of a viscous dissipative fluid flow past a infinite vertical plate. Ahmed [8] investigated the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Rajasekhar et al. [9] studied the unsteady MHD free convective flow past a semi-infinite vertical porous plate. Murali et al. [10] given a numerical solution to the problem of unsteady magneto hydrodynamic free convective flow past a vertical porous plate and it has been analysed. Ibrahim [11] analysed unsteady MHD free convective flow along a vertical porous plate embedded in a porous medium with heat generation, chemical reaction and variable suction effects. Loganathan et al. [12] investigated viscous dissipation effects on unsteady free convective flow past an infinite vertical plate with uniform heat and mass flux. Ahmed et al. [13] studied analytically on unsteady MHD free convection and mass transfer flow past a vertical porous plate.

The aim of this paper is to analyse the chemical reaction effects on unsteady magneto hydrodynamic flow past a vertical porous plate with viscous dissipation subject to heat and mass flux. The problem is governed by the system of coupled non-linear partial differential equations and solved by perturbation method.

II. FORMULATION OF THE PROBLEM

Consider an unsteady, two dimensional, laminar natural convective MHD flow over an infinite vertical plate in the presence of magnetic field through the porous medium under the influence of chemical reaction subject to uniform heat and mass flux. The plate is taken in the vertical direction which orients with x' axis and the y' axis is taken normal to it. Let u' and v' be the velocity components in their respective directions. All the fluid properties are assumed to be constant except that the influence of the density variation. In the energy equation viscous dissipation term is taken into account. Under usual Boussinesq's approximation the governing continuity, momentum, energy and concentration equations are given by

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

Momentum Equation:

$$\begin{aligned} \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} &= g\beta(T' - T'_\infty) \\ &+ g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \\ &- \frac{\nu}{k} u' - \frac{\sigma B_0^2}{\rho} u' \end{aligned} \quad (2)$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 \quad (3)$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - k_r (C' - C'_\infty) \quad (4)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned}
 t' > 0: u' = 0, \quad \frac{\partial T'}{\partial y'} = -\frac{q}{k^*}, \\
 \frac{\partial C'}{\partial y'} = -\frac{j'}{D} \quad \text{at } y' = 0 \\
 u' \rightarrow 0, \quad T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \\
 \text{as } y' \rightarrow \infty
 \end{aligned} \right\} \quad (5)$$

In the above equations all the physical variables are functions of y' and t' alone as the plate is infinite.

From equation (1), it is evident that v' is either a constant or a function of time. In this work, it is assumed that the suction velocity varies with time.

It can be written as

$$v' = -v_0 (1 + \varepsilon A e^{nt'}) \quad (6)$$

Where v_0 is a scale of suction velocity, A is real positive constant, ε and εA are very small ($\ll 1$)

On introducing the following non dimensional quantities

$$\left. \begin{aligned}
 u = \frac{u'}{v_0}, \quad t = \frac{t' v_0^2}{\nu}, \quad y = \frac{y' v_0}{\nu}, \quad T = \frac{T' - T'_\infty}{\left(\frac{qv}{k^* v_0}\right)}, \\
 Gr = \frac{\nu g \beta \left(\frac{qv}{k^* v_0}\right)}{v_0^3}, \quad Gc = \frac{\nu g \beta^* \left(\frac{j' \nu}{D v_0}\right)}{v_0^3}, \\
 C = \frac{C' - C'_\infty}{\left(\frac{j' \nu}{D v_0}\right)}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}, \\
 k = \frac{k' v_0^2}{\nu^2}, \quad n = \frac{n' \nu}{v_0^2}, \quad M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \quad Kr = \frac{k_r' \nu}{v_0^2}, \\
 Ec = \frac{k^* v_0^3}{c_p \nu q}
 \end{aligned} \right\} \quad (7)$$

With the help of (6) and (7), equations (2)- (5)

reduces to the following non dimensional form

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt'}) \frac{\partial u}{\partial y} = T Gr + C Gc + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{k}\right) u \quad (8)$$

$$\frac{\partial T}{\partial t} - (1 + \varepsilon A e^{nt'}) \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y}\right)^2 \quad (9)$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{nt'}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - Kr C \quad (10)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned}
 t > 0: u = 0, \quad \frac{\partial T}{\partial y} = -1, \quad \frac{\partial C}{\partial y} = -1 \quad \text{for } y = 0 \\
 u \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{for } y \rightarrow \infty
 \end{aligned} \right\} \quad (11)$$

III.SOLUTION OF THE PROBLEM

The above partial differential equations (8)-(10) are reduced to a system of ordinary differential equations in dimensionless form. In order to reduce this, the velocity, temperature and concentration are taken as follows

$$\left. \begin{aligned}
 u = u_0(y) + \varepsilon e^{nt'} u_1(y) + o(\varepsilon^2) \\
 T = T_0(y) + \varepsilon e^{nt'} T_1(y) + o(\varepsilon^2) \\
 C = C_0(y) + \varepsilon e^{nt'} C_1(y) + o(\varepsilon^2)
 \end{aligned} \right\} \quad (12)$$

Substituting (12) in equations (8)-(10) and equating the harmonic, non-harmonic terms, and also neglecting the ε^2 and higher order terms we get the following equations:

$$u_0'' + u_0' - \left(M + \frac{1}{k}\right) u_0 = -Gr T_0 - Gc C_0 \quad (13)$$

$$u_1'' + u_1' - \left(M + \frac{1}{k} + n\right) u_1 = -Gr T_1 - Gc T_1 - A u_0' \quad (14)$$

$$T_0'' + Pr T_0' = -Ec Pr (u_0')^2 \quad (15)$$

$$T_1'' + Pr T_1' - n Pr T_1 = -2Ec Pr u_0' u_1' - A Pr T_0' \quad (16)$$

$$C_0'' + Sc C_0' - Sc Kr C_0 = 0 \quad (17)$$

$$C_1'' + Sc C_1' - Sc (Kr + n) C_1 = -A Sc C_0' \quad (18)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} u_0=0, u_1=0, T_0'=-1, T_1'=0, C_0'=-1, C_1'=0 \text{ at } y=0 \\ u_0=0, u_1=0, T_0=0, T_1=0, C_0=0, C_1=0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (19)$$

Equations (13) to (18) are coupled and nonlinear.

To solve these equations, power series solution method is employed

$$\left. \begin{aligned} u_0(y) &= u_{01}(y) + Ec u_{02}(y) \\ u_1(y) &= u_{11}(y) + Ec u_{12}(y) \\ T_0(y) &= T_{01}(y) + Ec T_{02}(y) \\ T_1(y) &= T_{11}(y) + Ec T_{12}(y) \\ C_0(y) &= C_{01}(y) + Ec C_{02}(y) \\ C_1(y) &= C_{11}(y) + Ec C_{12}(y) \end{aligned} \right\} \quad (20)$$

Substituting (20) in (13) - (18) and equating the like and unlike terms and neglecting the terms containing Ec^2 and higher order terms, we get

$$u_{01}'' + u_{01}' - \left(M + \frac{1}{k}\right)u_{01} = -GrT_{01} - GcC_{01} \quad (21)$$

$$u_{02}'' + u_{02}' - \left(M + \frac{1}{k}\right)u_{02} = -GrT_{02} - GcC_{02} \quad (22)$$

$$u_{11}'' + u_{11}' - \left(M + \frac{1}{k} + n\right)u_{11} = -GrT_{11} - GcT_{11} - Au_{01}' \quad (23)$$

$$u_{12}'' + u_{12}' - \left(M + \frac{1}{k} + n\right)u_{12} = -GrT_{12} - GcT_{12} - Au_{01}' \quad (24)$$

$$T_{01}'' + PrT_{01}' = 0 \quad (25)$$

$$T_{02}'' + PrT_{02}' = -Pr(u_{01}')^2 \quad (26)$$

$$T_{11}'' + PrT_{11}' - nPrT_{11} = -APrT_{01}' \quad (27)$$

$$T_{12}'' + PrT_{12}' - nPrT_{12} = -2Pr u_{01}' u_{11}' - APrT_{02}' \quad (28)$$

$$C_{01}'' + ScC_{01}' - ScKrC_{01} = 0 \quad (29)$$

$$C_{02}'' + ScC_{02}' - ScKrC_{02} = 0 \quad (30)$$

$$C_{11}'' + ScC_{11}' - Sc(Kr+n)C_{11} = -AScC_{01}' \quad (31)$$

$$C_{12}'' + ScC_{12}' - Sc(Kr+n)C_{12} = -AScC_{02}' \quad (32)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} y=0: \quad u_{01}=0, \quad u_{02}=0, \quad u_{11}=0, \quad u_{12}=0 \\ \quad T_{01}'=-1, T_{02}'=0, \quad T_{11}'=0, \quad T_{12}'=0 \\ \quad C_{01}'=-1, C_{02}'=0, \quad C_{11}'=0, \quad C_{12}'=0 \\ y \rightarrow \infty: \quad u_{01}=0, \quad u_{02}=0, \quad u_{11}=0, \quad u_{12}=0 \\ \quad T_{01}=0, \quad T_{02}=0, \quad T_{11}=0, \quad T_{12}=0 \\ \quad C_{01}=0, \quad C_{02}=0, \quad C_{11}=0, \quad C_{12}=0 \end{aligned} \right\} \quad (33)$$

Solving the above equations from (21) – (32) with respect to the boundary conditions (33) we get the following solutions

$$u_{01} = a_4 e^{a_2 y} + a_5 e^{-a_3 y} + a_6 e^{-Pr y} + a_7 e^{-d_2 y} \quad (34)$$

$$\begin{aligned} u_{02} &= a_{28} e^{a_2 y} + a_{29} e^{-a_3 y} + a_{30} e^{-Pr y} \\ &+ a_{31} e^{-2a_3 y} + a_{32} e^{-2Pr y} + a_{33} e^{-2d_2 y} \\ &+ a_{34} e^{-b_{12} y} + a_{35} e^{-b_{14} y} + a_{36} e^{-b_{16} y} \end{aligned} \quad (35)$$

$$\begin{aligned} u_{11} &= a_{15} e^{a_{13} y} + a_{16} e^{-a_{14} y} + a_{17} e^{-b_4 y} \\ &+ a_{18} e^{-Pr y} + a_{19} e^{-a_3 y} + a_{20} e^{-d_2 y} \end{aligned} \quad (36)$$

$$\begin{aligned} u_{12} &= a_{56} e^{a_{13} y} + a_{57} e^{-a_{14} y} + a_{58} e^{-b_4 y} + a_{59} e^{-b_{26} y} \\ &+ a_{60} e^{-b_{28} y} + a_{61} e^{-b_{30} y} + a_{62} e^{-2a_3 y} + a_{63} e^{-b_{33} y} \\ &+ a_{64} e^{-b_{35} y} + a_{65} e^{-b_{37} y} + a_{66} e^{-2Pr y} + a_{67} e^{-b_{40} y} \\ &+ a_{68} e^{-b_{42} y} + a_{69} e^{-b_{44} y} + a_{70} e^{-2d_2 y} + a_{71} e^{-Pr y} \\ &+ a_{72} e^{-b_{12} y} + a_{73} e^{-b_{14} y} + a_{74} e^{-b_{16} y} + a_{75} e^{-a_3 y} \\ &+ a_{76} e^{-d_2 y} \end{aligned} \quad (37)$$

$$T_{01} = b_1 + b_2 e^{-Pr y} \quad (38)$$

$$\begin{aligned} T_{02} &= b_{17} + b_{18} e^{-Pr y} + b_{19} e^{-2a_3 y} + b_{20} e^{-2Pr y} \\ &+ b_{21} e^{-2d_2 y} + b_{22} e^{-b_{12} y} + b_{23} e^{-b_{14} y} + b_{24} e^{-b_{16} y} \end{aligned} \quad (39)$$

$$T_{11} = b_5 e^{b_3 y} + b_6 e^{-b_4 y} + b_7 e^{-Pr y} \quad (40)$$

$$\begin{aligned} T_{12} &= b_{50} e^{b_3 y} + b_{51} e^{-b_4 y} + b_{52} e^{-b_{26} y} + b_{53} e^{-b_{28} y} \\ &+ b_{54} e^{-b_{30} y} + b_{55} e^{-2a_3 y} + b_{56} e^{-b_{33} y} \\ &+ b_{57} e^{-b_{35} y} + b_{58} e^{-b_{37} y} + b_{59} e^{-2Pr y} \\ &+ b_{60} e^{-b_{40} y} + b_{61} e^{-b_{42} y} + b_{62} e^{-b_{44} y} \\ &+ b_{63} e^{-2d_2 y} + b_{64} e^{-Pr y} + b_{65} e^{-b_{12} y} \\ &+ b_{66} e^{-b_{14} y} + b_{67} e^{-b_{16} y} \end{aligned} \quad (41)$$

$$C_{01} = d_3 e^{d_1 y} + d_4 e^{-d_2 y} \quad (42)$$

$$C_{02} = d_{15}e^{d_{3,y}} + d_{16}e^{-d_{14,y}} \quad (43)$$

$$C_{11} = d_9e^{d_{7,y}} + d_{10}e^{-d_{8,y}} + d_{11}e^{-d_{2,y}} \quad (44)$$

$$C_{12} = d_{20}e^{d_{18,y}} + d_{21}e^{-d_{19,y}} \quad (45)$$

Substituting the above solutions in equations (34)–(45) and then the resulting expressions in eqn. (20) and then finally from equation (12), we get the expressions for velocity, temperature and concentration.

IV. RESULTS AND DISCUSSIONS

The problem of unsteady, two dimensional, laminar natural convective MHD flow over an infinite vertical plate in the presence of magnetic field through the porous medium under the influence of chemical reaction subject to uniform heat and mass flux has been formulated, analysed and solved by using perturbation technique. The effects of the flow parameters such as magnetic parameter (M), Grashof number for heat and mass transfer (Gr, Gc), Schmidt number (Sc), Chemical reaction parameter (Kr), Prandtl number (Pr), viscous dissipation parameter or Eckert number (Ec) and permeability parameter (k) on the velocity, temperature and concentration profiles of the flow field are presented with help of graphs.

Fig 1 depicts the velocity profiles (u) against y for different values of chemical reaction parameter Kr. From this figure we observe that as chemical reaction parameter (Kr) increases velocity decreases. It is noticed that the fluid velocity increases quickly up to some thin layer of the fluid adjacent to the plate and then decreases asymptotically as we move further away from the plate.

Fig.2 shows that the velocity profiles (u) against y for different values of Eckert number (Ec). From

this figure it is evident that as Eckert number increases velocity increases.

Fig.3 illustrate that the velocity profiles (u) against y for different values of Gr and Gc. From this figure we observe that as Gr and Gc increases velocity increases.

Fig.4 depicts the velocity profiles (u) against y for different values of Magnetic parameter (M). From this figure we observe that as Magnetic parameter increases velocity decreases due to application of transverse magnetic field will result a restrictive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity.

Fig.5 shows that the velocity profiles (u) against y for different values of Prandtl number (Pr). From this figure we observe that as Prandtl number increases velocity decreases. Also observe that for smaller values of Prandtl number the dispersion in the velocity profiles is small whereas significantly high for higher values of Pr.

Fig.6 exhibits the velocity profiles (u) against y for different values of permeability parameter (k). From this figure we observe that as Permeability parameter increases velocity increases.

Fig.7 depicts the velocity profiles (u) against y for different values of Schmidt number (Sc). From this figure we observe that as Schmidt number increases velocity decreases.

Fig.8 shows that the temperature profiles (T) against y for different values of Prandtl number (Pr). From this figure we observe that as Prandtl number increases velocity decreases.

Fig.9 illustrate that the temperature profiles (T) against y for different values of Eckert number (Ec). From this figure we observe that as Eckert number (Ec) increases temperature increases

Fig 10 depicts the concentration profiles (C) against y for different values of chemical reaction parameter Kr. From this figure we observe that as

GRAPHS

chemical reaction parameter (Kr) increases concentration decreases. The concentration level of the fluid falls due to increasing chemical reaction.

i.e., the consumption of chemical species leads to fall in the species concentration field.

Fig 11 portrays the concentration profiles (C) against y for different values of Schmidt number (Sc). From this figure we observe that as Schmidt number increases concentration decreases.

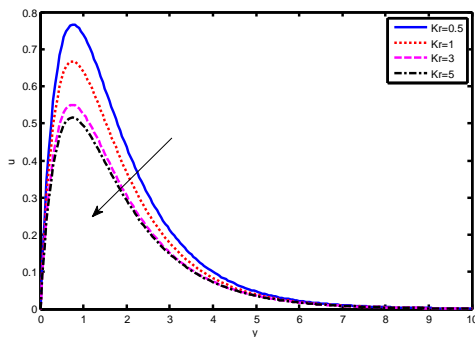


Fig. 1 – Velocity profiles for different values of chemical reaction parameter (Kr)

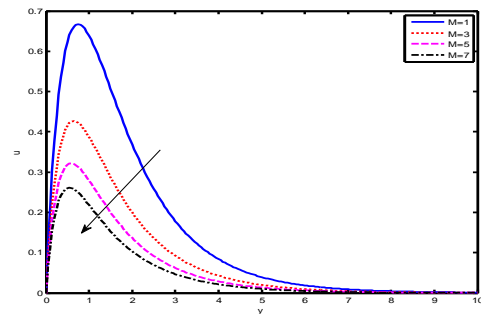


Fig. 4 – Velocity profiles for different values of magnetic parameter (M)

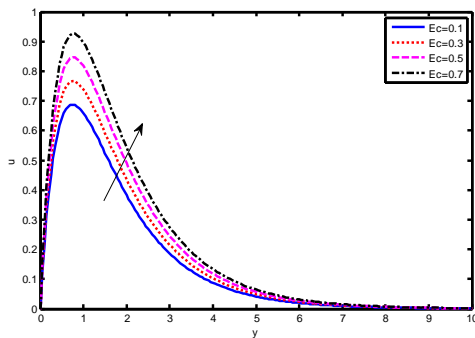


Fig. 2 – Velocity profiles for different values of Eckert number (Ec),

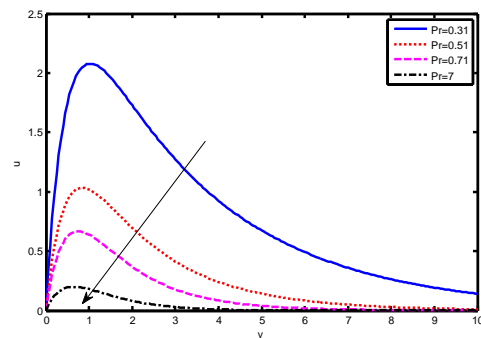


Fig. 5 – Velocity profiles for different values of Prandtl number (Pr)

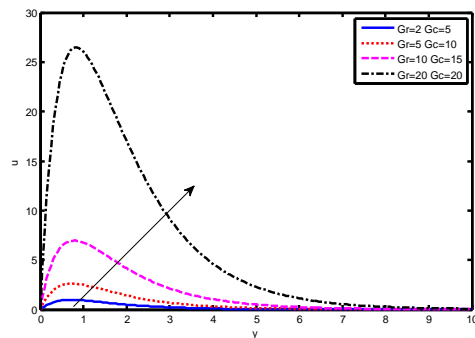


Fig. 3 – Velocity profiles for different values of Grashof number for heat and mass (Gr & Gc)

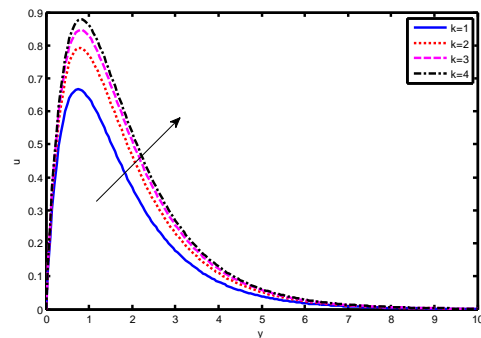


Fig. 6 – Velocity profiles for different values of permeability parameter (k)

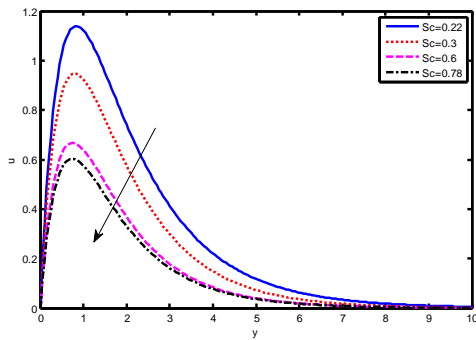


Fig. 7 – Velocity profiles for different values of Schmidt number (Sc)

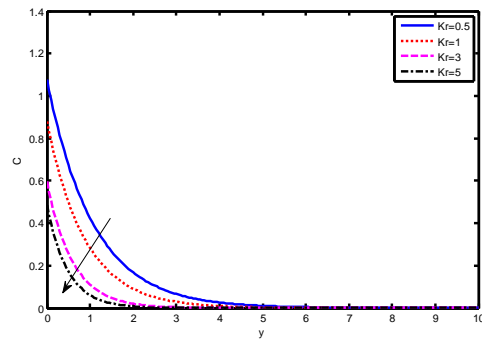


Fig. 10 – Concentration profiles for different values of chemical reaction parameter (Kr)

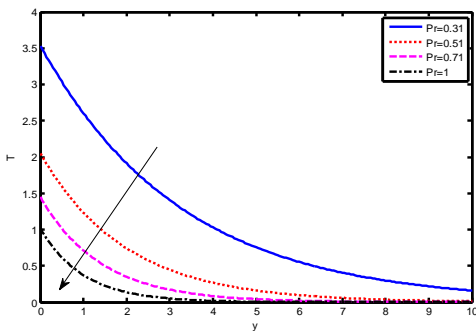


Fig. 8 – Temperature profiles for different values of Prandtl number (Pr)

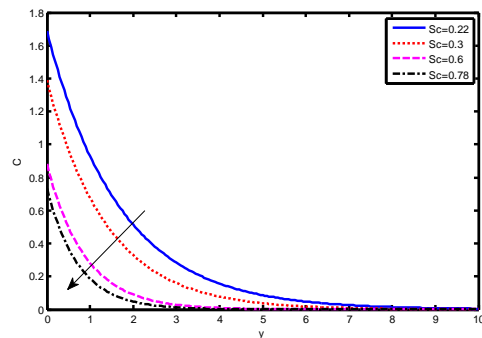


Fig. 11 – Concentration profiles for different values of Schmidt number (Sc)

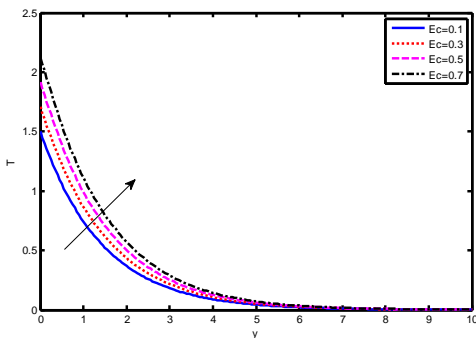


Fig. 9 – Temperature profiles for different values of Eckert number (Ec)

REFERENCES

1. E.M. Sparrow, R.D. Cess., The effect of a magnetic field on free convection heat transfer, *Int. J. Heat Mass Transfer*, vol.3, pp: 267-274, 1961.
2. V.M. Soundalgekar, Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction, *Int. J. Heat Mass transfer*, vol.15, pp. 1253-1261, 1972.
3. A.A. Raptis, Unsteady free convective flow through a porous medium, *Int. J. Heat Mass transfer*, vol.21 (4), pp. 345-348, 1983.
4. R.M. Lahurikar, S.G. Pohanerkar, M.D. Jahagirdar, Unsteady forced and free convective flow past an infinite vertical plate through a porous medium, *Indian J. Pure Appl. Math.*, vol. 21(4), pp. 396-402, 1990.
5. S.P. Anjalidevi, and Kandasamy R, Effects of a chemical reaction heat and mass transfer on MHD flow past a semi-infinite plate. *Z. Angew. Math. Mech.*, vol.80, pp. 697-701, 2000.
6. S.Sreekanth, A.S. Nagarajan, S.Venkata Ramana, Transient MHD free convection flow of an incompressible viscous dissipative fluid, *Indian J. Pure Appl. Math.*, vol.32 (7), pp. 1051-1058, 2001.

7. V.M. Soundalgekar, A.G. Uplekar, B.S. Jaiswal, Transient free convection flow of dissipative fluid past an infinite vertical porous plate, *Arch.Mech*, vol.56 (1), pp. 7-17, 2004.
8. Ahmed,S, Effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate, *Bull. Cal. Math. Soc.*, vol. 99 (5), pp. 511-522, 2007.
9. K. Rajasekhar, G. V. Ramana Reddy, B. D. C. N. Prasad, Unsteady MHD free convective flow past a semi-infinite vertical porous plate, *International Journal of Modern Engineering Research*, vol.2(5), pp.3123-3127, 2012.
10. Murali Gundaganian, , Sivaiah Sheria , Ajit Paul , M. C. K. Reddy, Unsteady Magnetohydrodynamic Free Convective Flow Past a Vertical Porous Plate, *International Journal of Applied Science and Engineering*, vol.11 (3), pp. 267-275, 2013.
11. S Mohammed Ibrahim, Unsteady MHD Free Convective Flow Along A Vertical Porous Plate Embedded In A Porous Medium With Heat Generation, Variable Suction And Chemical Reaction Effects, *Chemical and Process Engineering Research*, vol.21, 2014.
12. P. Loganathan and C. Sivapoornapriya, Viscous dissipation effects on unsteady natural convective flow past an infinite vertical plate with uniform heat and mass flux, *Wseas transactions on Heat and Mass transfer*, vol. 9, pp. 63-73, 2014.
13. S. F. Ahmmed, M. K. Das, L. E. Ali, Analytical Study on Unsteady MHD Free Convection and Mass Transfer Flow Past a Vertical Porous Plate, *American Journal of Applied Mathematics*, vol. 3(2), pp. 64-74, 2015.