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Phasor and frequency estimation in Phasor Measurement unit (PMU) using least square technique

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***Abstract*—In synchronized wide area measurements (WAMs), phasor measurement unit (PMU) is one of the technologies which measure instantaneous voltage, current, phase, frequency and rate of change of frequency of voltage & current signals. The paper describes least square phasor estimation technique for phasor as well as frequency measurement of voltage signal. As voltage & current signal is rarely free from noise, it may contribute to an error in phasor estimate. In the paper effect of electrical noise which can be picked up in wiring of input signal i.e. zero mean Gaussian noise has been considered. Least square estimation technique has been used in this paper to reduce the effect of noise. Frequency of the input voltage signal has also been estimated using least square estimation over reasonable data window.**

***Index Terms*— Gaussian noise, Least square, Phasor Measurement unit (PMU),Synchrophasor, Simulink.**

I. Introduction

PMU is the power quality monitor for electrical power grid which measures instantaneous voltage and current phasors in the transmission line. Measurements from a single PMU cannot be used to detect many errors in the power grid, thus different PMUs are installed across power grid and are synchronized using Global Positioning System (GPS). GPS source provides the required synchronization by time stamping the sampled signal within 1 microsecond accuracy [1]. Synchrophasor obtained from the PMUs can be used in detection of oscillations, and monitoring of voltage and frequency thus blackout conditions can be avoided [2]. PMUs provides wide-area grid visibility in real time and helps in power system monitoring, protection and control [3].

Various analogue measurements made by PMUs include signal magnitude, phase, frequency, rate of change of frequency (ROCOF), power and also digital measurements like circuit breaker status[4- 5]. Data from different PMUs is sent using high speed communication channels to a centralized location where Phasor data concentrator (PDC) is located. Various phasor estimation techniques based on Kalman filtering, Discrete Fourier Transform (DFT), Recursive Wavelet Transform (RWT), orthogonalfilters [6-9] are available in the literature. The noise present in the input signal often corrupt the phasor estimate. The noise source in the input signal can be harmonic and non-harmonic frequency components, electromagnetic interference, induced electrical noise or quantization errors. A zero mean Gaussian distribution characteristic can be approximated for the noise if the phasing of harmonic & non harmonic component is assumed to be random [4].

In the paper Least square estimation has been used to estimate phasor of the voltage signal in presence of zero mean gaussian noise that can be picked up in wiring of the signal. And thus least square estimation can be used for phasor estimation to minimize the error due to noise. For the optimum operation of power system, frequency is an important parameter. For understanding the dynamic behavior of the system instantaneous frequency measurements are required. Least square estimation algorithm is discussed in the paper to reduce the effect of random errors.

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II. BASIC PMU BLOCK

The basic block for PMU implemented in MATLAB SIMULINK is shown in Fig1. The block consists of the following units:

- Anti-aliasing filter: Input voltage signal is filtered using anti-aliasing low pass filter (LPF) before sampling to satisfy the Nyquist criteria and to approximately limit the bandwidth of a signal to about half of the sampling frequency. Thus the cut-off frequency of the analog LPF should be less than half the sampling frequency [10],[11]. Butterworth filter is chosen as low pass anti-aliasing filter because of its maximally flat response in passband region and smoother in stop band regions, and also it provides a more linear phase response[11]. In the Simulink modelbutterworth filter of 6 th order with cutoff frequency 750Hz is implemented and the sampling frequency is set as fs =1.6 KHz.
- Analog to digital converter (ADC): It is used to sample the analog input for further processing in the measurement unit. Sampling clock provides sampling rate to digitalize input data. The sampling frequency fsis fixed as N times the nominal fundamental frequency fo in the paper. The ADC implemented in the Simulink model consists of a sample and hold unit and a quantizer. A pulse generator unit has been used to provide sampling clock for the ADC unit
- Measurement unit: The measurement unit receives the sampled data and use phasor and frequency estimation algorithms to calculate RMS voltage magnitude, phase angle & frequency of input voltage signal. Least square technique has been implemented for phasor and frequency

measurement in this unit.

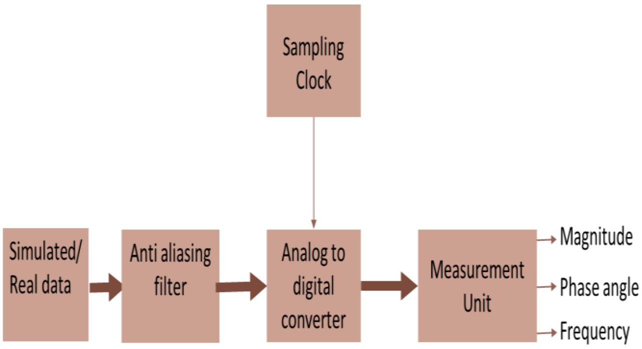


Figure 1Basic block of Phasor measurement unit

III. ESTIMATION TECHNIQUES

A. Least Square Phasor Estimation Algorithm

Consider a single phase voltage signal corrupted by Gaussian noise$\varepsilon(t)$:

x(t) = Xm cos(ωt + φ) + $\varepsilon(t)$(1)

And x(t) is uniformly sampled at N times per cycle of the signal to obtain:

x_n= Xm cos(2πn/N + φ) +ε_n (2)

Where Xm is the peak voltage magnitude and ω is the nominal frequency in radians,φ is the phase angle&$\varepsilon_n$is a zero-mean Gaussian noise process with a variance of σ^2.

Now phasor of the signal is:

$X = \frac{X_m}{2} e^{j\varphi} = X_r + jX_i$

x_n = Xm cos(φ)cos($n\theta$) - Xm sin(φ)sin($n\theta$); where $\theta = \frac{2\pi}{N}$

x_n= $X_r$cos($n\theta$) - $X_i$sin($n\theta$) (3)

The unknown phasor can be estimate from the sampled data using data window of M samples:

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ . \\ . \\ x_{M-1} \end{bmatrix} = \sqrt{2} \begin{bmatrix} \cos[0] & -\sin[0] \\ \cos[\theta] & -\sin[1\theta] \\ \cos[2\theta] & -\sin[2\theta] \\ . & . \\ . & . \\ \sin[(M-1)\theta] & -\sin[(M-1)\theta] \end{bmatrix} \begin{bmatrix} X_r \\ X_j \end{bmatrix} + \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_{M-1} \end{bmatrix}$$

(4)

In matrix notation:

[x]= [B][X]+[ε] (5)

Coefficients [X] are obtained by least square solution using the left pseudo-inverse of [B], which minimizes the sum $[[\varepsilon]^T[\varepsilon]]$ i.e. $Q = \sum_{n=0}^{N-1} {\varepsilon_n}^2$. The covariance matrix of error vector is assumed to be:$[W] = \sigma^2[\mathrm{I}]$.Thus the least square solution of equation (4) to provide estimate of phasor is:

$$[\widehat{X}] = [B^T W^{-1} B]^{-1} [B^T W^{-1}][x] = [B^T B]^{-1} [B^T][x] (6)$$

Unlike discrete Fourier transform one of the advantage of the least square technique is that it can be used for calculating phasors from fractional cycle data window which are often used in developing high speed relaying applications [4]. If M samples are used for estimating phasor of sinusoid input signal sampled at rate of N samples per cycle such that M is less than N then $[B^T B]$ does not forms a simple matrix and thereby increasing the computational burden.

$$[B^T B] = 2 \begin{bmatrix} \sum_{n=0}^{M-1} cos^2(n\theta) & \sum_{n=0}^{M-1} \cos(n\theta)\sin(n\theta) \\ \sum_{n=0}^{M-1} \cos(n\theta)\sin(n\theta) & \sum_{n=0}^{M-1} sin^2(n\theta) \end{bmatrix}$$

For phasor estimation using full cycle data window, M=N

$$[B^T B] = 2 \begin{bmatrix} N/2 & 0 \\ 0 & N/2 \end{bmatrix}$$

$$[\widehat{X}] = \frac{1}{N}[B^T][x] \qquad (7)$$

Thus estimated phasor can be obtained using equation (7).

The covariance matrix of error in phasor estimate is given by:

$$E[\,[\hat{X} - X][\hat{X} - X]^T] = [B^T W^{-1} B]^{-1} = \sigma^2/N \qquad (8)$$

The standard deviations of error in phasor estimate is thus equal to $\sigma/\sqrt{N}$ .Thus higher is the sampling rate or longer data windows are used lesser is the error in the phasor estimate. The phasor angle estimated above is not constant in time. Under steady state conditions, the estimated phasor angle changes at a fixed rate thus making one rotation every power system cycle. The phasor angle increases by $2\pi/N$radians with every sample. For analysis it is useful that the phase angle remains constant under steady state condition thus we may need to normalize the phase angle using some reference and thus reference $e^{-j\omega n}$ has been usedwhere $\omega = 2\pi/N$. It makes one rotation in every power system cycle in opposite direction than the phasor estimated. Each sample of the estimated phasors obtained in above method are multiplied by this reference phasor to obtain normalized phasor.

B. Least Square Frequency Estimation Algorithm

The phase angles computed in the above section are used to estimate change of frequency (COF) & thus the frequency.Frequency is defined as speed of rotation of the phasor [7].

The phase angle at any instant [6],[12] can be represented as:

$$\varphi(t) = \int \omega(t) = \varphi_0 + \Delta\omega t + t^2 \frac{\omega'}{2} \qquad (9)$$

Whereω(t) = frequency in radians, $\Delta\omega$ = change in frequency, and ω' represents the rate of change in frequency. With Tsbeing the sampling time then phase angle measurements of vector length N can be given by

$$\begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ . \\ . \\ \varphi_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & T_s & {T_S}^2 \\ 1 & 2T_S & 2^2{T_S}^2 \\ . & . & . \\ . & . & . \\ 1 & (N-1)T_S & (N-1)^2{T_S}^2 \end{bmatrix} \begin{bmatrix} \varphi_0 \\ \Delta\omega \\ \omega' \end{bmatrix} + \begin{bmatrix} \varepsilon_{\varphi 0} \\ \varepsilon_{\varphi 1} \\ \varepsilon_{\varphi 2} \\ . \\ . \\ \varepsilon_{\varphi N-1} \end{bmatrix}$$

(10)

The vector [φ] is assumed to be monotonically changing over the window of "N" sample.

In matrix notation: $[\varphi] = [A][F]$ +[ε] (11)

coefficients [F] are obtained by least square solution using the left pseudo-inverse of [A], which minimizes the sum $[[\varepsilon]^T[\varepsilon]]$

$[F] = [A^T A]^{-1}[A^T][\varphi]$ (12)

COF (Δf) =$\frac{\Delta\omega}{2\pi}$and thus frequency can be obtained as:

$f = f_0 + \Delta f$;
Where f_0 is the nominal fundamental frequency.

C. TOTAL VECTOR ERROR (TVE)

Total vector error is used as a tool to estimate the accuracy of the phasor measurement unit. It combines the magnitude and phase angle errors. TVE compares the estimated phasor with the theoretical value of phasor.
The Total Vector Error (TVE) [13] at any time instant k is:

$$TVE(k) = \frac{|X_{estimate}\ (k) - X_{theoretical\ (k)}|}{|X_{theoretical}\ (k)|} X100$$

As per the IEEE standard C37.118.1-2011 the TVE must be less than 1%[14].

IV. SIMULATION RESULTS

In the following simulation measured input voltage signal is set as:

$x(t) = A_m \cos(2\pi f_0 + \varphi) + \varepsilon$;

Where $A_m = 220\sqrt{2}\ V$, f_0=50Hz, $\varphi = \pi/3$

The given signal has been sampled at frequency(f_s) of 1.6 KHz using number of samples per cycle (N) of 32 for phasor estimate. The signal is corrupted with zero mean gaussian noise of standard deviation (σ) equal to 0.9.

The following figures 2-3 show voltage magnitude and phase angle computed using least square estimation technique at nominal frequency.

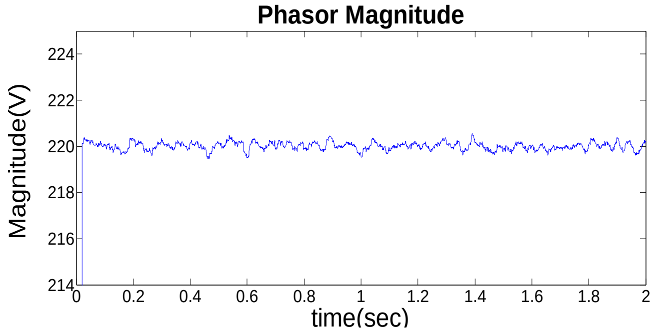


Figure 2 Voltage phasor magnitude

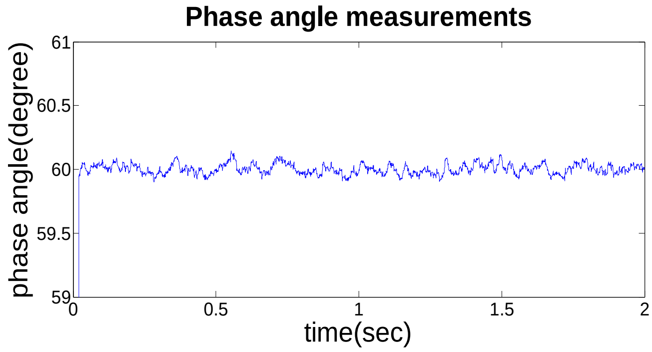


Figure 3 Voltage phase angle

The zero mean Gaussian noise considered in the paper affects voltage magnitude and phase angle computed using least square estimation technique as can be seen from Figures 4- 5. The maximum absolute error in computation of voltage magnitude& phase angle is found to be 0.5478 & 0.002486 respectively.

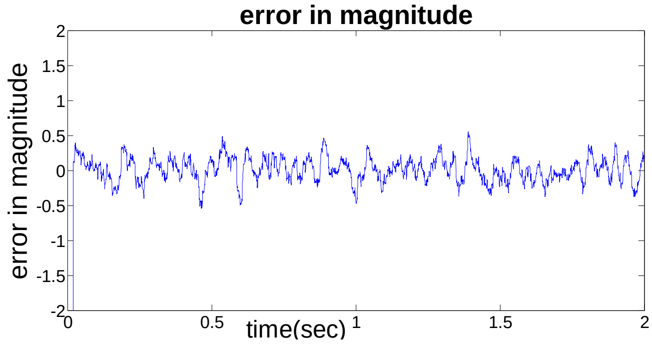


Figure 4 Error in Magnitude

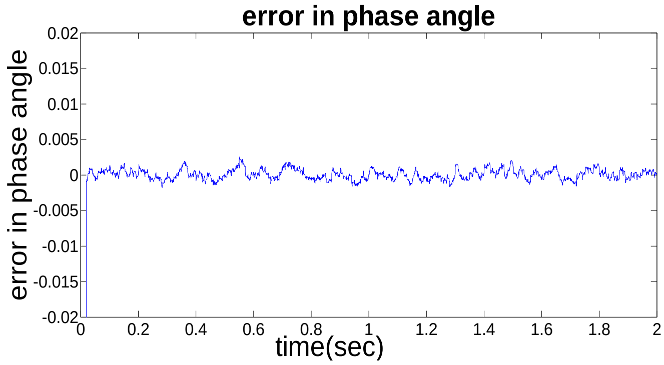


Figure 5 Error in phase angle

The frequency has been computed using phase angle estimates of signal obtained in fig 3. Figure 6 shows frequency estimation using least square technique.

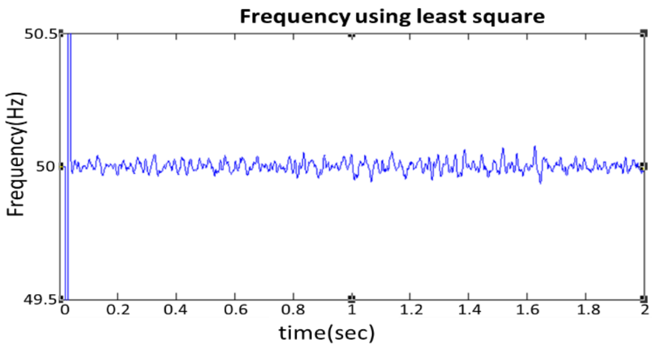


Figure 6 Frequency Measurement

TVE for the phasor estimate has been shown in figure 7. The TVE is found to be less than 0.2891%.

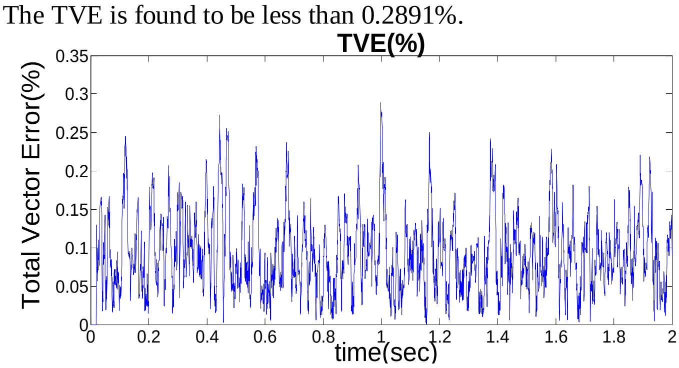
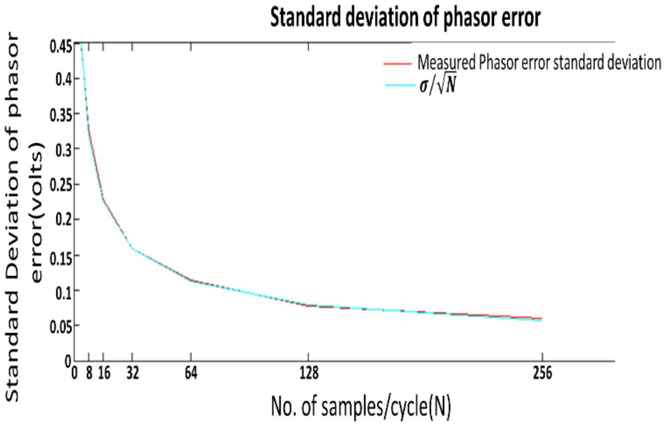


Figure 7 Total Vector Error

Figure 8 shows standard deviation of phasor estimate error computed for various number of samples/cycle (N) such as 8, 16, 32, 64, 128 etc. It has been observed that with increasing the number of samples per data window the standard deviation of phasor estimate error decreases.



V.CONCLUSION

It can be concluded that Least square estimation technique has been used to estimate phasor and frequency of the voltage signal at nominal frequency. Least square technique for frequency estimation suppresses the effect of random zero mean gaussian noise and thus gives better performance in the paper. It can also be concluded that the least square method for phasor estimation may be used for fractional or integer cycle window. It can be observed that with increasing the number of samples per data window, phasor estimate has improved.

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