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SEP Performance analysis of GMSK over Rayleigh, Rician and Nakagami-m Fading Environment Using MGF Approach

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***Abstract* - In this paper pdf of received SNR under Rician, Rayleigh, and Nakagami-m fading channel is used to derive SEP (Symbol Error Probability). Error Probability for GMSK with Moment generating function approach is obtained by averaging the conditional SEP over the Probability density function (pdf) of received SNR. Error Performance plots of GMSK modulation technique has been drawn for different value of Rician parameter K, diversity order N, modulation order M. These results provide information about error performance over fading channel.**

***Keywords* – Moment generating function, Gaussian minimum shift key (GMSK), Symbol Error Probability (SEP), Signal-to-Noise Ratio (SNR).**

I. Introduction

The wireless communications industry has been experiencing phenomenal annual growth rates over the past several years. This degree of growth reflects the tremendous demand for commercial communications services such as analog and digital cellular telephony, and Personal Communications Services (PCS) including high speed data, full motion video, Internet access, on-demand medical imaging, real-time road maps, and anytime, anywhere video conferencing. In many applications, wireless telecommunications can eliminate the high costs of installing and maintaining traditional wired systems. Wireless services make even the most rural community accessible through communications. The wireless revolution was triggered and is being sustained by several important factors: advances in microelectronics, high-speed intelligent networks, positive user response and an encouraging regulatory climate worldwide [1-2]. Beyond the arena of mobile communications, there are numerous wireless applications including Wireless local area networks (WLANs), Bluetooth, Local Multipoint Distribution Systems (LMDS), satellite communications and radio frequency identification (RFID) operating at frequencies extending into the millimeter-wave regime (>30 GHz). The move to higher (millimeter-wave) frequencies has been motivated by the need for more and more bandwidth for multimedia applications such as wireless “cable” TV and high-speed internet access, and by increased overcrowding of lower frequency bands. To meet this increasing demand, new wireless techniques and architectures must be developed to maximize capacity and quality of service (QoS) without a large penalty in the implementation complexity or cost. This provides many new challenges to engineers involved with system design, one of which is ensuring the integrity of the data is maintained during transmission. To study the M-ary digital modulation schemes on the basis of the error performance that are very important in today’s communication scenario. The demand for higher data rate and better bandwidth efficiency is increased day by day, but the total bandwidth allocation is limited. Therefore it is very much necessary to study the modulation schemes which give us the better result. M-ary modulation schemes achieve better bandwidth efficiency than other modulation techniques and give higher data rate.

II. Fading Channels

Multipath fading is due to the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components. This type of fading is relatively fast and is therefore responsible for the short term signal variations. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multipath fading envelope.

A Rayleigh Fading

The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. In this case, the channel fading amplitude α is distributed according to

$$p_{\alpha}(\alpha) = \left(\frac{2\alpha}{\Omega}\right)\exp\left(-\frac{\alpha^2}{\Omega}\right) \quad \text{when } \alpha \geq 0 \qquad (1)$$

And hence in Eq (2), the instantaneous SNR per symbol of

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the channel γ is distributed according to an exponential distribution given by

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad \gamma \geq 0 \tag{2}$$

The MGF corresponding to this fading model is given by

$$M_\gamma(s) = (1 - s\bar{\gamma})^{-1} \tag{3}$$

The Rayleigh fading model agrees very well with experimental data for mobile systems where no LOS path exists between the transmitter and receiver antennas [3]. It also applies to the propagation of reflected and refracted paths through the troposphere and ionosphere [4], and to ship-to-ship [5] radio links.

B Rician Distribution

When there is a dominant stationary (non-fading) signal component present, such as a LOS propagation path, the small-scale fading envelope distribution is Rician. The Rician distribution has a probability density function (PDF) given by

$$p_\alpha(a) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega} \exp\left(-\frac{(1+n^2)\alpha^2}{\Omega}\right) I_o\left(2n\alpha\sqrt{\frac{(1+n^2)}{\Omega}}\right) \quad \alpha \geq 0 \tag{4}$$

Where n is the Nakagami-*n* fading parameter, which ranges from 0 to ∞. This parameter is related to the Rician K factor by $K = n^2$ which corresponds to the ratio of the power of the LOS (secular) component to the average power of the scattered component. Eq (5) shows that the SNR per symbol of the channel, γ is distributed according to a non-central chi-square distribution given by

$$p_\gamma(\gamma) = \frac{(1+n^2)e^{-n^2}}{\bar{\gamma}} \exp\left(-\frac{(1+n^2)\gamma}{\bar{\gamma}}\right) I_o\left(2n\sqrt{\frac{(1+n^2)\gamma}{\bar{\gamma}}}\right) \gamma \geq 0 \tag{5}$$

It can also be shown that the MGF associated with this fading model is given by

$$M_\gamma(s) = \frac{1+K}{1+K-S\bar{\gamma}} \,.\exp\left(\frac{KS\bar{\gamma}}{1+K-S\bar{\gamma}}\right) \tag{6}$$

The Rician distribution spans the range from Rayleigh fading (K = 0) to no fading (constant amplitude) (K=∞). This type of fading is typically observed in the first resolvable LOS paths of microcellular urban and suburban land–mobile [6], picocellular indoor [7], and factory environments. It also applies to the dominant LOS path of satellite and ship-to-ship [5] radio links.

C. Nakagami-m Fading Model

The Nakagami-m distribution is the widely accepted statistical fading model due to both its good fit with experimental results and its versatility. It covers a wide range of fading scenarios by varying its fading severity index m. The model also includes the Rayleigh $(m = 1)$ and one-sided Gaussian $(m = 1/2)$ distribution as special cases, and closely approximates the Rician distribution via relationship $m = (k+1)^2/(2k+1)$.

The Nakagami-m PDF is in essence a central chi-square distribution given by

$$p_\alpha(a) = \frac{2m^m \alpha^{2m-1}}{rm\,\Omega^m} \exp\left(-\frac{m\alpha^2}{\Omega}\right) \qquad \alpha \geq 0 \tag{7}$$

Eq (8) shows that the SNR per symbol γ is distributed according to a gamma distribution given by

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{rm\,\bar{\gamma}^m} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) \qquad \gamma \geq 0 \tag{8}$$

It can also be shown that the MGF is given in this case by

$$M_\gamma(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m} \tag{9}$$

Finally, the Nakagami-*m* distribution often gives the best fit to land–mobile [8] and indoor–mobile multipath propagation, as well as scintillating ionosphere radio links [4,9].

III. SYSTEM PERFORMANCE MEASURES

A. Average Signal to noise Ratio

Signal-to-noise ratio (SNR) is the most common performance measure characteristic of a digital communication system. SNR is measured at the output of the receiver and is directly related to the data detection process. The term noise in average signal-to-noise ratio refers to the ever-present thermal noise at the input to the receiver and the word average means to statistical averaging over the probability distribution of the fading [10].

$$\bar{\gamma} = \int_0^\infty \gamma.\, p_\gamma(\gamma)\, d\gamma \tag{10}$$

Where γ denotes the instantaneous SNR at the receiver output and $p_\gamma(\gamma)$ denotes the probability density function (PDF) of γ.

B. Symbol error probability

This performance criterion average symbol error probability (SEP) is the one that is most revealing about the nature of the system behavior. The primary reason for the difficulty in evaluating average SEP lies in the fact that the conditional SEP is, in general, a nonlinear function of the instantaneous SNR, the nature of the nonlinearity being a function of the modulation/detection scheme employed by the system. The average SEP can be written as [10]

$$p_s(E) = \int_0^\infty P_S(E/\gamma) P_\gamma(\gamma) d\gamma \tag{11}$$

Where $p_s(E/\gamma)$ is the conditional symbol error probability.

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IV. MOMENT GENERATING FUNCTION

Moment-generating function of a random variable is an alternative representation of its probability distribution .Thus this approach provides an alternative way to compute results compared with working directly with probability density functions. There are particularly simple results for the moment-generating functions of. The MGF for a non-negative random variable γ with distribution $p_\gamma(\gamma), \gamma \geq 0$, is defined as

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma}\, d\gamma \qquad (12)$$

Where $p_\gamma(\gamma)$ denotes the probability density function (PDF) of γ. MGF for common Rician fading distribution with factor K and diversity order N is given by [11].

$$M_\gamma(s) = \left(\frac{N+K}{N+K-S\bar{\gamma}}\right)^N . \exp\left(\frac{KS\bar{\gamma}}{N+K-S\bar{\gamma}}\right) \qquad (13)$$

MGF for Nakagami-m fading distribution with factor m and diversity order N is given by

$$M_\gamma(s) = \left(1 - \frac{s\bar{\gamma}}{mN}\right)^{-mN} \qquad (14)$$

V. GMSK MODULATION SCHEME

Gaussian Minimum Shift Keying or Gaussian filtered Minimum Shift Keying is the form of modulation with no phase discontinuities used to provide data transmission with efficient spectrum usage. GMSK modulation is based on MSK [12], which is itself a form of continuous-phase frequency-shift keying. One of the problems with standard forms of PSK is that sidebands extend out from the carrier. To overcome this, MSK and its derivative GMSK can be used. Gaussian Minimum Shift Keying (GMSK) is a modification of MSK (i.e. CPFSK with h = 1/2). A filter used to reduce the bandwidth of a baseband pulse train prior to modulation is called a pre-modulation filter. The Gaussian pre-modulation filter smooths the phase trajectory of the MSK signal thus limiting the instantaneous frequency variations. The result is an FM modulated signal with a much narrower bandwidth. This bandwidth reduction does not come for free since the pre-modulation filter smears the individual pulses in pulse train [13]. As a consequence of this smearing in time, adjacent pulses interfere with each other generating what is commonly called inter-symbol interference or ISI. In the applications where GMSK is used, the trade-off between power efficiency and bandwidth efficiency is well worth the cost.

GMSK is implemented by Quadrature signal processing at baseband followed by Quadrature modulator. GMSK signal is an MSK signal with Gaussian shaped frequency pulse defined in equation (15) [14].

$$g(t) = \frac{1}{2T}\left[Q\left(2\pi B_b \frac{t-T/2}{\sqrt{\ln 2}}\right) + Q\left(2\pi B_b \frac{t+T/2}{\sqrt{\ln 2}}\right)\right] \qquad (15)$$

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^\infty \exp\left(-\frac{u^2}{2}\right) du \qquad (16)$$

Where B_b is the 3 dB bandwidth of a low pass Gaussian filter, T is the symbol period and $B_N = B_b T$ is the normalized 3dB bandwidth.

VI. PERFORMANCE ANALYSIS

The conditional symbol error probability of GMSK is given as [15].

$$P(E/\gamma) = \frac{1}{\pi} \int_0^{\pi/2} e^{-\left(\frac{\gamma}{1+\left(d_j-1\right)\cos^2\theta\, d\theta}\right)} \qquad (17)$$

$$\text{Where } \gamma = \frac{\frac{E}{N_0}}{B_n T} \qquad (18)$$

$$d_j = \left(\frac{\sigma}{Ws_j}\right)^2 \qquad (19)$$

$$\sigma^2 = \int_{-\infty}^{\infty} \frac{|2\pi f H(f)|^2 df}{B_n} \qquad (20)$$

$$\text{Where } B_n = \int_{-\infty}^{\infty} |H(f)|^2 df\ ,\ W_s(a) = 2\pi f_d \sum_{i=-2}^{2} a_k - ig_i \qquad (21)$$

H(f) is the transfer function of receiver filter.

Ws_j is the value of $W_s(a)$ for j^{th} sequence of binary number. and T is the bit duration. BT is the Gaussian filter bandwidth.

After substituting of $P(E/\gamma)$ and by considering the application of MGF we get-

$$P\left(E/\gamma\right) = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(-\frac{1}{1+(d_j-1)\cos^2\theta}\right) d\theta \qquad (22)$$

Using the MGF relation for Rician fading channel.

$$M_\gamma\left(-\frac{1}{1+(d_j-1)\cos^2\theta}\right) =$$

$$\left(\frac{N+K}{N+K+\left(\frac{\bar{\gamma}}{1+(d_j-1)\cos^2\theta}\right)}\right)^N . \exp\left[\frac{K}{\frac{(1+K)(1+d_j-1)\cos^2\theta}{\bar{\gamma}}+1}\right] \qquad (23)$$

Equation (23) shows the Symbol Error Probability of Coherent GMSK over Rician fading channel with Rician factor K and diversity order N.

The Symbol Error Probability for coherent GMSK over Rayleigh fading channel can be obtained by substitution of K=0 in equation (23) and we get-

$$P(E) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{N}{N+\left(\frac{\bar{\gamma}}{1+(d_j-1)\cos^2\theta}\right)}\right)^N d\theta \qquad (24)$$

Using the relation for Nakagami-m fading channel [4].

$$M_\gamma(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-mN} \qquad (25)$$

$$M_\gamma(-s) = \left(1 + \frac{\bar{\gamma}}{(1+(d_j-1)\cos^2\theta m)}\right)^{-mN} d\theta \qquad (26)$$

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$$P_s(E) = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{\bar{\gamma}}{(1+(d_j-1)\cos^2\theta m)}\right)^{-mN} d\theta \qquad (27)$$

$$P_s(E) = \frac{1}{\pi}\int_0^{\pi/2}\frac{1}{\left(1+\frac{\bar{\gamma}}{(1+(d_j-1)\cos^2\theta m)}\right)^{mN}} d\theta \qquad (28)$$

A. Numerical Results

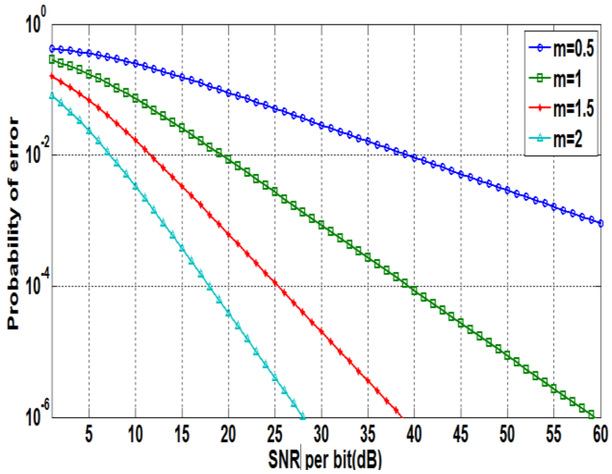

Fig. 1 SEP of GMSK over Nakagami-m fading channel.

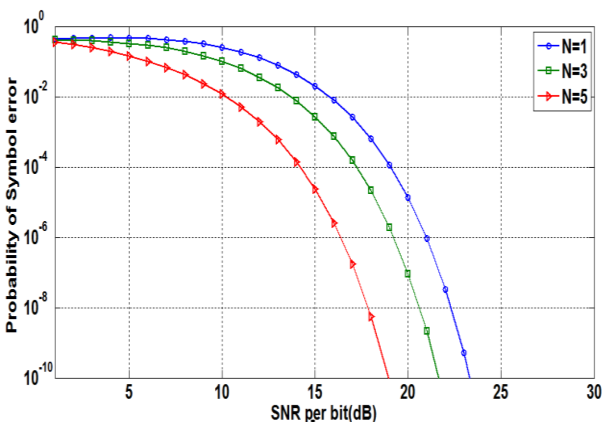

Fig. 2 SEP of GMSK over Rician fading channel with K=2dB.

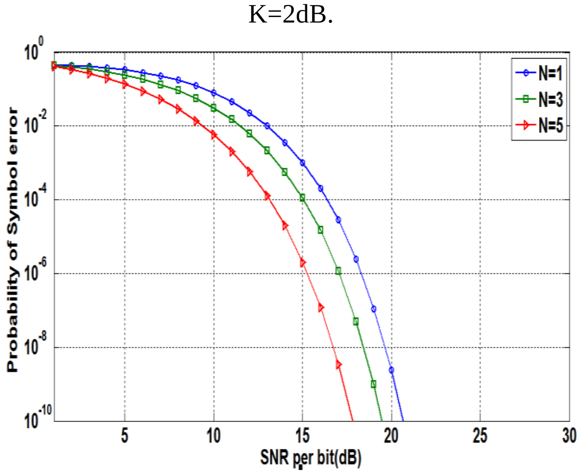

Fig. 3 SEP of GMSK over Rician fading channel with K=6dB.

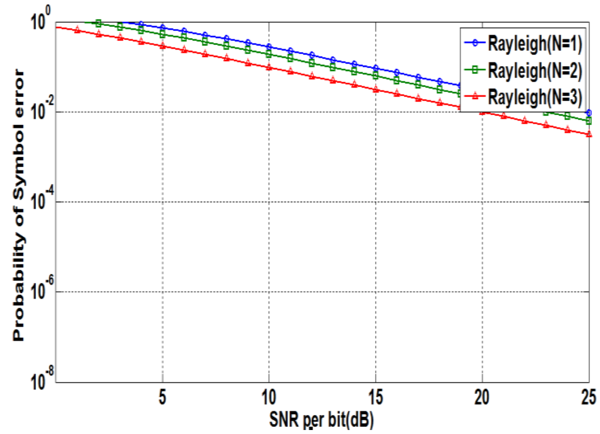

Fig.4 SEP of GMSK over Rayleigh fading channel

Fig 1, Fig 2, Fig 3 and Fig 4 shows the symbol error probabilities of GMSK modulation scheme over Rician and Rayleigh and Nakagami-m fading channel is plotted for different values of Rician parameter K and diversity order N. It is observed from the plots that the performance improves over the same SNR with increase in the value of diversity order while keeping the Rician fading parameter K and Modulation order M fixed.

VII. CONCLUSION

Digital modulation provides more information capacity, compatibility with digital data services, higher data security, better quality communications, and quicker system availability. To study the digital modulation schemes on the basis of the error performance that are very important in today's communication scenario. The demand for higher data rate and better bandwidth efficiency is increased day by day, but the total bandwidth allocation is limited. Therefore it is very much necessary to study the modulation schemes which give us the better result. Digital modulation schemes achieve better bandwidth efficiency and give higher data rate. It is necessary to study that how multipath and fading effect the modulated signals and the degradation of the symbol error rate due to these multipath and different fading channels. In this paper signal degradation due to Rayleigh, Rician and Nakagami-m distribution is to be analyzed. All these analysis is very much important, if it is considered for the wireless environment and every one known that today's world is wireless. Another necessary analysis is to how diversity will improve the degradation of the faded signals. By applying diversity to the receiver end the version of the incoming signal gives the best SNR.

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