Sphere Decoding in Multi-user Multiple Input Multiple Output with reduced complexity

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Abstract: **Multiple input multiple output (MIMO) supports greater data rate and higher reliability in wireless communication. MIMO technique uses the multiple antennas to achieve high transmission rate. A low complexity decoding technique is essential for Multi user- Multiple Input Multiple Output (MU-MIMO) systems. In this paper the Sphere Decoder (SD) techniques are discussed to be used in the Multi user-MIMO scenario where the available antennas are spread over a multitude of independent access points and independent radio terminals - each having one or multiple antennas and will emphasize on reducing the Bit Error Rate (BER) in Sphere Decoder (SD).**

Keywords: MIMO, MU-MIMO, BER, SD.

I. Introduction

Traditional wireless system such as SISO involves the transmission of the data packet from transmitter to receiver but they lack in the capacity of the single transmitting antenna and single receiving antenna. Unlike the traditional wireless systems, MIMO systems have attracted a wide range of attention due to its improved capacity, high spectral efficiency and good performance over fading channels. As there is a need of high SNR (Signal to Noise Ratio) in any system, MIMO systems could be the promising ones. MIMO system finds application in different scenarios. MIMO systems are aimed at providing the spatial diversity. In MIMO the antenna arrays could be exploited in two ways, they are spatial multiplexing and diversity.

Spatial multiplexing refers to the transmission of different signals or data at the same time. So spatial multiplexing results in the conservation of the resources which include bandwidth, also they are transmitted on the same transmission power. Diversity techniques provide the transmission of same data or signals through different antennas at the same time so that optimum signal would reach the receiver. This decreases the risk of the faded signal or the signal which has a large amount of interference to be received at the receiver.

After MIMO, there was a requirement of the system which could handle many users at the same time so an advance MIMO system was developed which is called as MU-MIMO (Multi-user MIMO). MU-MIMO system is used to provide the communication link between the base-station at the transmitter to the multi-user. In the case of MIMO, although many transmitters and receivers were used for the purpose of communication but only single antenna was used in a single transmitter/receiver and only one user could receive the signal. Whereas in case of MU-MIMO as many antennas could be used at the transmitter/receiver as per the requirement hence, the communication link could set up for many users at the same time.

II. System model of MU-MIMO

Multi-user MIMO could be defined as an advanced MIMO system having the capability of serving many users at one time. In communication, MU-MIMO technique is the set of improved MIMO technique. In this technique the antennas which are available, they are spread over a multitude of independent access points and each of the access point has its own one or multiple antennas. As compared to the single user MIMO which has a single transmitter with multiple antennas which communicates with a single receiver having multiple antennas. Multi-user MIMO can leverage

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multiple users as spatially distributed transmission resources, but it involves somewhat more expensive signal processing. On comparing with conventional or single-user MIMO only the local device multiple antenna dimensions are taken into consideration. MU-MIMO algorithms are used when the number of users, or connections, are greater than one. Also it enhances the MIMO systems.

(A) System Model

The system model of MU-MIMO represented here consists of a transmitter and the two receivers. The communication link between the transmitter and the receiver includes the transmission of data to many users at the same time. Also diversity techniques could also be applied at both the transmitter and receiver side.

The system equation for MU-MIMO could be given as :

$$y = hx + n \qquad (1)$$

In the matrix form the above expression could be represented as :

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1t} \\ h_{21} & h_{22} & \dots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ h_{r1} & \cdots & \dots & h_{rt} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}$$

where x is the transmitted signal or transmitted vector which contains the data. In other words this is the pure data in which there is no noise and interference. Here, y is the received signal at the receiver which includes the interference and noise. The received signal or the received vector is the sum of the transmitted signal through the channel and the interference which is added in the channel. Here, h is the channel matrix or it can be defined as the channel gain and n is the additive white gaussian noise which is added to the channel when the data/packet is transmitted from the transmitter after applying suitable precoding technique to the receiver.

(B) Introduction to SD

Sphere Decoder (SD) is presented initially by Finke and Pohst in [1] in 1985, this strategy plans to find the transmitted signal vector with least ML metric, that is, to find the ML resultant vector. It considers just a little arrangement of vectors inside of a given sphere instead of all conceivable transmitted signal vectors SD conforms the sphere range until there exists a solitary vector (ML resultant vector) inside that particular sphere. It expands the range when there exists no vector inside the sphere, and reduces the range when there exist numerous vectors inside the sphere. It is superior to every other method as it aides in enhancing the bit error rate and spatial complexity. Figure 1 demonstrates the thought behind the SD. Further in SD here are diverse calculations used to accomplish ideal results.

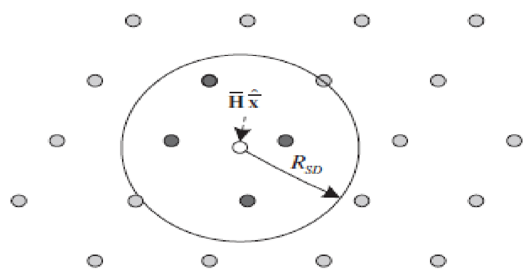

Fig.1 Original Sphere

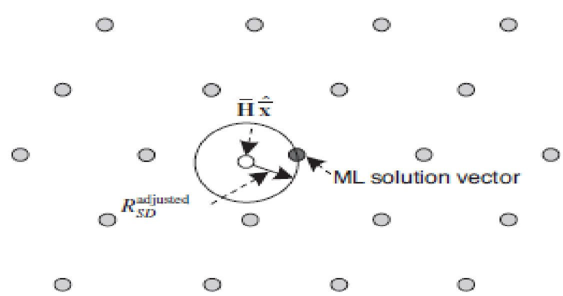

Fig.2 New sphere with the reduced radius

SD is expressed as:

$$\arg\min_{x} \|y-\bar{H}x\|^2 = \arg\min_{x} (x-\hat{x})^T H^T H (x-\hat{x}) \qquad \text{eqn1.}$$

The equation of SD can be derived by using the value of $\hat{x}$

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where $\hat{x}$ is unconstrained least squared solutions i.e.

$\hat{x} = (H^H H)^{-1} H^H y$, this relation exists for both real as well as complex systems.

H, y, x are used instead of $\overline{H}$, $\overline{y}$ & $\overline{x}$ respectively.

Considering:

$||y-Hx||^2 = ||y-Hx- H\hat{x} + H\hat{x}||^2$

$= (y-Hx- H\hat{x}+ H\hat{x})^T (y-Hx- H\hat{x}+ H\hat{x})$

$= \{(y- H\hat{x})^T+(H\hat{x}-Hx)^T\}\{(y- H\hat{x})+(H\hat{x}-Hx)$ eqn.2

$= (y- H\hat{x})^T (y- H\hat{x}) + (H\hat{x}-Hx)^T (H\hat{x}-Hx) + (H\hat{x}-Hx)^T (y- H\hat{x}) + (y- H\hat{x})^T (H\hat{x}-Hx)$

As, $\hat{x}$ is the Least Squared Solution,

$(H\hat{x}-Hx)^T (y- H\hat{x}) =(y- H\hat{x})^T(H\hat{x}-Hx)=0$

And thus the equation reduces to

$||y-Hx||^2=(y- H\hat{x})^T(y- H\hat{x})+(H\hat{x}-Hx)^T(H\hat{x}-Hx)$ eqn.3

Now substituting the value of $\hat{x}$ with

$(H^H H)^{-1}H^H y$

eqn.3 becomes

$=\{y-H (H^T H)^{-1}H^T y\}^T \{y-H(H^T H)^{-1}H^T y\}+(\hat{x} -x)^T H^T H(\hat{x} -x)$ eqn.4

Since,

$y-H (H^T H)^{-1}H^T y=\{I-H(H^T H)^{-1}\}y$,

the first term in the eqn.4 becomes

$=y^T \{I-H(H^T H)^{-1}H^T\}^T \{I-H(H^T H)^{-1}H^T\} y$

$=y^T \{I-H(H^T H)^{-1}H^T\} \{I-H(H^T H)^{-1}H^T\} y$

$=y^T\{I-H(H^T H)^{-1}H^T-H(H^T H)^{-T}H^T+H(H^T H)^{-T} H^T H(H^T H)^{-1}H^T\} y$

$=y^T \{I-H(H^T H)^{-1} H^T\} y$ eqn.5

Which turns out be constant with respect to x. From eqn.4 & 5, our relationship in eqn.1 immediately follows

$\arg\min_x ||y-Hx||^2=\arg\min_x(x - \hat{x} H^T H(x-\hat{x})$

This is the equation used for the calculation of the SD.

The ideal identification of the transmitted symbols can be seen as a discrete nearest point search issue [2]. The complexity of the ideal identification calculation develops exponentially with the quantity of transmitting antennas. This specific optimal solution can be carried out by sphere decoding algorithm using decreased complication.

This sphere decoding formula imposes the preprocessing referred to as QR decomposition to change over MIMO identification to a tree search issue.

In SD the computation of range is the most troublesome function, and it is to be done at the preprocessing level. In the event when the radius is too broad, normal processing cycle turns out to be very high, which makes operations in the real time impossible. Then again, even the ML arrangement can't fulfill the sphere constraint if the radius is too little. To effectively execute the SD the setting of the appropriate radius is vital, that too at the preprocessing level [3], [4], [5].

SD algorithms are a subset of decision feedback tree-search-decoders. SD calculations are a subset of choice input tree-look decoders. By emphasizing through a detection tree, data symbols are distinguished by MIMO, in which the tree levels, additionally alluded to as dimensions, relate to the components of the received symbol. Those detectors vary essentially in the way how they search along the tree. A tree search can be explained as

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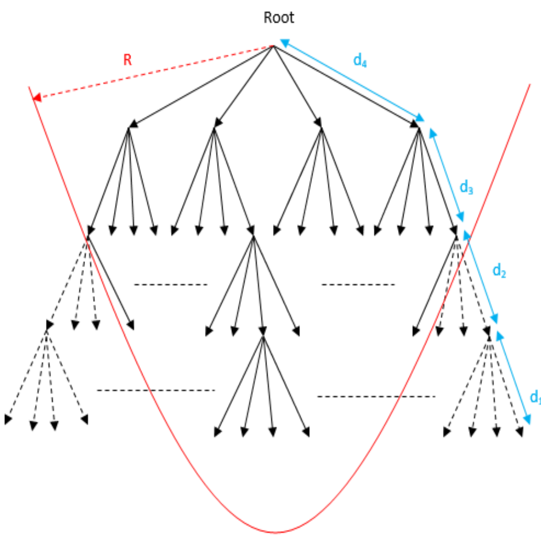

Fig.3 Tree structure of SD

Finke-Pohst Strategy

The Finke-Pohst enumeration [1] can be characterized as the procedure which considers the conceivable decisions s_i in the regular order, beginning with s_a until when s_b is found. Figure outlines the Finke-Pohst list system where four signal points are considered. It enumerates the symbols beginning from s_1 through to s_4 in the regular order.

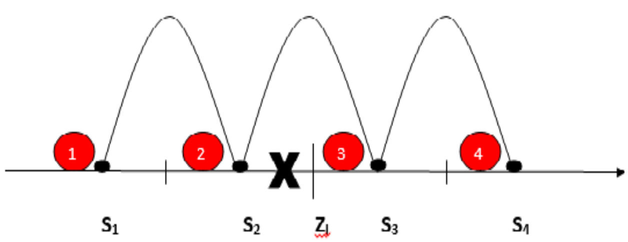

Fig.4 Illustration of Finke-Pohst

Schnorr-Euchner Strategy

On the other hand, the points can be ordered in ascending order of their Euclidean distance. Thus, they will be enumerated in a crisscross way beginning from the signal point nearest toward z_l, which is, s_2 as demonstrated in Figure. Demonstrates it then continues on to the following nearest point i.e.s_3, through to s_1 lastly to s_4. This procedure was initially grown by Schnorr and Euchner [6] and has later been re-evaluated in [7], [8]. Schnorr-Euchner list is intuitively favored over the Finke-Pohst enumeration, as it successfully executes largest branch metric first enumeration strategy for Euclidean distance part of the metric and also avoids having to explicitly calculate the branch metrics for all considered signal points to obtain the correct order of enumeration [9], [10],[11].

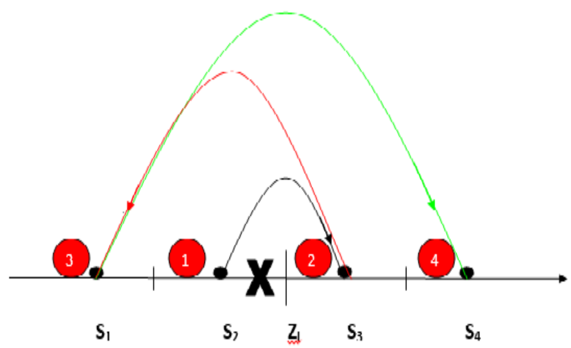

Fig.5 Illustration of Schnorr-Euchner

Flow Chart of SD

The input parameters are defined and then the QAM modulation is done to transmit the data. The QR decomposition is done to achieve the optimal results. Then the signal passes through the channel noise and interference acts upon the signal. At the receiver side the data is being demodulated after passing through the QPSK slicer and further the data is demodulated. The BER (The Bit Error Rate) is being calculated by the dividing the number of error bits with the total number of bits before that the error bits are calculated by subtracting the received bits from transmitted bits. The challenge is to increase the Signal to Noise ratio (SNR) and to keep the complexity low.

It is shown in the flow chart presented below

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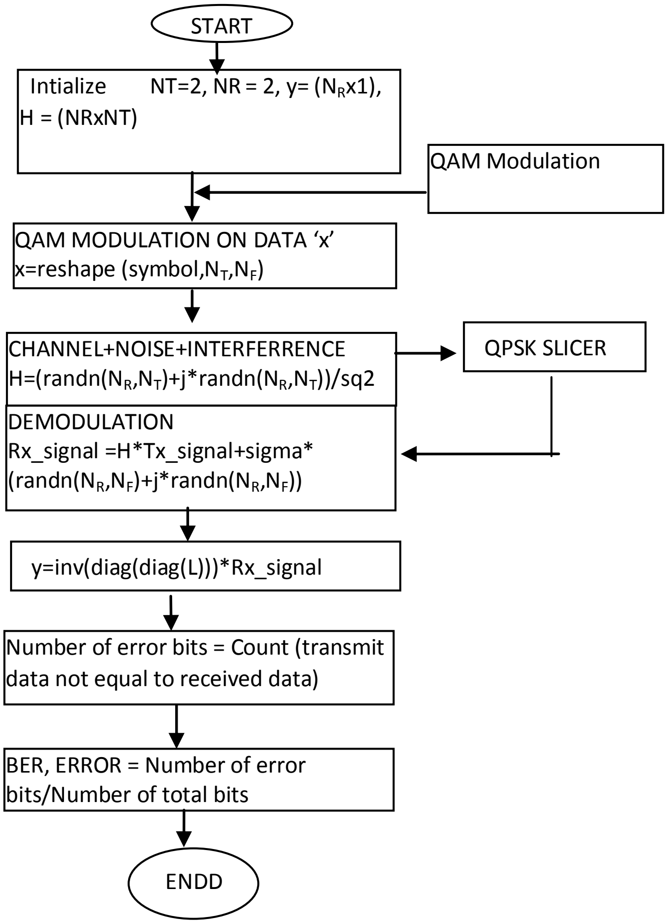

Results and Simulation

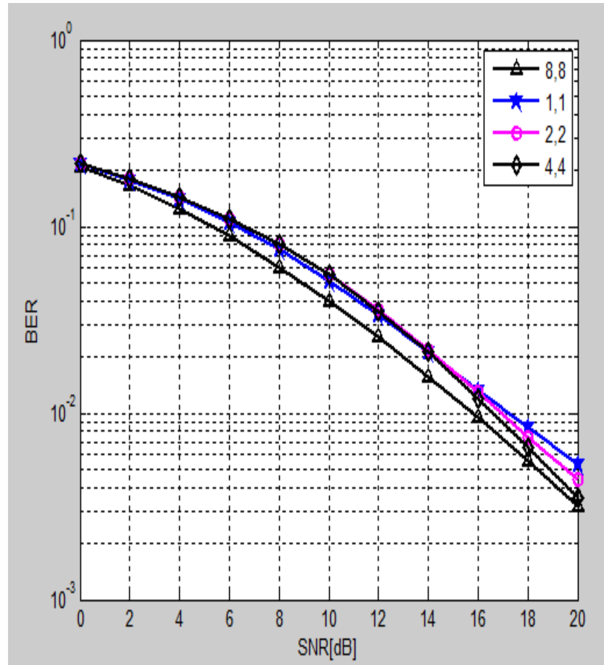

Fig. 6. Simulation results of Sphere Decoding

The result with all the possible antenna systems are shown. And in the 8x8 MIMO system the calculated BER is the lowest, as the SNR increases the BER decreases and the optimal results are obtained.

SD Algorithm

1. Find candidate $\overline{x}_4$ based on

$$\widehat{x_4} - \frac{R_{SD}}{r_{44}} \le \overline{x_4} \le \widehat{x_4} + \frac{R_{SD}}{r_{44}}$$

Let it be $\widetilde{\widetilde{x_4}}$.

If no such value is found, increase radius of sphere.

2. Now, calculate $\overline{x_3}$ using

$$|r_{44}(\widetilde{\widetilde{x_4}} - \widehat{x_4})|^2 + |r_{33}(\overline{x_3} - \widehat{x_3}) + r_{34}(\widetilde{\widetilde{x_4}} - \widehat{x_4})|^2 \le R_{SD}^2$$

Or

$$\widehat{x_3} - \sqrt{\frac{R_{SD}^2 - |r_{44}(\widetilde{\widetilde{x_4}} - \widehat{x_4})| - r_{34}(\widetilde{\widetilde{x_4}} - \widehat{x_4})}{r_{33}}} \le \overline{x_3} \le \widehat{x_3} + \sqrt{\frac{R_{SD}^2 - |r_{44}(\widetilde{\widetilde{x_4}} - \widehat{x_4})| - r_{34}(\widetilde{\widetilde{x_4}} - \widehat{x_4})}{r_{33}}}$$

Using $\widetilde{\widetilde{x_4}}$ from step 1.

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If no such value satisfies$\overline{x_3}$, select $\overline{x}_4$ again by further increasing the radius in step 1.

Final value is now$\widetilde{\overline{x_3}}$.

3. Now choose$\overline{x_2}$ using

$|r_{44}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 + |r_{33}(\widetilde{\overline{x_3}}- \widehat{\overline{x_3}}) + r_{34}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 + |r_{22}(\widetilde{\overline{x_2}}- \widehat{\overline{x_2}}) + r_{23}(\widetilde{\overline{x_3}}- \widehat{\overline{x_3}}) + r_{24}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 \leq R_{SD}^2$ $\widetilde{\overline{x_4}}$ and $\widetilde{\overline{x_3}}$ is used from step 2.

If no such candidate value is found, then repeat step 3 with increased radius.

Still if no value satisfies, repeat step 1 with further increased radius.

The result is$\widetilde{\overline{x_2}}$.

4. Choose$\overline{x_1}$ using

$|r_{44}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 + |r_{33}(\widetilde{\overline{x_3}}- \widehat{\overline{x_3}}) + r_{34}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 + |r_{22}(\widetilde{\overline{x_2}}- \widehat{\overline{x_2}}) + r_{23}(\widetilde{\overline{x_3}}- \widehat{\overline{x_3}}) + r_{24}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 + |r_{11}(\widetilde{\overline{x_1}}- \widehat{\overline{x_1}}) + r_{12}(\widetilde{\overline{x_2}}- \widehat{\overline{x_2}}) + r_{13}(\widetilde{\overline{x_3}}- \widehat{\overline{x_3}}) + r_{14}(\widetilde{\overline{x_4}}- \widehat{\overline{x_4}})|^2 \leq R_{SD}^2$

If no value satisfies repeat step 3 with increased radius. If still no value is found, repeat step 2, then step 1.

The result is

5. $\widetilde{\overline{x_1}}$,$\widetilde{\overline{x_2}}$, $\widetilde{\overline{x_3}}$ and $\widetilde{\overline{x_4}}$ turns out to be a single point inside the sphere. The procedure stops.

$\widetilde{\overline{x_1}}$,$\widetilde{\overline{x_2}}$, $\widetilde{\overline{x_3}}$ and $\widetilde{\overline{x_4}}$ turns out to be a single point inside the sphere. The procedure stops.

This is the solution vector.

Conclusion

A method for fixing the complexity of the SD used for MIMO detection has been presented in this paper. The results with all the simulations are shown. The complexity is reduced a bit. The technique used in SD provides optimal results. The work is still on to find much better technique with lesser complexity. Furthermore, the different techniques are being used in SD. The use of MU-MIMO has been increased a lot.

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