Simulation of MUSIC and RLS Algorithm for Adaptive Antenna Array

Dhaval M. Modi, Jayesh S. Patel, Ashwin M. Patel

***Abstract*—Radio frequency spectrum is considered to be an expensive resource to the users. Its efficient utilization is possible using Smart or Adaptive Antenna array system to explore capabilities of wireless systems for optimization of service quality and capacity. Two major issues in smart antennas are Direction of Arrival (DOA) estimation and beamforming. The Direction of Arrival of all the incoming signals including the interfering signals is estimated using DOA algorithms. Some of the DOA algorithms are the MUSIC (Multiple Signal Classification), Root MUSIC, and ESPIRIT (Estimation of Signal Parameters via Rotational Invariance Techniques). Beam is steered in the direction of the desired signal and null are placed in the direction of the interfering signal. So, different beamforming algorithms are used like LMS, RLS. In this paper we have analyzed MUSIC Direction of Arrival (DOA) estimation and RLS algorithm for beamforming.**

***Index Terms* — Beamforming, Direction of Arrival (DOA), Eigen value, MUSIC**

I. INTRODUCTION

The demand for mobile communication services is increasing at a rapid pace throughout the globe. The increasing demand for mobile communication services in a limited RF spectrum motivates the need for better techniques to improve spectrum utilization. A smart antenna system combines multiple antenna elements with a signal-processing capability to optimize its radiation pattern automatically in response to the signal environment [1]. Direction of Arrival (DOA) estimation and Beamforming is a key technology in smart antenna systems so that many different adaptive beamforming algorithms have been the subject of active research.

The goal of direction-of-arrival (DOA) estimation is to estimate the directions of the signals from the desired users as well as the directions of interference signals [2], [3]. The results of DOA estimation are used to adjust the weights of the adaptive beam former so that the radiated power is maximized towards the desired users, and radiation nulls are placed in the directions of interference signals.

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Hence, a successful design of an adaptive array depends highly on the choice of the DOA estimation algorithm which should be highly accurate and robust.

After DOA estimation different beamforming algorithms are used to generate the beam in the desired direction. From the literature we can say that beamforming algorithms are mainly classified into two categories a reference signal based algorithms (Non-Blind) and Blind algorithms [2], [3], [4]. In non-blind algorithms, we required a reference signal at the receiver. LMS, RLS, NLMS are the non-blind algorithms. In blind algorithms a reference single is not required at the receiver.

II. SIGNAL MODEL FOR DOA AND BEAMFORMING

Let a uniform linear array consist of N elements and it receives M narrowband source signals $S_m(t)$ from the desired user arriving at directions θ_1, θ_2, θ_M, as shown in the figure 1.

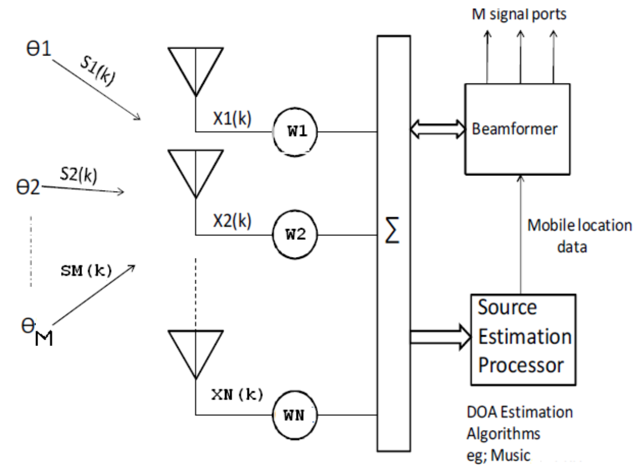

Figure 1 N element antenna array with M arriving signals.

Array also receives interfering signals. The desired signal vector $X_M(t)$ can be defined as

$$x_M(t) = \sum_{m=1}^{M} S_m(t) a(\theta_m) \quad (1)$$

Where $a(\theta_i$ is steering vector and it is given by equation

$$a(\theta_m) = [1, e^{-j\emptyset}, \dots \dots \dots \dots \dots \dots \dots, e^{-(N-1)\emptyset}]^T \quad (2)$$

Where T is a Transposition operator and $\emptyset$ represent phase shift between antenna elements and it is given by

$$\emptyset = (\tfrac{2\pi d}{\lambda})\cos(\theta) \quad (3)$$

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Where d is the spacing between elements and λ is the wavelength of the received signal.

Now the signal vector can be re-written as

$$X_M(t) = A_M.S(t) \tag{4}$$

Where A_M is a steering vector for user signal S(t)

$$X_I(t) = A_I.I(t) \tag{5}$$

Where A_I is a steering vector for interferene signals I(t)

Array output consist of the signal component, interfering signal and noise components

$$X(t) = X_M(t) + X_I(t) + n(t) \tag{6}$$

Here n(t) represent gaussian noise. The estimate of the covariance matrix defined as

$$R = E\{ X(t)\, X^H(t) \} \tag{7}$$

E[] is the expectaion value and H is conjugate transpose.

Equation 7 can be approximated by applying temporal averaging over k samples taken fron the signals incident on the sensor array. So , R is given by[5]

$$R = \frac{1}{K}\sum_{k=1}^{K} X(K)\, X^H(K) \tag{8}$$

Substituting x(t) from equation 5 we can get

$$R = A_M R_{SS} A_M^H + A_I R_{ii} A_I^H + n(k)n^T(k) \tag{9}$$

Where Rss $= E\{ S(t)\, S^H(t) \}$ is desired signal covarince matrix and $R_{ii} = E\{ I(t)\, I^H(t) \}$ is interfering signal covariance matrix.

A. MUSIC (Multiple Signal classification) Algorithm

The term Multiple Signal Classification (MUSIC) is a technique where the parameters of multiple wavefronts arriving at an antenna array are determined from measurements made on the signals received at array elements. MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. Estimation of DOA is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated.

Let the spatial correlation matrix R is eigen decomposed. That is, let λi be the i^{th} eigen value of matrix R with the corresponding eigenvector Vi. Here we assume that each Vi is a column vector. Hence, by definition, we have

$$RVi = \lambda iVi \qquad i = 1, 2, L \tag{10}$$

Where L is the total number of eigenvalues of R. Let H be the multiplicity of the minimum eigenvalue of R. Also, we define the "modal matrix" V as a matrix containing its columns as the eigenvectors of R[6]. That is,

$$V = [v_1, v_2, v_L] \tag{11}$$

We define a "projection matrix" U as

$$U = [u_1, u_2, u_H] \tag{12}$$

Where

$$U_k = \sum_{i=1}^{L-k} V_i V_i^T \quad k = 1, 2, H \tag{13}$$

The matrix U is an L×H matrix whose columns are the H noise-eigenvectors. Let D be the number of signals incident on the antenna array. Then

$$D = L - H \tag{14}$$

This definition implies that U_k is a column vector which is exactly same as the (D + 1)th column of V. Then, the MUSIC method says that the energy of the signal is [6]

$$P(\theta) = \frac{1}{A^H UU^H A} \tag{15}$$

The above equation is known as the multiple signal classification (MUSIC) "spectrum". The MUSIC algorithm has good performance and can be used with a variety of array geometries. The disadvantage of the MUSIC algorithm is that it is not able to identify DOAs of correlated signals and is computationally expensive because it involves a search over the function $P(\theta)$ for the peaks.

1) Steps of the MUSIC Algorithm

Step1: Collect the data samples X

Step2: Estimate the correlation matrix R from Equation $R = E\{ X(t)\, X^H(t)\}$

Step3: Find the eigen values and eigenvectors of X. Hence determine L and H

Step4: Estimate the number of signals from D = L − H

Step5: Find the noise subspace U from $U = [u_1, u_2, u_H]$ and $U_k = \sum_{i=1}^{L-k} V_i V_i^T$ k = 1, 2, H

Step6: Evaluate P(θ) using $P(\theta) = \frac{1}{A^H UU^H A}$

B. RLS (Recursive Least Square) ALGORITHM

The Recursive least squares (RLS) adaptive is an algorithm which recursively finds the filter coefficients that minimize a weighted linear least squares cost function relating to the input signals. This algorithm takes advantage of all the array data information that obtained after the initiation of the algorithm and using the iteration method to realize the inverse operation of the matrix, so the convergence rate is rapid and can realize the trade-off between the rate of the convergence and the computing complexity. This algorithm is not sensitive to the eigen value distribution, but compared to the normal LMS algorithm, its computational complexity is high [7], [8].

The convergence speed of the LMS algorithm depends on the Eigen values of the auto correlation matrix. In an environment yielding an auto correlation matrix with large eigen value spread the algorithm converges with a slow speed this problem is solved with the RLS algorithm by replacing the gradient step size µ with a gain matrix $R^{-1}(n)$ at the nth iteration [8, 9]. Producing the weight update equation.

$$w(n+1) = w(n) + R^{-1}(n)x(n)\varepsilon^*(n) \tag{10}$$

Where $R(n)$ is given by

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$$R(n) = \lambda R(n-1) + x(n)x^H(n) \quad (11)$$

Where λ is known as forgetting factor that represents a real scalar quantity which is small but close to one and used for exponential weighting of the past data.

The RLS adaptive beamforming algorithm updates the required inverse of R (n) using previous inverse and present sample as

$$R^{-1}(n) = \frac{1}{\lambda}\left[R^{-1}(n-1) - \frac{R^{-1}(n-1)x(n)x^H(n)R^{-1}(n-1)}{\lambda + x^H(n)R^{-1}(n-1)x(n)}\right] \quad (12)$$

The matrix R^{-1} is initialized as

$$R^{-1}(0) = \frac{1}{\delta}\mathrm{I}, \delta > 0$$

1) Steps for RLS Algorithm

Step 1: Initialize $R^{-1}(0) = \frac{1}{\delta}$ I, where δ small positive constant is and I is the identity matrix.
Step2: Enter the no. of Iteration and forgetting factor (λ)
Step3: Find y(n) using the equation $y(n) = w^H x(n)$
Step 4: find error e(n) using desired signal and y(n), $e(n) = d(n) - y(n)$
Step5: Find $R^{-1}(n)$ using equation

$$R^{-1}(n) = \frac{1}{\lambda}\left[R^{-1}(n-1) - \frac{R^{-1}(n-1)x(n)x^H(n)R^{-1}(n-1)}{\lambda + x^H(n)R^{-1}(n-1)x(n)}\right]$$

Step 6: Find weight vector w using

$$w(n+1) = w(n) + R^{-1}(n)x(n)\varepsilon^*(n)$$

III. MATLAB SIMULATION AND RESULTS

MUSIC algorithm for DOA estimation and RLS algorithm for Beamforming is simulated in MATLAB. The simulation of the MUSIC algorithm is done for three signal coming from different angles -40°, 0° and 7° for different number of array elements and samples. From the figure 2 shows the MUSIC algortithm simulation for 100,500,1000 samples and from the figure we can say that as the number of samples increases the resolution capabilty increase and signals are clearly identified. In figure 3 MUSIC algorithm simulation for different number of array elements is shown and from that we can say that as the number of arrray elements are more the MUSIC spectrum becomes sharp and and resoltuion capability of MUSIC increases.

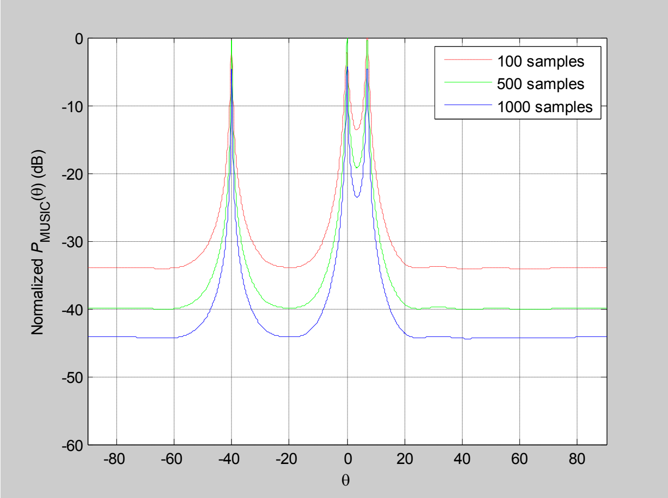

Figure 2 MUSIC spectrum for DOA=-40°, 0° and 7°.

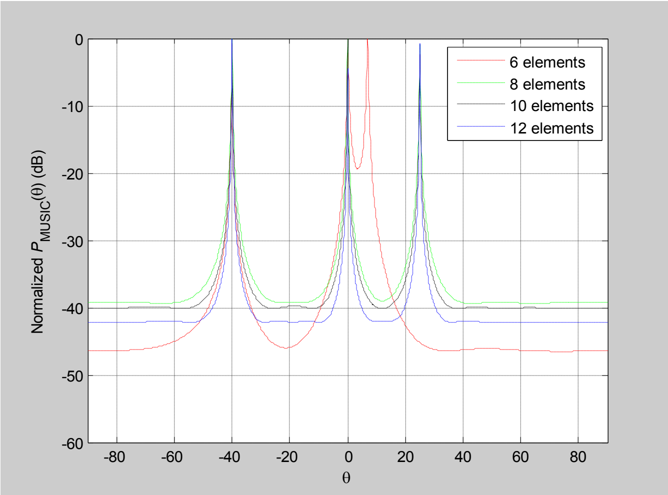

Figure 3 MUSIC spectrum for different number antenna elements and DOA=-40°, 0° and 7°.

RLS beamforming algorithm simulation is done for 4 and 8 antenna elements and seperation in between them is taken as 0.5λ and 500 iteration.We have taken three incoming signal arriving at -40°, 0° and 25° from this Signal at 0° is taken as user signal and other two signal are taken as interfering signals. We have shown the beam pattern, Array factor and Mean Square error plot in the simulation results. Figure 4 and 5 shows the beam pattern, Array factor for 4 array elements and figure 6 shows the mean square error. From the figure 4 we can say that it provides main beam in the direction of user and provides nulls in the direction of interfering signals. Figure 7 and 8 shows the simulation result for 8 array elements and from that we can say that as the number of array elements increases beam becomes sharper and it provids depper nulls in the direction of interfering signals.

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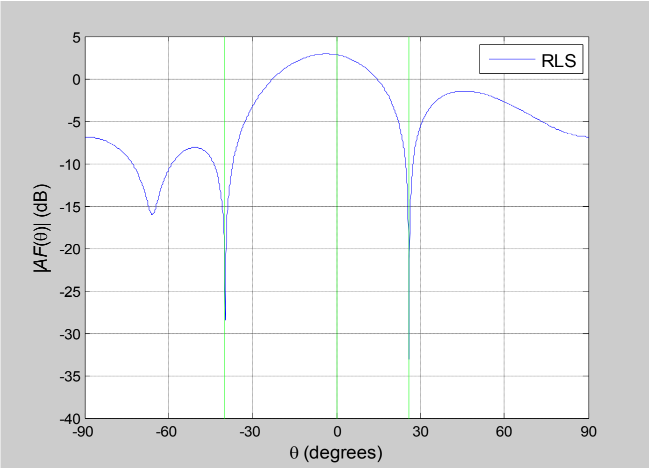

Figure 4 Beam pattern using RLS algorithm for 4 element antenna array d = 0.5λ, user at 0^0, N =500

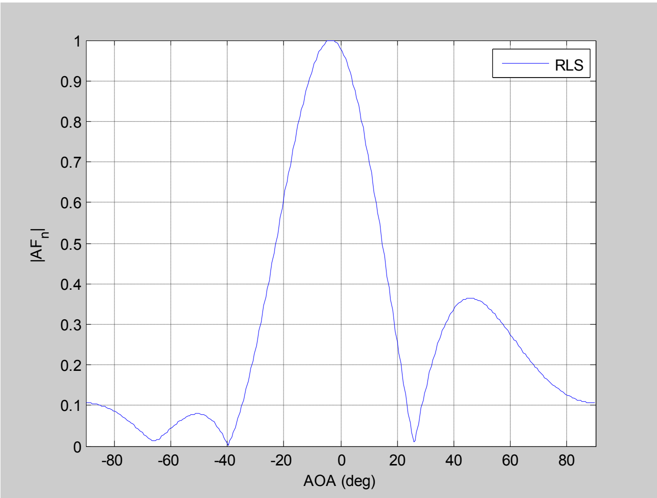

Figure 5 Array factor using RLS algorithm for 4 element antenna array for d = 0.5λ, user at 0^0, N =500

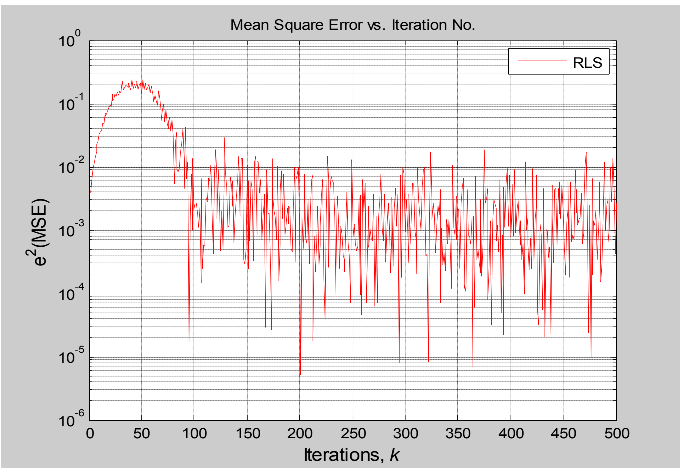

Figure 6 Mean square error plot for 4 element array

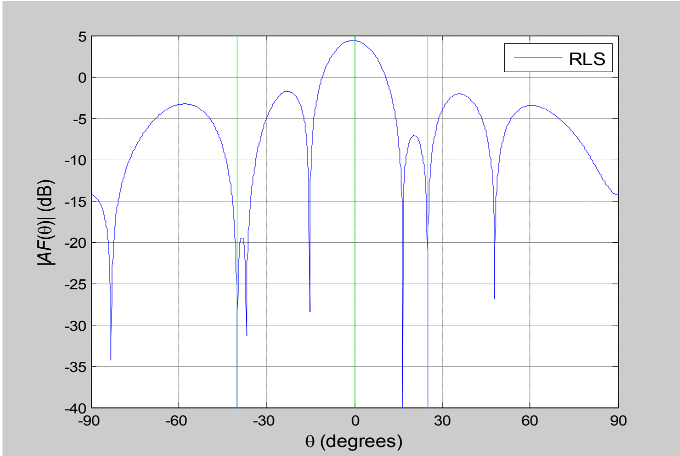

Figure 7 Beam pattern using RLS algorithm for 8 element antenna array d = 0.5λ, user at 0^0, N =500

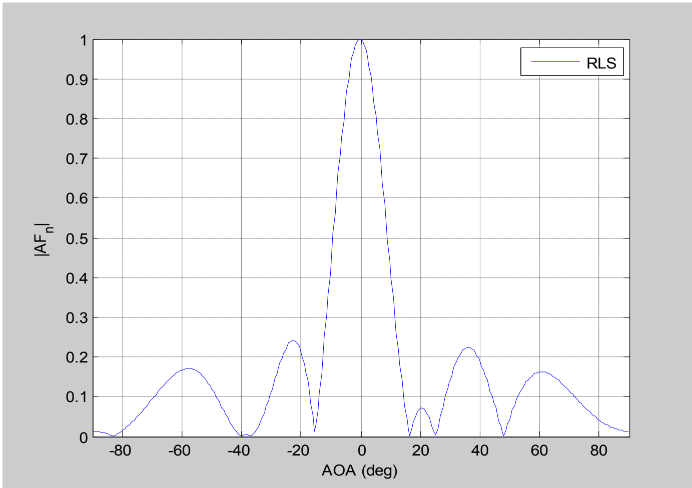

Figure 8 Array factor using RLS algorithm for 8 element antenna array for d = 0.5λ, user at 0^0, N =500

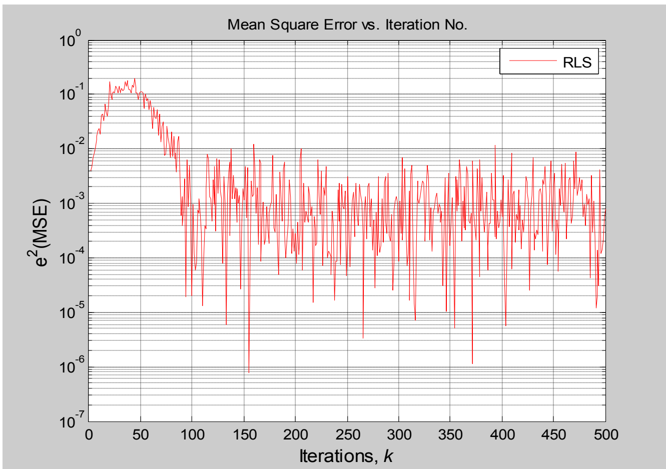

Figure 9 Mean square error plot for 8 element array

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IV. CONCLUSION

In this paper we have presented MUSIC algorithm for DOA estimation and RLS algorithm for adaptive beamforming. Simulation results shows that the performance of the MUSIC algorithm improves with more elements in the array and with increasing the number of samples. This improvements are analyzed in the form of sharper peaks in the MUSIC spectrum. From the simulation results of the RLS algorithm we can say that as we increases the number of array elements then it provides narrower beam in the direction of user signal and depper null in the direction of interferring singnals.

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