Fuzzy Optimal Replenishment Policy for Weibull Deteriorating Items with Ramp Type Demand and Partial Backlogging Under Permissible Delay in Payments

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Abstract -- **In this paper, we developed a fuzzy optimal replenishment policy for Inventory models of Weibull Deterioration items with Ramp type Demand under permissible delay in payments. Deterioration of items begins on their arrival in stock. The shortages are allowed and partially backlogged. All the inventory costs involved in this model are taken as triangular fuzzy numbers. Replenishment cycle length, the time at which shortage begins are taken as decision variables. Graded mean integration representation method is used to defuzzify the model. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.**

Keywords-- **Fuzzy numbers and fuzzy concepts, partial backlogging, Ramp Type Demand, Weibull distribution of two parameters**.

I. INTRODUCTION

The inventory system is taking an important part of cost controlling in business. For the last few years, researchers in this area have extended investigation into various models with considerations of item shortage, item deterioration, demand patterns, item order cycles and their combinations. Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods undergo decay or deterioration over time.

Shortages are classified either completely backlogged shortages or partially backlogged shortages. Completely backlogged shortages are the shortages which are duly fulfilled by the vendor during the shortage period. Sometimes the shortages may not be completely backlogged and only a part of the demand can be met by the vendor during the shortage period, which is termed as partially backlogged. In some inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate should be variable and depends on the waiting time for the next replenishment. Chang and Dye [1] developed a model for deteriorating items with time varying demand and shortages in which the backlogging rate is assumed to be inversely proportional to the waiting time for the next replenishment. Skouri and Papachristos [2] studied a multi period inventory model using the exponentially decreasing backlogging rate proposed by Abad. Recently many researchers have modified inventory policies by considering the time proportional partial backlogging rate such as Wang [3], Teng and Yang [4], San Jose et al. [5] and Wu et al. [6]

The demands of fashionable goods increase up to the certain level and after that the demand becomes steady. Such type of demand functions are known as ramp type of demand. We know that demand is not always a monotonic function (i.e., items like fashionable goods) over a planning period and after a particular time it becomes steady. Hill [7] resolved the indiscipline of time dependent demand pattern by considering the demand as the combination of two different types of disciplined demand in two successive time periods over the entire time horizon and termed it as ramp type time dependent demand pattern. Wu et al. [8] developed an EOQ model with ramp type demand rate for items with Weibull deterioration. Mandal and Pal [9] developed an order-level inventory model for deteriorating items with ramp type demand. Wu and Ouyang [10] extended their model by incorporating the concept of shortages followed by

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inventory. Singh et al. [11] developed an EOQ inventory model with Weibull distribution deterioration, ramp type demand and partial backlogging. Giri et al. [12] developed an economic order quantity model with Weibull deterioration distribution, shortages and ramp type demand.

The suppliers offer delay in payment to the retailers to buy more items and the retailers can sell the item before the closing of the delay time. As a result, the retailers sell the items and earn interests. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. This provides opportunities to the retailers to accumulate revenue and earn interest by selling their items during the delay period. This permissible delay in payment provides benefit to the supplier by attracting new customers who consider it to be a type of price reduction and reduction in sells outstanding as some customers make payments on time in order to take advantage of permissible delay more frequently. In this direction, Goyal [13] extended the EOQ model under the conditions of permissible delay in payments. Chung – Yuan Dye [14] developed an inventory model for deteriorating items with stock dependent demand and partial backlogging under conditions of permissible delay in payments. R.Uthayakumar et al [15] have developed a deterministic inventory model with stock and time dependent demand under permissible delay in payments.

In the crisp inventory models, all the parameters in the total cost are known and have definite values. But in the practical situation it is not possible. Hence fuzzy inventory models fulfil that gap. Different fuzzy inventory models occur due to fuzzy various cost parameters in the total cost. Researchers related to this area are: Zimmermann [16] and Kaufmann and Gupta [17] , Kacprzyk and Staniewski [18] , Bellman and Zadeh [19] , Lee and Yao [20] , Yao and Su [21] , Chen and Ouyang [22] , Mahata and Goswamy [23], Vijayan and Kumaran [24] , etc.

In this paper, an optimal replenishment policy for inventory model of deteriorating items with ramp type demand under permissible delay in payments has been developed. Two parameters Weibull distribution deterioration rate has been taken in this study. Shortages are allowed with partial backlogging. Backlogging rate is an exponential decreasing function of time. The inventory costs are taken as triangular fuzzy number. Graded mean Integration representation method is applied for defuzzification. The proposed model is illustrated with numerical examples.

II. FUZZY PRELIMINARIES

Definition 1: Let X denotes a universal set. Then the fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], to each element x∈ X where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x

Definition 2: A fuzzy set $\tilde{A}$ on R is convex if $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right]$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

Definition 3: A fuzzy set $\tilde{A}$ in the universe of discourse X is called as a fuzzy number in the universe of discourse X Triangular fuzzy number

We consider the situation where fuzzy numbers are represented by triangular membership functions. The fuzzy number $\tilde{A}$ is said to be triangular fuzzy number if it is fully determined by (a_1, a_2, a_3) of crisp numbers such that $(a_1 < a_2 < a_3)$ whose membership function, representing triangle, can be denoted by

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

The Function Principle

The function principle was introduced by Chen to treat fuzzy arithmetical operations. This principle is used for the operation for addition, subtraction, multiplication and division of fuzzy numbers. Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) Addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

where a_1,a_2,a_3,b_1,b_2,b_3 are any real numbers.

(ii)Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_3, a_1 - b_2, a_3 - b_1)$

where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) Multiplication: $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3$

Where a_1,a_2,a_3,b_1,b_2,b_3 are all non zero positive real numbers

(iv) Division: $\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$ where b_1, b_2, b_3 are all non zero positive real numbers

(v) Scalar Multiplication: For any real number K,

$$K\tilde{A} = (Ka_1, Ka_2, Ka_3), K \geq 0$$

$$K\tilde{A} = (Ka_3, Ka_2, Ka_1), K < 0$$

Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3)$ is triangular fuzzy number then the graded mean integration representation of $\tilde{A}$ is given by $P(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$

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III. NOTATIONS AND ASSUMPTIONS

The proposed inventory model having following n otations and assumptions:

3.1 Notations

$\tilde{C}$ = Fuzzy unit purchase cost in \$.

$\tilde{C}_1$ =Fuzzy inventory holding cost per unit per time unit excluding interest charges in \$.

$\tilde{C}_2$ = Fuzzy replenishment cost per cycle in \$.

$\tilde{C}_3$ = Fuzzy shortage cost \$, per unit per year.

$\tilde{l}$ = Fuzzy opportunity cost due to last sales, \$ per unit.

$\tilde{I}_e$ = Fuzzy interest which can be earned, \$ per unit time.

$\tilde{I}_c$ = Fuzzy interest charges payable per \$ per time unit $\left(\tilde{I}_c > \tilde{I}_e\right)$

I = inventory holding charges per \$ unit per year.

T = The length of the order cycle .

t_1 = Time at which shortage starts, $0 \le t_1 \le T$

M = permissible delay period for settling accounts in time units.

μ = The life time of the items per cycle.

$Q(t)$ = The inventory level at time $t \in [0, T]$

S = Initial inventory level of each ordering cycle.

Q = The order size per cycle.

$T\tilde{C}_1$ = Fuzzy total inventory cost when $0 < M \le \mu$

$T\tilde{C}_2$ =Fuzzy total inventory cost when $\mu < M \le t_1$

$T\tilde{C}_3$ = Fuzzy Total Inventory cost when $M > t_1$

3.2 Assumptions

1. The lead-time is zero.
2. The time-horizon of the system is infinite.
3. There is no repair or replacement of the deteriorated inventory during a given cycle.
4. Ramp-type demand rate *f(t)* is given by

$$f(t) = D_0\left[t - (t-\mu)H(t-\mu)\right] \ ;D_0 > 0$$

Where $H(t-\mu) = \begin{cases} 0 \ for \ t < \mu \\ 1 \ for \ t \geq \mu \end{cases}$ is well known Heaviside's unit function

5. During the fixed credit period *M*, the unit cost of generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day-to-day expenses of the system. At the end of the credit period, the account is settled and interest charges are payable on the account in stock.

6. The deterioration of time as follows by Weibull parameters (two) distribution $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ is the scale parameter and $\beta > 0$ is the shape parameter.

7. Shortages are allowed and unsatisfied demand is backlogged at a rate $e^{-\delta t}$, where the backlogging parameters δ is a positive constant.

IV. MODEL FORMULATION

The inventory system is developed as follows: Q units of items arrive at the inventory system at the beginning of each cycle. During the time interval [0, μ], the inventory level is decreasing only due to demand rate. The inventory level is dropping to zero owing to demand and deterioration during the time interval [μ , t_1]. Finally, a shortage occurs due to demand and partial backlogging during the time interval [t_1, T]. The behaviour of the inventory model is demonstrated in Fig.1.

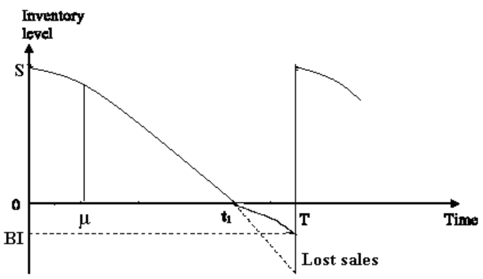

Fig.1 Graphical representation of the inventory system

Let Q (t) denote the on hand inventory of the system at any time t, $(0 \le t \le T)$ Depletion due to demand and deterioration will occur simultaneously. The differential equation that describes the instantaneous state of Q (t) is given by

$$\frac{dQ}{dt} + \alpha\beta t^{\beta-1}Q(t) = -D_0 t \ ; \ 0 \le t \le \mu \qquad \text{----- (1)}$$

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$$\frac{dQ}{dt}+\alpha\beta t^{\beta-1}Q(t)=-D_0\mu;\ \mu\le t\le t_1 \qquad \text{------ (2)}$$

$$\frac{dQ}{dt}=-D_0e^{-\delta t}\ ;\ t_1\le t\le T \qquad \text{----- (3)}$$

with the initial and boundary conditions

$Q(0) = S$ and $Q(t_1) = 0$ ------ (4)

where S is the initial inventory .The solutions of equations (1), (2) and (3) subject to the conditions (4) are respectively

$$Q(t)=Se^{-\alpha t^{\beta}}-D_0e^{-\alpha t^{\beta}}\left[\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}\right];$$
$$0\le t\le\mu \qquad \text{------- (5)}$$

$$Q(t)=D_0\mu e^{-\alpha t^{\beta}}\left[t_1-t+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)\right]$$
$$;\quad \mu\le t\le t_1 \qquad \text{-------- (6)}$$

$$Q(t)=\frac{D_0}{\delta}\left[e^{-\delta t}-e^{-\delta t_1}\right]\ ;\ t_1\le t\le T \qquad \text{----- (7)}$$

Substituting $t=\mu$ in equations (5) and (6) and then equating , we get

$$S=D_0\left[t_1\mu-\frac{\mu^2}{2}+\frac{\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha\mu}{\beta+1}\left(t_1^{\beta+1}-\mu^{\beta+1}\right)\right] \qquad \text{-------- (8)}$$

The maximum backlogged inventory (BI) is obtained at t = T, then from (7)

$$BI=-Q(T)=-\frac{D_0}{\delta}\left[e^{-\delta T}-e^{-\delta t_1}\right] \qquad \text{------ (9)}$$

Thus the order size during total time interval $[0,T]$ is $Q=S+BI$

$$Q=D_0\left[\begin{array}{l}t_1\mu-\frac{\mu^2}{2}+\frac{\alpha\mu^{\beta+2}}{\beta+2}+\\ \frac{\alpha\mu}{\beta+1}\left(t_1^{\beta+1}-\mu^{\beta+1}\right)\end{array}\right] \qquad \text{------ (10)}$$
$$-\frac{D_0}{\delta}\left[e^{-\delta T}-e^{-\delta t_1}\right]$$

(i) Total inventory fuzzy holding cost during the period $[0,T]$ is given by

$$H\tilde{C}=\tilde{C}_1\int_0^{t_1}Q(t)dt$$
$$=\tilde{C}_1\int_0^{\mu}Q(t)dt+\tilde{C}_1\int_{\mu}^{t_1}Q(t)dt$$

$$=\tilde{C}_1D_0\mu\left\{\begin{array}{l}\frac{t_1^2}{2}-\frac{\mu^2}{6}-\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}+\frac{\alpha t_1^{\beta+2}}{\beta+2}\\ +\frac{\alpha\mu^{\beta+2}}{2(\beta+3)}+\\ \frac{\alpha^2}{(\beta+1)^2}\left[\frac{\mu^{2(\beta+1)}}{2}-\frac{t_1^{2(\beta+1)}}{2}\right]\\ -\frac{\alpha}{(\beta+1)(\beta+2)}\left[\alpha\mu^{2(\beta+1)}+t_1^{\beta+2}-\mu^{\beta+2}\right]\\ -\frac{\alpha\mu^{\beta+2}}{(\beta+2)(\beta+3)}+\frac{\alpha^2\mu^{2(\beta+1)}}{(\beta+2)(2\beta+3)}\end{array}\right\}$$

------------- (11)

(ii) Number of units D(T) that deteriorate during the period $[0,T]$ is given by

$$D(t)=Q_0-\left[\int_0^{\mu}D_0t\,dt+\int_{\mu}^{t_1}D_0\mu\,dt\right]$$

$$=D_0\left\{\frac{\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha}{\beta+1}\left(\mu t_1^{\beta+1}-\mu^{\beta+2}\right)\right\}$$

The fuzzy cost due to deterioration during the period $[0,T]=\tilde{C}D(t)$

$$D\tilde{C}=\tilde{C}D_0\left\{\frac{\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha}{\beta+1}\left(\mu t_1^{\beta+1}-\mu^{\beta+2}\right)\right\}$$

-------------- (12)

(iii) Fuzzy replenishment cost per cycle = $\tilde{C}_2$

------------- (13)

(iv) The fuzzy shortage cost during the period $[0,T]$ is given by

$$S\tilde{C}=-\tilde{C}_3\int_{t_1}^{T}Q(t)dt$$
$$=-\tilde{C}_3D_0t_1(T-t_1) \qquad \text{-------- (14)}$$

(v) The fuzzy opportunity cost due to lost sales during the period $[0,T]$ is given by

$$L\tilde{C}=\tilde{l}\int_{t_1}^{T}D_0(1-e^{-\delta t})dt$$
$$=\tilde{l}D_0\left[T-t_1+\frac{1}{\delta}(e^{-\delta T}-e^{-\delta t_1})\right] \qquad \text{------ (15)}$$

Case (i) $(0<M\le\mu)$:

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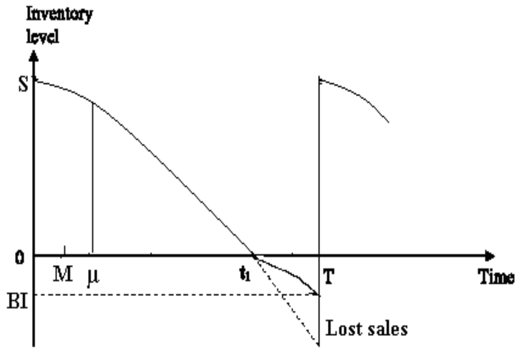

Fig. 2. Inventory level as a function of time for case: (i) $(0 < M < \mu)$

During the permissible delay period when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with fuzzy rate $\tilde{I}_e$. The interest earned denoted $I\tilde{E}_1$. Therefore, the total interest earned in the cycle period $[0,T]$ is

$$I\tilde{E}_1 = \tilde{I}_e \int_0^{\mu} \tilde{C} D_0 t\,dt = \frac{\tilde{C} D_0 \tilde{I}_e \mu^2}{2} \quad \text{-------} (16)$$

After the credit period the buyer has to pay the interest for the goods still in stock with fuzzy rate $\tilde{I}_c$. The interest payable denoted $I\tilde{C}_1$. Therefore, the interest payable in any cycle $[0,T]$ is

$$I\tilde{C}_1 = \tilde{I}_c \int_M^{t_1} \tilde{C} Q(t)\,dt$$

$$= \tilde{I}_c \tilde{C} \left\{ \int_M^{\mu} Q(t)\,dt + \int_{\mu}^{t_1} Q(t)\,dt \right\}$$

$$= \tilde{I}_c \tilde{C} D_0 \left\{ \begin{array}{l} \frac{t_1^2 \mu}{2} - \frac{\mu^3}{6} + \frac{M\mu^2}{2} + \frac{M^3}{6} - t_1 \mu M + \\ \frac{\alpha\mu}{(\beta+1)} \left[\begin{array}{l} M\mu^{\beta+1} + M^{\beta+1} t_1 - M t_1^{\beta+1} \\ - \frac{M^{\beta+1}\mu}{2} \\ - \frac{\mu^{\beta+2}}{2} \end{array} \right] \\ + \frac{\alpha}{(\beta+2)} \left[\mu t_1^{\beta+2} - M\mu^{\beta+2} \right] + \frac{\alpha}{2(\beta+3)} \left[\begin{array}{l} \mu^{\beta+3} \\ - M^{\beta+3} \end{array} \right] \\ + \frac{\alpha\mu}{(\beta+1)(\beta+2)} \left[\begin{array}{l} \alpha M^{\beta+1} \mu^{\beta+1} - \alpha \mu^{2(\beta+1)} \\ + \mu^{\beta+2} - t_1^{\beta+2} \end{array} \right] \\ - \frac{\alpha}{(\beta+2)(\beta+3)} \left[\mu^{\beta+3} - M^{\beta+3} \right] + \\ \frac{\alpha^2}{(\beta+2)(2\beta+3)} \left[\mu^{2\beta+3} - M^{2\beta+3} \right] \\ + \frac{\alpha^2 \mu}{(\beta+1)^2} \left[\begin{array}{l} M^{\beta+1} t_1^{\beta+1} - M^{\beta+1} \mu^{\beta+1} \\ - \frac{t_1^{2(\beta+1)}}{2} + \frac{\mu^{2(\beta+1)}}{2} \end{array} \right] \end{array} \right\}$$

---------- (17)

The total average fuzzy cost of the inventory system per unit time is given by

$$T\tilde{C}_1(t_1, T) = \frac{\mathrm{RC} + \mathrm{HC} + \mathrm{DC} + \mathrm{SC} + \mathrm{LSC} + \mathrm{IC}_1 - \mathrm{IE}_1}{\mathrm{T}}$$

$$= \frac{\tilde{C}_2}{T} + \frac{\tilde{C}_1 D_0 \mu}{T} \left\{ \begin{array}{l} \frac{t_1^2}{2} - \frac{\mu^2}{6} - \frac{\alpha\mu^{\beta+2}}{2(\beta+1)} + \frac{\alpha t_1^{\beta+2}}{\beta+2} \\ + \frac{\alpha\mu^{\beta+2}}{2(\beta+3)} + \frac{\alpha^2}{(\beta+1)^2} \left[\begin{array}{l} \frac{\mu^{2(\beta+1)}}{2} \\ - \frac{t_1^{2(\beta+1)}}{2} \end{array} \right] \\ - \frac{\alpha}{(\beta+1)(\beta+2)} \left[\begin{array}{l} \alpha\mu^{2(\beta+1)} \\ + t_1^{\beta+2} - \mu^{\beta+2} \end{array} \right] \\ - \frac{\alpha\mu^{\beta+2}}{(\beta+2)(\beta+3)} + \frac{\alpha^2 \mu^{2(\beta+1)}}{(\beta+2)(2\beta+3)} \end{array} \right\}$$

$$+ \frac{D_0 \tilde{C}}{T} \left\{ \frac{\alpha\mu^{\beta+2}}{\beta+2} + \frac{\alpha}{\beta+1} \left(\mu t_1^{\beta+1} - \mu^{\beta+2} \right) \right\}$$

$$- \tilde{C}_3 D_0 t_1 (T - t_1) + \frac{\tilde{l} D_0}{T} \left[T - t_1 + \frac{1}{\delta} (e^{-\delta T} - e^{-\delta t_1}) \right]$$

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$$+\frac{\tilde{I}_c\tilde{C}D_0}{T}\left\{\begin{array}{l}\frac{t_1^2\mu}{2}-\frac{\mu^3}{6}+\frac{M\mu^2}{2}+\frac{M^3}{6}-t_1\mu M\\ +\frac{\alpha\mu}{(\beta+1)}\left[\begin{array}{l}M\mu^{\beta+1}+M^{\beta+1}t_1-Mt_1^{\beta+1}\\ -\frac{M^{\beta+1}\mu}{2}-\frac{\mu^{\beta+2}}{2}\end{array}\right]\\ +\frac{\alpha}{(\beta+2)}\left[\mu t_1^{\beta+2}-M\mu^{\beta+2}\right]\\ +\frac{\alpha}{2(\beta+3)}\left[\mu^{\beta+3}-M^{\beta+3}\right]\\ +\frac{\alpha\mu}{(\beta+1)(\beta+2)}\left[\begin{array}{l}\alpha M^{\beta+1}\mu^{\beta+1}-\alpha\mu^{2(\beta+1)}\\ +\mu^{\beta+2}-t_1^{\beta+2}\end{array}\right]\\ -\frac{\alpha}{(\beta+2)(\beta+3)}\left[\mu^{\beta+3}-M^{\beta+3}\right]\\ +\frac{\alpha^2}{(\beta+2)(2\beta+3)}\left[\mu^{2\beta+3}-M^{2\beta+3}\right]\\ +\frac{\alpha^2\mu}{(\beta+1)^2}\left[\begin{array}{l}M^{\beta+1}t_1^{\beta+1}-M^{\beta+1}\mu^{\beta+1}\\ -\frac{t_1^{2(\beta+1)}}{2}+\frac{\mu^{2(\beta+1)}}{2}\end{array}\right]\end{array}\right\}$$

$$-\frac{\tilde{C}D_0\tilde{I}_e\mu^2}{2T}$$

------------ (18)

To minimize total average cost per unit time, the optimal values of $t_1 = t_1^*$ and $T = T^*$ can be obtained by solving the following equations simultaneously.

$$\frac{\partial T\tilde{C}_1(t_1,T)}{\partial T}=0 \quad \text{---------- (19)}$$

$$\frac{\partial T\tilde{C}_1(t_1,T)}{\partial t_1}=0 \quad \text{---------- (20)}$$

Provided, they satisfy the following conditions :

$$\frac{\partial^2 T\tilde{C}_1(t_1,T)}{\partial T^2}>0,\ \frac{\partial^2 T\tilde{C}_1(t_1,T)}{\partial t_1^2}>0$$

--------- (21)

And

$$\left(\frac{\partial^2 T\tilde{C}_1(t_1,T)}{\partial T^2}\right)\left(\frac{\partial^2 T\tilde{C}_1(t_1,T)}{\partial t_1^2}\right)-\left(\frac{\partial^2 T\tilde{C}_1(t_1,T)}{\partial T\partial t_1}\right)>0$$

---------- (22)

The optimal solution of the equations (18) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

Case (ii): $\mu < M \le t_1$

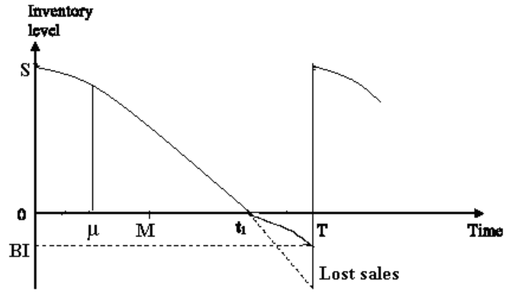

Fig. 3. Inventory level as a function of time for case: (ii) $(\mu < M \le t_1)$

In this case the period of delay in payment (M) is more than the period with no deterioration (μ) but less than the period with positive inventory (t_1). We can interest payable denoted by $I\tilde{C}_2$ as follows and fuzzy annual rate is $\tilde{I}_c$ .The interest payable in any cycle [0 , T] is

$$I\tilde{C}_2=\tilde{C}\tilde{I}_c\int_M^{t_1}Q(t)dt$$

$$=D_0\tilde{C}\tilde{I}_c\left\{\begin{array}{l}\frac{t_1^2}{2}+\frac{M^2}{2}-t_1M+\frac{\alpha}{\beta+1}\left[M^{\beta+1}t_1-Mt_1^{\beta+1}\right]\\ +\frac{\alpha}{\beta+2}\left[t_1^{\beta+2}-M^{\beta+2}\right]\\ -\frac{\alpha}{(\beta+1)(\beta+2)}\left[t_1^{\beta+2}-M^{\beta+2}\right]\\ +\frac{\alpha^2}{(\beta+1)^2}\left[M^{\beta+1}t_1^{\beta+1}-\frac{t_1^{2(\beta+1)}}{2}-\frac{M^{2(\beta+1)}}{2}\right]\end{array}\right\}$$

-------- (23)

Interest earned in the cycle period [0, T] is

$$I\tilde{E}_2=\tilde{C}\tilde{I}_e\int_0^{t_1}D_0(t)tdt=\frac{\tilde{C}\tilde{I}_eD_0t_1^2}{2} \quad \text{-------- (24)}$$

The total average fuzzy cost of the inventory system per unit time is given by

$$T\tilde{C}_2(t_1,T)=\frac{\mathrm{RC+HC+DC+SC+LSC+IC_2\text{ - }IE_2}}{\mathrm{T}}$$

$$=\frac{\tilde{C}_2}{T}+\frac{\tilde{C}_1D_0\mu}{T}\left\{\begin{array}{l}\frac{t_1^2}{2}-\frac{\mu^2}{6}-\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}\\ +\frac{\alpha t_1^{\beta+2}}{\beta+2}+\frac{\alpha\mu^{\beta+2}}{2(\beta+3)}\\ +\frac{\alpha^2}{(\beta+1)^2}\left[\frac{\mu^{2(\beta+1)}}{2}-\frac{t_1^{2(\beta+1)}}{2}\right]\\ -\frac{\alpha}{(\beta+1)(\beta+2)}\left[\begin{array}{l}\alpha\mu^{2(\beta+1)}\\ +t_1^{\beta+2}-\mu^{\beta+2}\end{array}\right]\\ -\frac{\alpha\mu^{\beta+2}}{(\beta+2)(\beta+3)}+\frac{\alpha^2\mu^{2(\beta+1)}}{(\beta+2)(2\beta+3)}\end{array}\right\}$$

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$$+\frac{D_0\tilde{C}}{T}\left\{\frac{\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha}{\beta+1}\left(\mu t_1^{\beta+1}-\mu^{\beta+2}\right)\right\}$$
$$-\tilde{C}_3 D_0 t_1 (T-t_1)+\frac{\tilde{l} D_0}{T}\left[T-t_1+\frac{1}{\delta}(e^{-\delta T}-e^{-\delta t_1})\right]$$
$$+\frac{D_0\tilde{C}\tilde{I}_c}{T}\left\{\begin{array}{l}\frac{t_1^2}{2}+\frac{M^2}{2}-t_1 M+\frac{\alpha}{\beta+1}\left[M^{\beta+1}t_1-Mt_1^{\beta+1}\right]\\ +\frac{\alpha}{\beta+2}\left[t_1^{\beta+2}-M^{\beta+2}\right]\\ -\frac{\alpha}{(\beta+1)(\beta+2)}\left[t_1^{\beta+2}-M^{\beta+2}\right]\\ +\frac{\alpha^2}{(\beta+1)^2}\left[M^{\beta+1}t_1^{\beta+1}-\frac{t_1^{2(\beta+1)}}{2}-\frac{M^{2(\beta+1)}}{2}\right]\end{array}\right\}$$
$$-\frac{\tilde{C}D_0\tilde{I}_e {t_1}^2}{2T}$$

---------- (25)

To minimize total average cost per unit time, the optimal values of $t_1 = t_1^*$ and $T = T^*$ can be obtained by solving the following equations simultaneously.

$$\frac{\partial T\tilde{C}_2(t_1,T)}{\partial T}=0$$ --------- (26)

$$\frac{\partial T\tilde{C}_2(t_1,T)}{\partial t_1}=0$$ --------- (27)

Provided, they satisfy the following conditions :

$$\frac{\partial^2 T\tilde{C}_2(t_1,T)}{\partial T^2}>0,\ \frac{\partial^2 T\tilde{C}_2(t_1,T)}{\partial t_1^2}>0$$

---------- (28)

And

$$\left(\frac{\partial^2 T\tilde{C}_2(t_1,T)}{\partial T^2}\right)\left(\frac{\partial^2 T\tilde{C}_2(t_1,T)}{\partial t_1^2}\right)-\left(\frac{\partial^2 T\tilde{C}_2(t_1,T)}{\partial T\partial t_1}\right)>0$$

----------- (29)

The optimal solution of the equations (25) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

Case (iii) : $M > t_1$

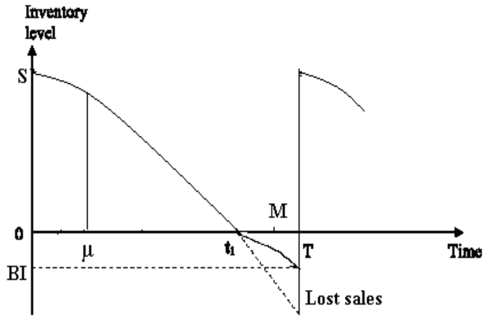

Fig. 4. Inventory level as a function of time for case (iii): $(M > t_1)$

In this case, the period of delay in payment (M) is more than period with positive inventory (t_1). In this case the retailer earns interest on the sales revenue with an fuzzy annual rate $\tilde{I}_e$ to the permissible delay period and no interest is payable during the period for the item kept in stock. Interest earned for the time period [0, T].

The interest earned denoted by $I\tilde{E}_3$ is given by

Total interest earned during the cycle = Interest earned up to t_1 + Interest earned during $(M-t_1)$

$$I\tilde{E}_3=\tilde{C}\tilde{I}_e\left[\int_0^{t_1}D_0(t)t\,dt+(M-t_1)\int_0^{t_1}D_0(t)dt\right]$$
$$=\tilde{C}I_e D_0 t_1\left(M-\frac{t_1}{2}\right)$$ -------- (30)

The total average fuzzy cost of the inventory system per unit time is given by

$$T\tilde{C}_3(t_1,T)=\frac{\mathrm{RC+HC+DC+SC+LSC-IE_3}}{\mathrm{T}}$$
$$=\frac{\tilde{C}_2}{T}+\frac{\tilde{C}_1 D_0\mu}{T}\left\{\begin{array}{l}\frac{t_1^2}{2}-\frac{\mu^2}{6}-\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}+\frac{\alpha t_1^{\beta+2}}{\beta+2}\\ +\frac{\alpha\mu^{\beta+2}}{2(\beta+3)}+\frac{\alpha^2}{(\beta+1)^2}\left[\frac{\mu^{2(\beta+1)}}{2}-\frac{t_1^{2(\beta+1)}}{2}\right]\\ -\frac{\alpha}{(\beta+1)(\beta+2)}\left[\begin{array}{l}\alpha\mu^{2(\beta+1)}+t_1^{\beta+2}\\ -\mu^{\beta+2}\end{array}\right]\\ -\frac{\alpha\mu^{\beta+2}}{(\beta+2)(\beta+3)}+\frac{\alpha^2\mu^{2(\beta+1)}}{(\beta+2)(2\beta+3)}\end{array}\right\}$$
$$+\frac{D_0\tilde{C}}{T}\left\{\frac{\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha}{\beta+1}\left(\mu t_1^{\beta+1}-\mu^{\beta+2}\right)\right\}$$
$$-\tilde{C}_3 D_0 t_1(T-t_1)$$
$$+\frac{\tilde{l} D_0}{T}\left[T-t_1+\frac{1}{\delta}(e^{-\delta T}-e^{-\delta t_1})\right]-\frac{\tilde{C}\tilde{I}_e D_0 t_1}{T}\left(M-\frac{t_1}{2}\right)$$

----------- (31)

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To minimize total average cost per unit time, the optimal values of $t_1 = t_1^*$ and $T = T^*$ can be obtained by solving the following equations simultaneously.

$$\frac{\partial T\tilde{C}_3(t_1, T)}{\partial T} = 0 \quad \text{-------- (32)}$$

$$\frac{\partial T\tilde{C}_3(t_1, T)}{\partial t_1} = 0 \quad \text{-------- (33)}$$

Provided, they satisfy the following conditions :

$$\frac{\partial^2 T\tilde{C}_3(t_1, T)}{\partial T^2} > 0, \ \frac{\partial^2 T\tilde{C}_3(t_1, T)}{\partial t_1^2} > 0$$

----------- (34)

And

$$\left(\frac{\partial^2 T\tilde{C}_3(t_1, T)}{\partial T^2}\right)\left(\frac{\partial^2 T\tilde{C}_3(t_1, T)}{\partial t_1^2}\right) - \left(\frac{\partial^2 T\tilde{C}_3(t_1, T)}{\partial T \partial t_1}\right) > 0$$

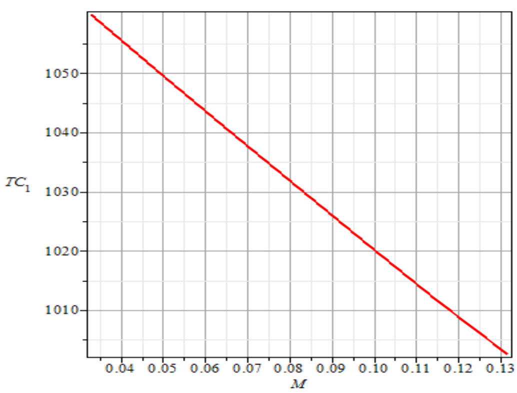
----------- (35)

The optimal solution of the equations (31) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

V. NUMERICAL EXAMPLES

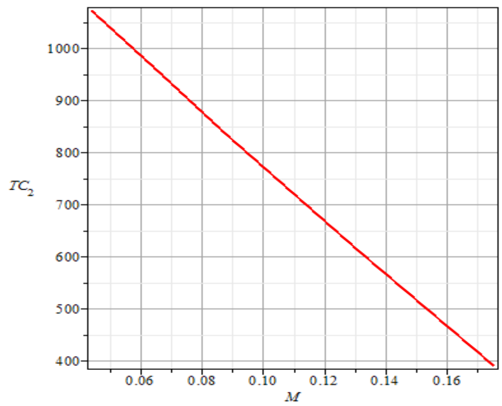
Example-1: Consider an inventory system with the following data: α = 0.1; β = 3 ; D = 500; $\tilde{C}_3$ =(15,20,25) ; $\tilde{I}$ =(17,18,19) ; $\tilde{C}$ =(90,100,110) ; $\tilde{C}_1$ =(5,10,15) ; $\tilde{C}_2$ = (500,600,700) ; $\tilde{I}_e$ =(0.11,0.12,0.13) ; $\tilde{I}_c$ = (0.14, 0.15, 0.16) ; μ = 0.11; δ = 1; M = 30/365 in appropriate units. Then we get the optimal values as t_1^* = 0.7515 ; T^* = 0.9495 ; Q^* = 137.751 ; TC_1^* = 1030.533 in appropriate units.

Effects of Total cost function over Trade Credit Period



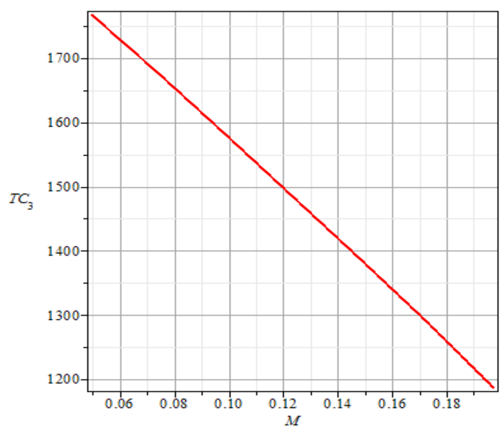
Example-2: Consider an inventory system with the following data : α = 0.1; β = 3 ; $\tilde{C}_3$ = (10,20,30) ; $\tilde{I}$ = (17,18,19) ; $\tilde{C}$ = (90,100,110) ; $\tilde{C}_1$ = (5,10,15) ; $\tilde{C}_2$ = (500,600,700) ; $\tilde{I}_e$ = (0.11, 0.12, 0.13) ; $\tilde{I}_c$ = (0.14, 0.15, 0.16) ; D = 500 ; μ = 0.11 ; δ = 1 ; M = 40/365 in appropriate units. Then we get the optimal values as t_1^* = 0.6896 ; T^* = 0.8466 ; Q^* = 113.714 ; TC_2^* = 723.348 in appropriate units .

Effects of Total cost function over Trade Credit Period



Example-3: Consider an inventory system with the following data : α = 0.1; β = 3 ; $\tilde{C}_3$ =(15,20,25); $\tilde{I}$ =(17,18,19); $\tilde{C}$ =(90,100,110); $\tilde{C}_1$ =(5,10,15) ; $\tilde{C}_2$ = (500,600,700) ; $\tilde{I}_e$ =(0.11, 0.12, 0.13) ; $\tilde{I}_c$ = (0.14, 0.15, 0.16) ; D = 500 ; μ = 0.11 ; δ = 1 ; M = 45/365 in appropriate units. Then we get the optimal values as t_1^* = 0.3770 ; T^* = 0.5906 ; Q^* = 124.572 ; TC_3^* = 1546.068 in appropriate units .

Effects of Total cost function over Trade Credit Period



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VI. SENSITIVITY ANALYSIS

We have performed sensitivity analysis by changing parameters α, β, μ and M as -60%, -40%, -20%, +20%, +40%, +60% and keeping the remaining parameters at their original values. The corresponding changes in the optimal length of time in which there is no inventory shortage t_1^*, length of the order cycle T*, the optimal order quantity per cycle Q* and the optimal total average cost TC*.

Table -1: Sensitivity analysis for Case – (i) $(0 < M \le \mu)$

Parameters	% change	t_1^*	T^*	Q^*	TC_1^*
α	-60	0.8197	1.0214	143.160	995.698
	-40	0.7926	0.9928	140.956	1008.448
	-20	0.7704	0.9693	139.199	1019.971
	+20	0.7352	0.9324	136.527	1040.315
	+40	0.7207	0.9174	135.475	1049.448
	+60	0.7078	0.9040	134.557	1058.029
β	-60	0.7242	0.9298	140.821	1126.100
	-40	0.7301	0.9311	138.442	1078.910
	-20	0.7405	0.9394	137.743	1049.796
	+20	0.7621	0.9597	138.072	1017.088
	+40	0.7718	0.9694	138.534	1007.306
	+60	0.7806	0.9784	139.051	999.956
μ	-60	1.1069	1.3111	126.809	731.218
	-40	0.9435	1.1438	130.836	858.897
	-20	0.8340	1.0329	134.715	955.585
	+20	0.6848	0.8819	139.704	1088.036
	+40	0.6284	0.8238	140.459	1130.295
	+60	0.5787	0.7717	139.931	1158.411
M	-60	0.7474	0.9482	138.511	1059.960
	-40	0.7487	0.9486	138.519	1050.049
	-20	0.7501	0.9490	138.128	1040.231
	+20	0.7530	0.9501	137.391	1021.002
	+40	0.7546	0.9509	137.052	1011.658
	+60	0.7563	0.9517	136.737	1002.544

Table -2: Sensitivity analysis for Case – (ii) $(\mu < M \le t_1)$

Parameters	% change	t_1^*	T^*	Q^*	TC_2^*
α	-60	0.7411	0.8998	117.251	687.225
	-40	0.7209	0.8788	115.807	700.344
	-20	0.7041	0.8615	114.658	712.307
	+20	0.6770	0.8337	112.922	733.634
	+40	0.6657	0.8223	112.247	743.284
	+60	0.6556	0.8121	111.662	752.390
β	-60	0.6626	0.8282	117.218	827.668
	-40	0.6683	0.8289	114.679	777.375
	-20	0.6786	0.8368	113.825	745.135
	+20	0.7002	0.8567	113.951	708.009
	+40	0.7099	0.8662	114.351	696.859
	+60	0.7187	0.8751	114.823	688.542
μ	-60	0.7648	0.9163	92.280	598.751
	-40	0.7368	0.8901	100.121	642.970
	-20	0.7119	0.8671	107.248	684.412
	+20	0.6693	0.8282	119.558	759.973
	+40	0.6507	0.8113	124.809	794.422
	+60	0.6335	0.7958	129.485	826.784
	-60	0.6537	0.8455	129.109	1073.051

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M	-40	0.6651	0.8451	123.807	954.613
	-20	0.6771	0.8454	118.674	838.015
	+20	0.7027	0.8487	108.927	610.683
	+40	0.7163	0.8515	104.300	499.819
	+60	0.7305	0.8551	99.839	390.891

Table -3: Sensitivity analysis for Case – (iii) $(M > t_1)$

Parameters	**% change**	t_1^*	T^*	Q^*	TC_3^*
α	-60	0.3793	0.5929	124.647	1543.158
	-40	0.3785	0.5921	124.621	1544.135
	-20	0.3772	0.5914	124.596	1545.105
	+20	0.3762	0.5899	124.548	1547.025
	+40	0.3755	0.5892	124.525	1547.976
	+60	0.3748	0.5885	124.503	1548.920
β	-60	0.3633	0.5802	125.685	1589.123
	-40	0.3699	0.5847	124.820	1562.811
	-20	0.3743	0.5882	124.597	1551.325
	+20	0.3786	0.5921	124.598	1543.583
	+40	0.3795	0.5931	124.629	1542.380
	+60	0.3801	0.5936	124.654	1541.787
μ	-60	0.4014	0.6103	106.044	1478.702
	-40	0.4004	0.6095	106.923	1481.552
	-20	0.3994	0.6087	107.017	1484.389
	+20	0.3973	0.6070	109.309	1490.045
	+40	0.3963	0.6062	110.314	1492.815
	+60	0.3953	0.6054	111.130	1495.593
M	-60	0.3766	0.6149	122.842	1768.475
	-40	0.3842	0.6132	118.253	1676.854
	-20	0.3915	0.6109	113.520	1583.093
	+20	0.4048	0.6041	103.624	1389.234
	+40	0.4109	0.5997	98.452	1288.986
	+60	0.4165	0.5946	93.129	1186.531

From the above tables we can conclude the following :

1. It is interesting to observe that increases in deterioration parameter α decrease ordering quantity Q^* and increase total cost TC^* of an inventory system.
2. And also increases in deterioration parameter β decrease ordering quantity Q^* and total cost TC^* of an inventory system.
3. Increase the parameter μ, increase ordering quantity Q^* and total cost TC^* of an inventory system.
4. Increase in delay period results in decrease ordering quantity Q^* and total cost TC^* of an inventory system.

VII. CONCLUSION

We have developed a fuzzy inventory model. Which is more suitable to day-to-day life.Weibull distribution deterioration has also been discussed. Moreover the concepts of partial backlogging of shortages and trade credit period are also taken in our model. Trade credit period is an important aspect for the smooth atmosphere in any business. By comparing the time, at which shortage begins with trade credit period, three cases have been studied for minimizing the total average inventory cost .It is observed that incentive of credit period is advantageous to the retailer for lowering the total cost of an inventory system. All the costs incurred in this model are adopted in fuzzy nature which is more general. The inventory costs are taken as triangular fuzzy number. Graded mean Integration representation method is applied for defuzzification. Sensitivity analysis is carried out to elucidated the proposed model.

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