Effect of Radiation on Free Convection Heat and Mass Transfer Flow through Porous Medium in a Vertical Channel with Heat Absorption/ Generation

A K Mishra, Rajesh Menon K.and Shaima Abdullah Amer Al- Shanfari
Math Section, Department of IT, Salalah College of Technology, Oman

Abstract

The present paper analyses a solution for the transient free convective flow of a viscous and incompressible fluid between two vertical walls as a result of heat and mass transfer. The perturbation technique has been used to find the solutions for the velocity and temperature fields by solving the governing partial differential equations. The temperature of the one plate is assumed to be fluctuating. The effects of the various parameters entering into the problem, on the velocity and the temperature are depicted graphically. The impact of various parameters (Da, Rv, Pr, R and S) on velocity and temperature fields are shown graphically. The expressions for skin friction at both walls are also obtained.

Key Words: Vertical Channel, Heat and Mass transfer, Porous medium, perturbation technique.

1. Introduction

There are many transport processes in nature and in many industries where flows with free convection currents caused by the temperature differences are affected by the differences in concentration or material constitutions. In a number of engineering applications foreign gases are injected to attain more efficiency, the advantage being the reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer.

The recent books of Ingham and Pop [1] and Nield and Bejan [2] have documented exhaustive work on this area. The mass transfer phenomenon in unsteady free convective flow past infinite vertical porous plate was also studied by Hossain and Begum [11].The combined effects of thermal and mass diffusion in channel flows has been studied in the recent times by a few authors notably Nelson and Wood [18]. Nelson and Wood [19] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. Singh, [25] studied effect of mass transfer on free convection in MHD flow of a viscous fluid. Effect of mass transfer on transient MHD free convection flow of an incompressible viscous fluid examined by Singh and Gupta [26]. Muthucumaraswamy, et. al. [16] have investigated Mass transfer effects on exponentially accelerated isothermal vertical plate. Rajesh [22] heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium. Later heat and mass transfer free convection between two vertical walls filled with porous materials having periodic temperature on a wall investigated by Mishra and Mustapha [14].

Radiation convective flows are frequently encountered in many scientific and environmental processes, such as astrophysical flows, water evaporation from open reservoirs, heating and cooling of chambers, and solar power technology. Novotny, Lloyd, and Bankston, [20] studied radiation convection interaction in an absorbing emitting liquid in natural convection boundary layer flows. Several researchers have investigated radiative effects on heat transfer in nonporous and porous medium utilizing the Rosseland or other radiative flux model, such as Raptis [3], Hall et al. [9], Raptis [4], Bakier [6], Raptis and Perdikis [5], Sanyal and Adhikari [8], Rao [7], Prasad and Reddy [21]. Effects of radiation on free convection flow due to heat and mass transfer through a porous medium bounded by two vertical walls studied by Mishra at. al.[15]. Deka and Bhattacharya [10] explained unsteady

ISSN: 2278 – 1323

free convective Couette flow of heat generation/absorbing fluid in porous medium. Mebine, [13] studied radiation effects on MHD Couette flow with heat transfer between two parallel plates. Rajput and Sahu [23] described radiation effects on steady hydromagnetic flow of a viscous fluid through a vertical channel in a porous medium with heat generation or absorption. Narahari,[17] investigated effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary. Uwanta, Sani and Ibrahim [27] described transient convection fluid flow with heat and mass flux in a fixed vertical plate with radiation. Jha and Mussa [12] studied unsteady natural convection Couette flow of heat generating/absorbing fluid between vertical parallel plates filled with porous material. Radiative and free convective effects on MHD flow through a porous medium with periodic wall temperature and heat generation and absorption investigated by Sharma et. al. [24]

The main goal here is to study the effects of radiation on free convection steady fully developed flow through porous medium in a vertical channel. The closed form solutions for velocity, temperature, skin friction and concentration are presented. The effects of pertinent parameters on fluid flow and heat and mass transfer characteristics are studied in detail. This work is presented as follows.

2. Mathematical Formulation

The free convection flow of a viscous incompressible fluid has been considered in a porous region bounded by two vertical walls. The viscous and Darcy resistances are taken into account to model the momentum transfer in the porous region by taking permeability of the medium as constant. The $x'-$axis is taken along one of the vertical plate while $y'-$axis taken normal to the plate. The temperature of one plate is high enough to induce radiative heat.

Under the usual Boussinesq's approximation the natural convection flow is governed by following equations of conservation of momentum, thermal energy and concentration in non-dimensional forms

$$\frac{\partial u}{\partial t} = Rv\frac{\partial^2 u}{\partial y^2} - \frac{u}{Da} + Gr\theta + GmC \quad (1)$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - (R^2 + S)\theta \quad (2)$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \quad (3)$$

In non- dimensionalization, following quantities have been used:

ISSN: 2278 – 1323

$$\text{Da}=\frac{K}{h^2}, \qquad \text{Rv}=\frac{\mu_{\text{eff}}}{\mu_{\text{f}}}, \quad y=\frac{y'}{h}, \quad u=\frac{u'}{v_0},$$

$$R^2=\frac{4\alpha^2 h^2}{\kappa}, \qquad \theta=\frac{(T'-T'_c)}{(T'_w-T'_c)}, \qquad \text{n}=\frac{n'h^2}{\nu}, \quad \text{t}=\frac{t'\nu}{h^2}$$

$$C=\frac{C'-C'_c}{C'_w-C'_c}, \quad \text{Gr}=\text{g}\beta^*\nu(T'_w-T'_c)\frac{h^3}{\upsilon_f^2}, \quad R^2=\frac{Q_0 h^2}{\kappa}$$

$$\text{Gm}=\text{g}\beta(C'_w-C'_c)\frac{h^2}{\upsilon_f^2}, \quad \text{Pr}=\frac{\mu_{\text{f}} C_p}{k}, \text{Sc}=\frac{\upsilon}{D} \text{ -------------------------(4)}$$

where R, radiation parameter, S, heat generation/ absorption parameter, Da, Darcy number, Rv, ratio of viscosities, α, radiation/ absorption coefficient, u', is the velocity components in y' directions respectively. K is the permeability of the porous medium μ_{eff}, effective viscosity of the porous medium; μ_f dynamic viscosity of the fluid; υ_f kinematic viscosity of the fluid; u' fluid velocity; g acceleration due to gravity; β, coefficient of thermal expansion the temperature of the fluid; density; k, the thermal conductivity of the fluid, D, thermal diffusivity of the fluid; ρ, is density; T' and T'_c are the respective temperature of the fluid and cold wall; C' and C'_c are the respective concentration of the fluid and cold wall and C_p, is the specific heat at the constant pressure.

The boundary conditions for above set of equations in none—dimensional form are obtained as

$$u(y,t)=1, \qquad \theta(y,t)=1+\varepsilon e^{nt}, \quad C(y,t)=1, \qquad \text{at } y=0$$
$$u(y,t)=0, \qquad \theta(y,t)=0, \qquad C(y,t)=0, \qquad \text{at } y=1 \text{ --------(5)}$$

2.1 Solution of the Problem

Equation (1), (2) and (3) are second order coupled partial differential equations. We apply the regular perturbation method to solve them, since $\varepsilon << 1$, therefore, fluid velocity, temperature and concentration can be expanded as follows:

$$u(y,t)=u_0(y)+\varepsilon e^{nt}u_1(y)+o(E^2)\cdots$$
$$\theta(y,t)=\theta_0(y)+\varepsilon e^{nt}\theta_1(y)+o(E^2)\cdots$$
$$C(y,t)=C_0(y)+\varepsilon e^{nt}C_1(y)+o(E^2)\cdots \qquad (6)$$

Substituting the values from equation (6) in to equation (1), (2) and (3), neglecting the coefficients of $o(E^2)$, we obtain the following equations

ISSN: 2278 – 1323

$$Rv\frac{d^2u_0}{dy^2}-\frac{u_0}{Da}+Gr\theta_0+GmC_0=0 ----------------(7)$$

$$Rv\frac{d^2u_1}{dy^2}-(\frac{1}{Da}+n)u_1+Gr\theta_1+GmC_1=0 ------------(8)$$

$$\frac{d^2\theta_0}{dy^2}-(R^2+S)\theta_0=0 ----------------------(9)$$

$$\frac{d^2\theta_1}{dy^2}-(R^2+S+n\Pr)\theta_1=0 -------------------(10)$$

$$\frac{d^2C_0}{dy^2}=0 ------------------------------(11)$$

$$\frac{d^2C_1}{dy^2}-nScC_1=0 --------------------------(12)$$

The corresponding boundary conditions are as follows:

$$u_0=1,\quad u_1=0,\quad \theta_0=1,\quad \theta_1=1\quad C_0=1,\quad C_1=0,\quad \text{at } y=0$$
$$u_0=0,\quad u_1=0,\quad \theta_0=0,\quad \theta_1=1\quad C_0=0,\quad C_1=0,\quad \text{at } y=1 \qquad (13)$$

By solving equations (7) to (12) subject to boundary conditions (13), the solutions $u_0(y),\ u_1(y), \theta_0(y), \theta_1(y), and\ C_0(y),$ are obtained as follows:

$$u_0(y)=B_2e^{\sqrt{m_3}y}+B_3e^{-\sqrt{m_3}y}+$$
$$\frac{Gr}{Rv(m_1+m_3)}(B_1e^{\sqrt{m_1}y}+(1-B_1)e^{-\sqrt{m_1}y})+\frac{Gm}{Rvm_3}(1-y) ------(14)$$

$$u_1(y)=B_6e^{\sqrt{m_4}y}+B_7e^{-\sqrt{m_4}y}-\frac{1}{(m_1+m_2)}(A_1e^{\sqrt{m_2}y}+A_2e^{-\sqrt{m_2}y}) ---------(15)$$

$$\theta_0(y)=B_1e^{\sqrt{m_1}y}+(1-B_1)e^{-\sqrt{m_2}y} ------------------------(16)$$

$$\theta_1(y)=(\frac{e^{-2\sqrt{m_2}}}{e^{-2\sqrt{m_2}}-1})e^{\sqrt{m_2}y}+(\frac{1}{1-e^{-2\sqrt{m_2}}})e^{-\sqrt{m_2}y} ------------------(17)$$

$$C(y)=1-y ------------------------------------(18)$$

$$m_1=R^2+S,\ m_2=m_1+n\Pr,\ m_3=1/DaRv,\ m_4=m_3+n/Rv,$$

$$B_3=1/(1-e^{2\sqrt{m_1}}),\quad B_1=1/(1-e^{2\sqrt{m_1}}),\quad B_2=1-B_3-B_4B_3=\frac{(1-B_4)e^{\sqrt{m_3}}+B_5}{e^{\sqrt{m_3}}-e^{-\sqrt{m_3}}},$$

$$B_4=\frac{Gr}{Rv(m_1+m_2)}+\frac{Gm}{m_3Rv},\ B_5=\frac{Gr}{Rv(m_1+m_2)}\left\{B_1e^{\sqrt{m_1}}+(1-B_1)e^{-\sqrt{m_1}}\right\}+\frac{Gm}{m_3Rv}$$

$$B_6=\frac{A_1+A_2}{m_1+m_2}-B_7,\quad B_7=\frac{1}{(m_1+m_2)}\left\{(A_1+A_2)e^{\sqrt{m_4}}-(A_1e^{\sqrt{m_2}}+A_2e^{-\sqrt{m_2}})\right\}$$

ISSN: 2278 – 1323

The expressions of skin friction at both the plates are given by

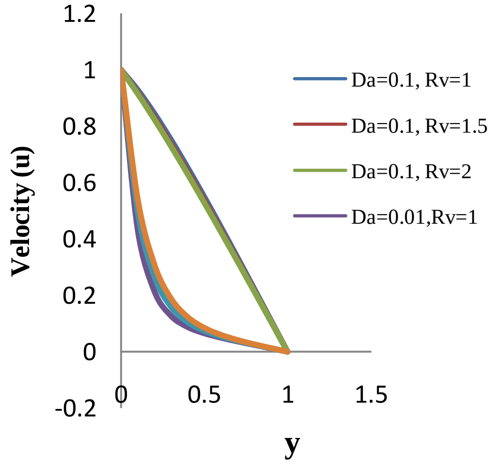
$$\tau_1 = \left.\frac{du}{dy}\right|_{y=0} = \left(\sqrt{m_3}B_2 - \sqrt{m_3}B_3 + \frac{2B_1\sqrt{m_2}Gr}{Rv(m_1+m_3)} - \frac{Gm}{Rvm_3}\right)$$
$$+\varepsilon e^{nt}\left(\sqrt{m_4}(B_6 - B_7) - \frac{\sqrt{m_2}}{(m_1+m_2)}(A_1 - A_2)\right) ----(19)$$

$$\tau_2 = \left.\frac{du}{dy}\right|_{y=1} = \left(\sqrt{m_3}(B_2 e^{\sqrt{m_3}} - B_3 e^{-\sqrt{m_3}}) + \frac{Gr\sqrt{m_1}B_1}{Rv(m_1+m_3)}(e^{\sqrt{m_1}} + e^{-\sqrt{m_1}}) - \frac{Gm}{Rvm_3}\right)$$
$$+\varepsilon e^{nt}\left(\sqrt{m_4}(B_6 e^{\sqrt{m_4}} - B_7 e^{-\sqrt{m_4}}) - \frac{\sqrt{m_2}}{(m_1+m_2)}(A_1 e^{\sqrt{m_2}} - A_2 e^{-\sqrt{m_2}})\right) -(20)$$

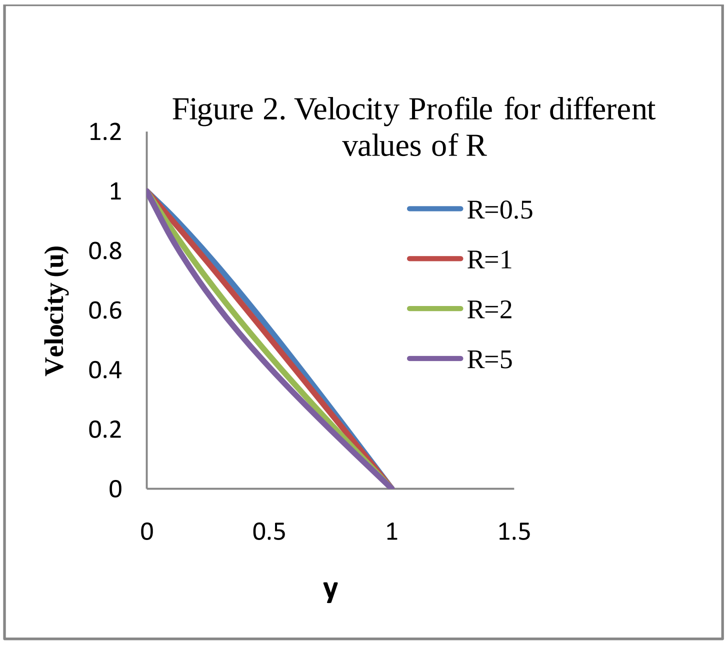
3. Result and Discussion

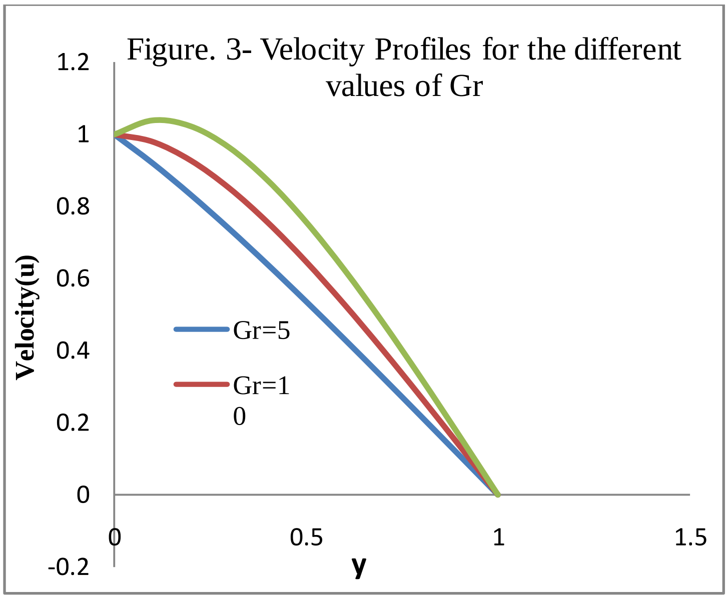
In order to understand the physical importance of the flow between the two plates, calculations have been carried out for velocity, temperature, concentration and skin friction. Effects for different values of Darcy number (Da), ratio of viscosities (Rv), Grashof number (Gr), radiation parameter (R) and absorption/generation parameter (S) on velocity and temperature profiles between plate y = 0, and plate y = 1, are shown graphically while all other parameters are kept constant.

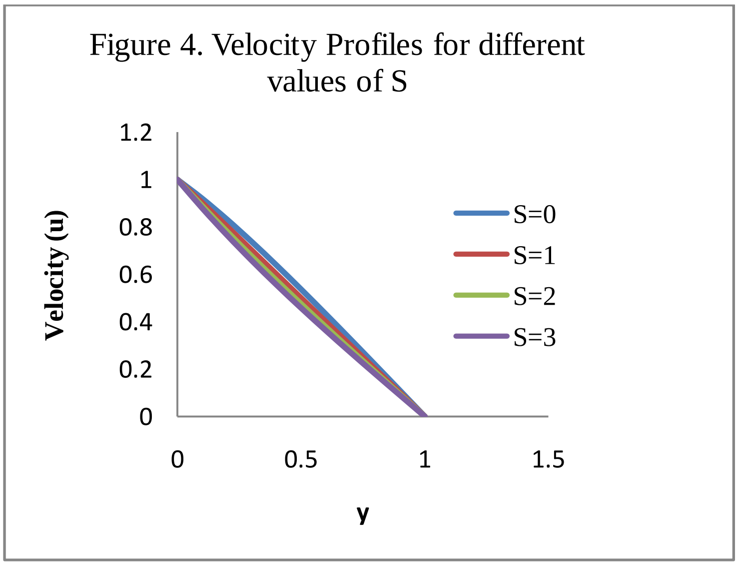
Figure- 1. Velocity profiles for diffrent values of Da and Rv



ISSN: 2278 – 1323

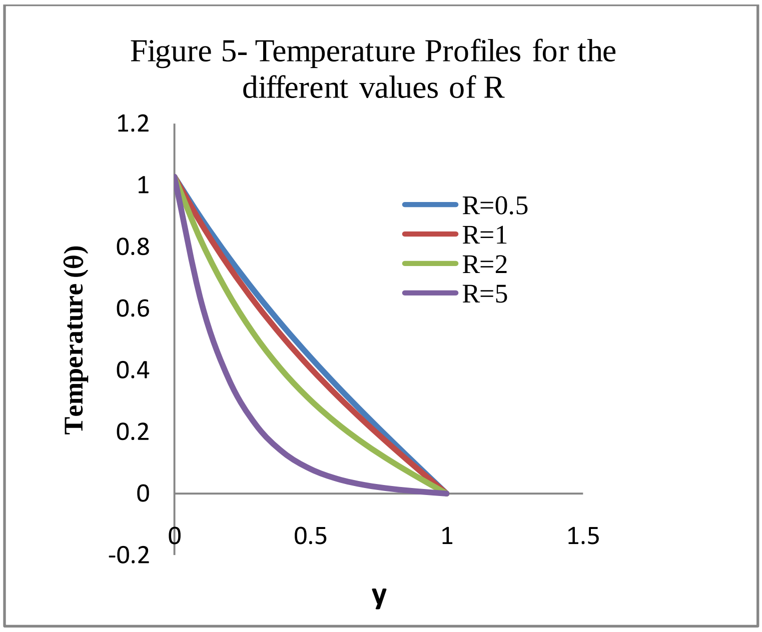






ISSN: 2278 – 1323

The velocity profiles are shown in figures 1 to 4 for different values of Darcy number (Da), ratio of viscosities (Rv), Grashof number (Gr), radiation parameter (R) and absorption/generation parameter (S) while all other parameters are kept constant. It is noticed that the velocity of fluid increases with the increase of Da and Gr while decreases with the increase of Rv and R.Interpreting physically, increase in the permeability parameter (K) increases the flow which leads to increase in velocity. Velocity decreases due to the increase of Rv for all values of Da, which is due to the high effective viscosity of the porous medium to that of fluid viscosity.It is observed there is a fall in velocity in the presence of high thermal radiation.



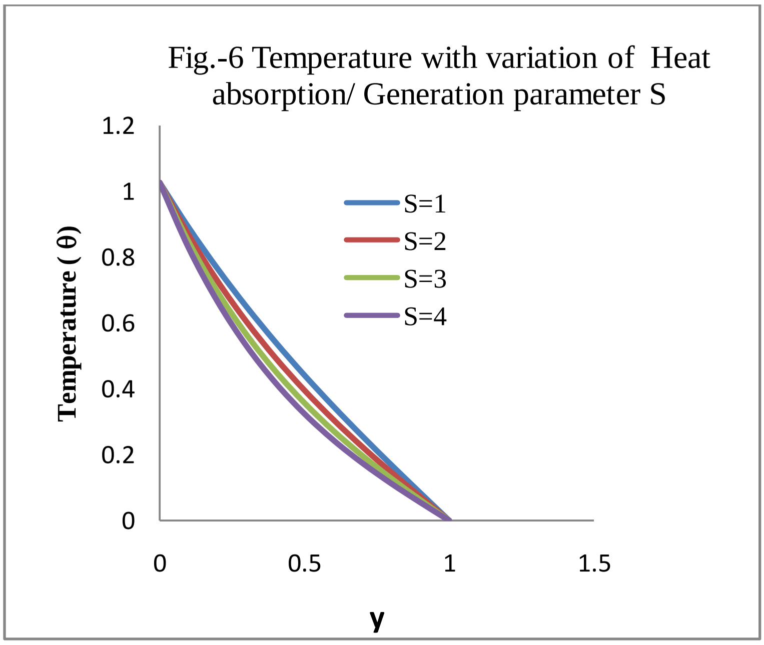


Figure 5 and 6 represents the temperature profiles for different values of radiation parameter R and heat generation/ absorption parameter S, while all the other parameters are kept

ISSN: 2278 – 1323

constant. It is observed from the figures that the temperature decreases with the increase of radiation parameter R and heat generation/ absorption parameter S.

4. Conclusion

The fully developed free convection heat and mass transfer flow between two vertical plates through porous medium has been investigated. Perturbation technique is used to solve the equations. It is observed that velocity of fluid increases with increase of Darcy number (Da) , Grashof number (Gr) while decreases with increase of radiation parameter (R), ratio of viscosities (Rv) and heat generation/ absorption parameter (S). It is also obtained that temperature decreases with increase of radiation parameter (R) and heat generation/ absorption parameter (S). Further, it is found that radiation causes to decrease the rate of heat transfer to the fluid thereby reducing the effect of natural convection. The rate of flow decreases with an increase of radiation.

References

[1] Ingham, B. D., and Pop, I., Convection in porous media, (2nd ed.) Springer,New York, (1990).
[2] Nield, D. A., and Bejan, A., Transport phenomena in porous media, Oxford, (1998).
[3]A. Raptis, Radiation and free convection flow through a porous medium, Int. Commun. Heat Mass Transf., 25(1998), 289-295.
[4] A. Raptis, Radiation and flow through a porous medium, J. Porous Media, 4(2001), 271-273.
[5] A. Raptis and C. Perdikis, Unsteady flow through a highly porous medium in the presence of radiation, Transport in Porous Media, 57(2004), 171-179.
[6] A.Y. Bakier, Thermal radiation effects on mixed convection from vertical surfaces in saturated porous media, Int. Comm. Heat and Mass Transfer, 28(2001), 243-248.
[7] C.G. Rao, Interaction of surface radiation with conduction and convection from a vertical channel with multiple discrete heat sources in the left wall, Numer. Heat Transfer Part A: Appl., 52(2007), 831-848.
[8] D.C. Sanyal and A. Adhikari, Effects of radiation on MHD vertical channel flow, Bull. Cal. Math. Soc., 98, 5(2006), 487-497.
[9] D. Hall, G.C. Vliet and T.L. Bergman, Natural convection cooling of vertical rectangular channels in air considering radiation and wall conduction, J. Electron. Pack. 121(1999), 75-84.
[10] Deka, R. K. and Bhattacharya, A., "Unsteady free convective Couette flow of heat generating/absorbing fluid in porous medium". Int. J. Math. Archive, Vol. 2, 2011, pp. 853-863.
[11] Hossain,M.A and Begum,R.A: The mass transfer phenomenon in unsteady free convective flow past infinite vertical porous plate Appl. Space Sci,V.15,p.145(1985)
[12] Jha, B. K. and Musa, M. K., "Unsteady natural convection Couette flow of heat generating/absorbing fluid between vertical parallel plates filled with porous material". Appl. Math. and Mech., Vol. 33, 2012, pp. 303-314.
[13] Mebine, P., "Radiation effects on MHD Couette flow with heat transfer between two parallel plates". Global J. Pure applied Mathematics, Vol. 3, 2007, pp. 191-202.
[14] Mishra, A. K., and Mustaph, Heat and mass transfer free convection between two vertical walls filled with porous materials having periodic temperature on a wall, Ijmr, Volume 1, Number 1 (2013), pp. 1-12.
[15] Mishra,A. K., Djam X., and Manjak N. H., Effects of radiation on free convection flow due to heat and mass transfer through a porous medium bounded by two vertical walls, Ijater, Volume 3, Issue 3, May 2013, pp 120-125.
[16] Muthucumaraswamy, R.; Sathappan, K. E. and Natarajan, R., "Mass transfer effects on exponentially accelerated isothermal vertical plate". Int. J. of Appl. Math.and Mech., Vol. 4, 2008, pp. 19-25.
[17]Narahari, M., "Effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary". Wseas Transactions on Heat and Mass Transfer, Vol. 5, 2010, pp. 21-30.
[18] Nelson D.J and Wood, B.D,: Combined heat and mass transfer natural convection between vertical parallel plates, Intl. J. Heat and Mass transfer, V.32, PP.1779-1787 (1989).
[19] Nelson D.J. and Wood B.D: J. Heat transfer V.4, PP-1587-1582. (1986)
[20] Novotny, J. L.; Lloyd, J. R. and Bankston, J. D., "Radiation convection interaction in an absorbing emitting liquid in natural convection boundary layer flows". ASME, J. Heat Transfer, Vol. 99, 1971, pp. 125-127.
[21] Prasad, V. R.; Muthucumaraswamy, R. and Vasu, B. "Radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in a porous medium". Int. J. of Appl. Math.and Mech., Vol. 6, 2010, pp. 1-21.

ISSN: 2278 – 1323

[22]. Rajesh, V., “Heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium”. Int. J. Engineering, Tome, Vol. XI, 2011, pp. 205-212.
[23]. Rajput, U. S. and Sahu, P. K., “Radiation effects on steady hydromagnetic flow of a viscous fluid through a vertical channel in a porous medium with heat generation or absorption”. International Journal of Mathematical Archive, Vol. 2, 2011, pp. 1898- 1904.
[24] Sharma, P. R., Sharma, K. and Trpathi M., Radiative and free convective effects on MHD flow through a porous medium with periodic wall temperature and heat generation and absorption investigated, IJMA 5(9), 2014, 119-128.
[25]. Singh, A. K., “Effect of mass transfer on free convection in MHD flow of a viscous fluid”. Ind. J. Pure Appl. Phys., Vol. 41, 2003, pp. 262-274.
[26]. Singh, J. and Gupta, C.B., “Effect of mass transfer on transient MHD free convection flow of an incompressible viscous fluid”. ‘GanitaSandesh’, J. Raj. GanitaParishad, India, Vol. 17, 2003, pp. 33 40.
[27] Uwanta, I. J.; Sani, M. and Ibrahim, M. O., “Transient convection fluid flow with heat and mass flux in a fixed vertical plate with radiation”. Int. J. of computer Appl., Vol. 36, 2011, pp. 33-40.

ISSN: 2278 – 1323