DEVELOPMENT OF EFFECTIVE TIME SERIES FORECASTING MODEL

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Abstract. **This article is dedicated to the development of time series forecasting scheme. It is created based on the forecasting models system that determines the trend of time series and its internal rules. The developed scheme is synthesized with the help of basic forecasting models "competition" on a certain time interval. As a result of this "competition", for each basic predictive model there is determined the corresponding weighting coefficient, with which it is included in the forecasting scheme. Created forecasting scheme allows simple implementation in neural basis. The developed flexible scheme of forecasting of economic, social, environmental, engineering and technological parameters can be successfully used in the development of substantiated strategic plans and decisions in the corresponding areas of human activity.**

Keywords**. Trend, forecasting model, time series, functional, step of forecast, autoregression, neural element, neural network.**

I. INTRODUCTION

Analysis of problems that are solved by specialists various scientific and applied fields in the course of them of their professional activities indicate the expediency of specialness use mathematical tools for solving a variety of applications.

In our days, it is very important, to be able to forecast the main indicators, such as: economical, social, medical, technical and so on.

Estimates and forecasts of the financial condition of the company make it possible to find additional resources, to increase its profitability and solvency.

Problems of the analysis and the forecast of financial condition of the company by means of corresponding indicators are an actual task, because on the one hand this is the result of the company, on the other it defines the preconditions for the development of the company. Qualitative forecast gives us an opportunity to develop reasonable strategic plans for economic activity of enterprises.

To determine strategies for enterprise development, calculation of forecasts of economic indicators and factors of organizations plays an important role. If there is reliable information about the company in the past, mathematical methods can be applied to obtain necessary forecasts. These methods depend on the objectives and detailed forecast factors; they also depend on the environment.

Various aspects of the theory, practice, and forecast of financial condition of a company have been the subject of research of many domestic and foreign scientists, such as Blank I.A [1], Heyets V.M. [2], Zaychenko Y.P. [3], Ivakhnenko V.M. [4], Ivakhnenko O.G. [5], Yarkina N.M. [6], Tymashova L. [7], Stepanenko O.P. [8], Matviichuk A.V. [9].

When forecasting the indicators by which the financial position or efficiency of the company's production resources use are determined, it is impossible to point out a single "the best" method of prediction because the internal laws (trends) of various indicator systems are different and there arises the problem of choosing the method of forecasting the studied indicator system.

Forecasting the medical data is also an important task as it make possible to give accurate diagnoses and predict and prevent disease.

One of more difficult and serious problems in medicine is quantitative prediction characteristics of officially registered HIV-infected persons in the region.

Therefore, the development of new forecasting models of corresponding systems of indicators is an actual and important problem.

The aim of the study is to develop an efficient scheme of time series prediction that automatically (in the course of its training) adjusts to the appropriate system of economic, social, environmental, and engineering parameters, and it can be successfully used in the development of high-quality strategic plans in the branch of economy, environment, medicine and for forecast of different natural processes.

The research methodology includes the method of least squares, exponential smoothing method, iterative techniques of minimization of functionals, and methods of synthesis of neural-network schemes.

II. SYNTHESIS OF FORECASTING SCHEMES OF TIME SERIES BASED ON CLASSIC FORECASTING METHODS.

Let $v_1, v_2, \ldots v_t, \ldots, v_n$ be a time series. Prognostic value $\tilde{v}_t$ of the element v_t at the instant of time t can be written as follows [10]

$$\tilde{v}_t = f(a_1,\ldots,a_r, v_{t-1},\ldots,v_{t-k}, t), \qquad (1)$$

where $a_1,\ldots,a_r$ are the model parameters, k is the depth of prehistory. To find the parameters $a_1,\ldots,a_r$, we constructed the functional

$$L(a_1,\ldots,a_r) = \sum_{t=1}^{n} \left(v_t - \tilde{v}_t\right)^2, \qquad (2)$$

which is usually to be minimized. Let $a_1^*,\ldots,a_r^*$ are the values of parameters $a_1,\ldots,a_r$ for which the functional L takes its minimum value. Then the prognostic value $\tilde{v}_{n+\tau}$ of the model f with optimal parameters $a_1^*,\ldots,a_r^*$ is determined as follows

$$\tilde{v}_{n+\tau} = f(a_1^*,\ldots,a_r^*, v_{t-1},\ldots,v_{t-k}, n+\tau), \qquad (3)$$

where τ is the step of the forecast. Depending on the type of the function f with the parameters $a_1^*,\ldots,a_r^*$, we have different optimal forecasting models of time series.

To build a predictive scheme, at the beginning let us consider the autoregression method by means of which we define the optimal step of the prehistory k_τ^* for the given time series v_t with the fixed step of the forecast τ. In the autoregression model, it is assumed that the indicator value v_t at the instant of time t depends on $v_{t-\tau}, v_{t-\tau-1},\ldots,v_{t-\tau-k_\tau+1}$, where k_τ is the parameter of the prehistory with fixed τ. The prognostic value $\tilde{v}_{n+\tau}$ by the autoregression method is found according to the following model

$$\tilde{v}_{n+\tau} = a_1^{(\tau)} v_n + a_2^{(\tau)} v_{n-1} + \ldots + a_{k_\tau}^{(\tau)} v_{n-k_\tau+1}. \quad (4)$$

To determine the optimal values of the parameters $a_t^{*(\tau)} (t = 1,2,\ldots,k_\tau)$ for a fixed τ $(t = t_0)$, we minimize the functional

$$L(a_1^{(\tau)},\ldots,a_{k_\tau}^{(\tau)}) = \sum_{t=k_\tau+\tau}^{n} \left(v_t - a_1^{(\tau)} v_{t-\tau} - \ldots - a_{k_\tau}^{(\tau)} v_{t-\tau-k_\tau+1} \right)^2, \quad (5)$$

i.e. we solve the system of equations

$$\frac{\partial L}{\partial a_i^{(\tau)}} = 0,\ t = 1,2,\ldots,k_\tau. \quad (6)$$

Let $a_1^{*(\tau)},\ldots,a_{k_\tau}^{*(\tau)}$ be a solution of the system (6). Then, according to (4) we have

$$\tilde{v}_t = a_1^{*(\tau)} v_{t-\tau} + a_2^{*(\tau)} v_{t-\tau-1} + \ldots + a_{k_\tau}^{*(\tau)} v_{t-\tau-k_\tau+1}, \quad (7)$$

where $t \ge k_\tau + \tau$.

It is obvious that the variable $\tilde{v}_t$ for a fixed value of τ $(\tau = \tau_0)$ depends on the parameter k_τ $(1 \le k_\tau \le n - \tau)$. To determine the optimal value of the prehistory parameter k_τ for $\tau = \tau_0$ for the given time series v_τ, let us consider the variables

$$\delta_1 = \frac{1}{n-\tau} \sum_{t=\tau+1}^{n} \left(v_t - a_1^{*(\tau)} v_{t-\tau} \right)^2,$$

$$\delta_2 = \frac{1}{n-\tau-1} \sum_{t=\tau+2}^{n} \left(v_t - a_1^{*(\tau)} v_{t-\tau} - a_2^{*(\tau)} v_{t-\tau-1} \right)^2,$$

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$$\delta_{n-\tau} = \left(v_n - a_1^{*(\tau)} v_{n-\tau} - \ldots - a_{n-\tau}^{*(\tau)} v_1 \right)^2$$

Thus we obtain $\min\{\delta_1, \delta_2, \ldots, \delta_{n-\tau}\} = \delta_{k_\tau^*}$. The variable k_τ^* determines the optimal value of the prehistory parameter in the autoregression model for a fixed τ $(\tau = \tau_0)$.

After determining the k_τ^* for a fixed t $(\tau = \tau_0)$, consider the main base forecasting models $M_1, M_2, \ldots M_q$ of time series with the fixed step of the forecast τ, i.e. models on the bases of which a new forecasting scheme are synthesized. Using the results of the forecasting models mentioned above on the time interval $t = n - k_\tau^* + 1, n - k_\tau^* + 2, \ldots, n$, we draw the following table

Table 1. The Prognostic Values of Time Series

Forecasting Models	**Elements of Time Series** v_t			
	$v_{n-k_\tau^*+1}$	$v_{n-k_\tau^*+2}$	$\cdots$	v_n
M_1	$\tilde{v}_{n-k_\tau^*+1}^{(1)}$	$\tilde{v}_{n-k_\tau^*+2}^{(1)}$	$\cdots$	$\tilde{v}_n^{(1)}$
M_2	$\tilde{v}_{n-k_\tau^*+1}^{(2)}$	$\tilde{v}_{n-k_\tau^*+2}^{(2)}$	$\cdots$	$\tilde{v}_n^{(2)}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
M_q	$\tilde{v}_{n-k_\tau^*+1}^{(q)}$	$\tilde{v}_{n-k_\tau^*+2}^{(q)}$	$\cdots$	$\tilde{v}_n^{(q)}$

In each column $v_{n-k_\tau^*+1}, v_{n-k_\tau^*+2}, \ldots, v_n$ of Table 1, we can find the least squared difference of the prognostic and the actual values of the corresponding time series terms. Mathematically this can be written as following:

let $j_1 = n - k_\tau^* + 1$ and

$$\varepsilon_1 = \min\left\{ (v_{j_1} - \tilde{v}_{j_1}^{(1)})^2, (v_{j_1} - \tilde{v}_{j_1}^{(2)})^2, \ldots, (v_{j_1} - \tilde{v}_{j_1}^{(q)})^2 \right\},$$

$j_2 = n - k_\tau^* + 2$ and

$$\varepsilon_2 = \min\left\{ (v_{j_2} - \tilde{v}_{j_2}^{(1)})^2, (v_{j_2} - \tilde{v}_{j_2}^{(2)})^2, \ldots, (v_{j_2} - \tilde{v}_{j_2}^{(q)})^2 \right\},$$

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$j_{k_\tau^*} = n$ and

$$\varepsilon_{k_\tau^*} = \min\left\{ (v_n - \tilde{v}_n^{(1)})^2, (v_n - \tilde{v}_n^{(2)})^2, \ldots, (v_n - \tilde{v}_n^{(q)})^2 \right\}$$

Define the sets $I_1, I_2, \ldots, I_{k_\tau^*}$ as follows

$$I_1 = \left\{ i \in \{1,2,\ldots,q\} \middle| \varepsilon_1 = (v_{j_1} - v_{j_1}^{(i)})^2 \right\},$$

$$I_2 = \left\{ i \in \{1,2,\ldots,q\} \middle| \varepsilon_2 = (v_{j_2} - v_{j_2}^{(i)})^2 \right\},$$

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$$I_{k_\tau^*} = \left\{ i \in \{1,2,\ldots,q\} \middle| \varepsilon_{k_\tau^*} = (v_n - v_n^{(i)})^2 \right\}$$

and draw the table

Table 2. Parameters for Determining the Weighting Coefficients of the Model

Forecasting Models	j_1	j_2	$\cdots$	$j_{k_\tau^*}$	**Resultant Column**
M_1	a_{11}	a_{12}	$\cdots$	$a_{1k_\tau^*}$	S_1
M_2	a_{21}	a_{22}	$\cdots$	$a_{2k_\tau^*}$	S_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
M_q	a_{q1}	a_{q2}	$\cdots$	$a_{qk_\tau^*}$	S_q

where

$$a_{ps} = \begin{cases} \beta^{k_\tau^* - s}, & if\ s \in I_s, \\ 0, & if\ s \notin I_s, \end{cases}$$

$$S_p = \sum_{j=1}^{k_\tau^*} a_{pj}, 0 < \beta \le 1, \quad (p = 1,2,\ldots,q,\ s = 1,2,\ldots,k_\tau^*).$$

With the help of $S_p = S_p(\beta)$ and $S(\beta) = \sum_{p=1}^{q} S_p(\beta)$ we determine the weighting coefficients of the forecasting models $M_p(p \le q)$, with which these models are included in the following forecasting scheme

$$\tilde{v}_{n+\tau} = \frac{S_1(\beta)}{S(\beta)} \tilde{v}_{n+\tau}^{(1)} + \frac{S_2(\beta)}{S(\beta)} \tilde{v}_{n+\tau}^{(2)} + \ldots + \frac{S_q(\beta)}{S(\beta)} \tilde{v}_{n+\tau}^{(q)}. \qquad (8)$$

The coefficients of the forecasting models in the scheme (8) depend on the parameter β that determines the influence of the element v_t upon the prognostic value $\tilde{v}_{n+\tau}$. The more remote element v_t is from the prognostic point $\tilde{v}_{n+\tau}$, the less is its influence on the prognostic value $(0 < \beta < 1)$. In the case of $\beta = 1$, all points of time series v_t are equivalent, i.e. in the model (8) the distance of the element v_t from the prognostic point $\tilde{v}_{n+\tau}$ is not taken into account.

Synthesis of the predictive scheme (8) will be completed in the course of training its concerning β. For this purpose, we construct the functional

$$L(\beta) = \sum_{i=1}^{k_\tau^*} (v_{j_i} - \frac{S_1(\beta)}{S(\beta)} \tilde{v}_{j_i}^{(1)} - \ldots - \frac{S_{q+r}(\beta)}{S(\beta)} \tilde{v}_{j_i}^{(q)})^2, \ (j_i = n - k_\tau^* + i),$$

and minimize it by varying the value β. The interval (0,1] we divide into m equal subintervals and find the value $L(\beta_i)$ at the points $\beta_i = \frac{i}{m}$ $(i = 1,2,\ldots,m)$. It is obvious that m gives the accuracy of the finding the minimum of the functional $L(\beta)$. Let $L(\beta_m^*) = \min L(\beta_i)$. Then the forecast of time series we conduct according to the scheme (8), substituting β_m^* for β.

Implementation of Forecasting Schemes of Time Series in Artificial Neural Basis

The basis of all forecasting methods is an idea of extrapolation of patterns of the development of the process, which was formed by the time when the forecast came true for future period of time.

Let $v_1, v_2, \ldots v_t, \ldots, v_n$ is time series. For the synthesis of artificial neural-network forecasting scheme, there must exist a method (methods) of synthesis of neural elements that implement appropriate forecasting models, on whose basis a neural scheme should be constructed. For example, the following artificial neural element with linear activation function implements the autoregression model $\tilde{v}_{n+\tau} = w_1^{(\tau)} v_n + w_2^{(\tau)} v_{n-1} + \ldots \ldots + w_{k_\tau^*}^{(\tau)} v_{n-k_\tau^*+1}$, with the optimal step k_τ^* of the prehistory and the step of the forecast τ if $w_1^{(\tau)} = a_1^{*(\tau)}, \ldots, w_{k^*}^{(\tau)} = a_{k_\tau^*}^{*(\tau)}$ $a_1^{*(\tau)}, \ldots, a_{k_\tau^*}^{*(\tau)}$ are optimal values of parameters of the autoregressive model.

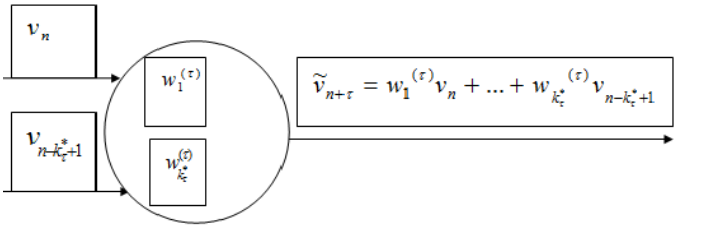

Fig. 1. Neuron of the optimal autoregressive model

After the development of methods for the synthesis of neural elements that implement the optimal forecasting models in the corresponding classes of models, to predict the values $v_i (i = 1,2,\ldots,n)$ at instants of time $t = n+\tau$, let us design the following neural- network scheme

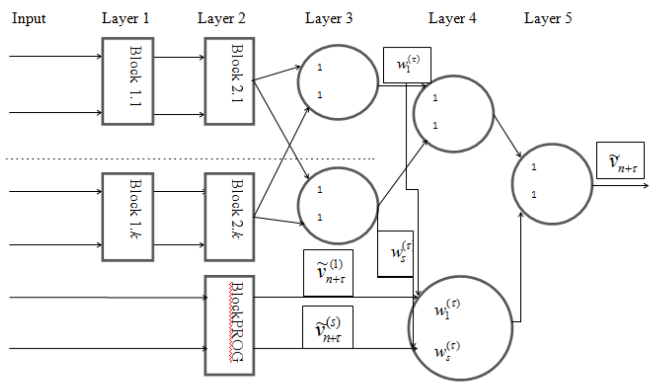

Fig. 2. Neuro-scheme for Time Series Prediction

All the blocks of the 1st layer contain the same number S of neurons, where each neuron implements one of the forecasting models (autoregressive model, polynomial, exponential, linear ones, Brown's linear model, etc.). Neurons that implement the same model in different blocks of this layer have the same serial number. Each Block 2. m ($m = 1,2,\ldots,k$; $k = k_\tau^*$ **Error! Reference source not found.**) of the 2nd layer contains as much neurons as in Block 1. m. In Block 2. m each neuron has two inputs and a weight vector (1,1), where the value v_{n-k+m} is given to the first input**Error! Reference source not found.**, and the prognostic value $\tilde{v}_{n-k+m,i}^{(\tau)}$ is given to the 2nd input,**Error! Reference source not found.** which is the output signal of the ith neuron of Block 1.m. Activation function of the ith neuron of Block 2. m is set as follows $\exp(-(v_{n-k+m} - \tilde{v}_{n-k+m,i}^{(\tau)})^2)$ **Error! Reference source not found.**. The neuron of the serial number i of Block 2. m is related to i th neuron of the 3rd layer in the following way: from the i th neuron of Block 2. m to the m th input of the i th neuron of the 3rd layer there is given the signal $f_{m,i}^{(\tau)}$ **Error! Reference source not found.**, where

$$f_{m,i}^{(\tau)} = \begin{cases} 1, \text{ if } i = \arg\max(\exp(-(v_{n-k+m} - \tilde{v}_{n-k+m,i}^{(\tau)})^2)), \\ 0, \quad \text{otherwise}. \end{cases}$$

Neurons of the 3rd layer have the linear activation function, and each of the weighting coefficients of each neuron is equal to 1. At the output of the ith neuron of the 3rd layer for the fixed τ we obtain the number $w_i^{(\tau)}$. The 3rd layer, except for neurons with linear activation function, has one more BlokPROG containing exactly as many neurons as a Block of the 1st layer contains. Neurons of this block implement corresponding forecasting model with the depth τ and their serial numbers coincide with the numbers of neurons of Blocks of Layer 1.

The 4th layer contains two linear neurons. The first neuron has s inputs, all its weighting coefficients are equal to 1, and it has activation function $w_1^{(\tau)} + w_2^{(\tau)} + \ldots + w_s^{(\tau)}$.

The second neuron of this layer has weighting coefficients $w_1^{(\tau)}, w_2^{(\tau)}, \ldots, w_s^{(\tau)}$ **Error! Reference source not found.**. If **Error! Reference source not found.** the forecast result of the i^{th} model of BlockPROG is denoted by $\tilde{v}_{n+\tau}^{(i)}$, then at the output of the second neuron of Layer 4 we have $w_1^{(\tau)}\tilde{v}_{n+\tau}^{(1)} + \ldots + w_s^{(\tau)}\tilde{v}_{n+\tau}^{(s)}$ **Error! Reference source not found.**.

The 5th layer contains one neuron that has two inputs, a weight vector (1.1), and the activation function $\tilde{v}_{n+\tau} = \dfrac{w_1^{(\tau)}\tilde{v}_{n+\tau}^{(1)} + \ldots + w_s^{(\tau)}\tilde{v}_{n+\tau}^{(s)}}{w_1^{(\tau)} + w_2^{(\tau)} + \ldots + w_s^{(\tau)}}$.

Blocks 2. m ($m = 1,2,\ldots,k_\tau^*$ **Error! Reference source not found.**) determine the most effective basic forecasting models. At the output of the scheme we have a convex linear combination of the best forecasting models.

III. FORECASTING THE ECONOMICAL INDICATORS.

To compare the quality of forecasting, it is often used the average relative error (MRE - Mean Relative Error) is often used

$$MRE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{v_t - \tilde{v}_t}{v_t}\right|, \quad (9)$$

and the average square error (RMSE -Root Mean Square Error) is also used

$$RMRE = \sqrt{\frac{\sum_{t=1}^{n}(v_t - \tilde{v}_t)^2}{n}}, \quad (10)$$

where v_t are the terms of the time series, $\tilde{v}_t$ are the prognostic values of v_t. RMSE and MRE are relative errors, i.e. they can be used to compare two (or more) different time series prediction the best is the forecast whose value of MRE (9) or RMSE (10) is less.

According to the average relative error criterion, the quality of the forecast of the constructed predicting scheme is estimated by comparing its results with the results of main forecasting models on base of which it is synthesized. To perform this, we use data from the following Table 3[11].

Table 3. The Original and Forecasted Volumes of Passenger Traffic

Year	Railway	Sea	River	Automobile (coaches)[1]	Aircraft	Undeground railway
1980	648869	28478.4	24789	7801058	12492.4	430040
1981	653177	30705.6	27531.6	7794859	12720	473437
1982	656485	29362.2	26629.4	7874069	12728.7	515382
1983	668287	29690.2	26810.8	7876161	12711.6	520700
1984	687645	29228.8	24979.6	7998739	12777.8	551851
1985	695129	28660.6	23817.4	8076846	12616	602671
1986	734204	28681	21008.5	8230409	12797.5	598022
1987	717461	27567.3	18750.2	8383820	12670.4	590513
1988	711123	27961.5	20345.5	8552803	13065.3	634616
1989	704078	26524.3	20199.7	8382872	14299.6	648816
1990	668979	26256.7	19090.3	8330512	14833	678197
1991	537407	20786.5	18285.8	7450322	13959.6	595313
1992	555356	13139.5	11158	6464891	5669.3	610668
1993	501495	10497	8064.4	4795664	1947.4	644417
1994	630959	10358.2	6967.9	4039917	1673.3	684480
1995	577432	7817	3594.1	3483173	1914.9	561012
1996	538569	5044.6	2735.9	3304600	1724	536304
1997	500839	4311.3	2443.1	2512147	1484.5	507897
1998	501429	3838.3	2356.5	2403425	1163.9	668456
1999	486810	3084.3	2269.4	2501708	1087	724426

2000	498683	3760.5	2163.3	2557515	1164	753540
2001	467825	5270.8	2034.2	2722002	1289.9	793197
2002	464810	5417.9	2211.9	3069136	1767.5	831040
2003	476742	6929.4	2194.1	3297505	2374.7	872813
2004	452226	9678.4	2140.2	3720326	3228.5	848176
2005	445553	11341.2	2247.6	3836515	3813.1	886598
2006	448422	10901.3	2021.9	3987982	4350.9	917700
2007	447094	7690.8	1851.6	4173034	4928.6	931512
2008	445466	7361.4	1551.8	4369126	6181	958694
2009	425975	6222.5	1511.6	4014035	5131.2	751988
2010	427241	6645.6	985.2	3726289	6106.5	760551
2011	429785	7064.1	962.8	3611830	7504.8	778253
2012	429115	5921	722.7	3450173	8106.3	774058
2013	425217	6642	631.1	3343660	8107.2	774794
2014	**424272.5**	**3490.2**	**453.8915**	**3059461.2**	**9308.8**	**816682.9**
2015	**414375.8**	**5373.2**	**406.4361**	**2645239.9**	**7243.7**	**876984.7**
2016	**425925.3**	**3847.1**	**369.0345**	**2641221.6**	**10609.4**	**972098.3**
2017	**420469.8**	**2975.1**	**233.0464**	**2395820.5**	**10870.7**	**1073108.1**
2018	**426849.1**	**3061.2**	**403.4616**	**2606148.8**	**12330.5**	**1205853.8**

Table 4. Forecast Errors of Passenger Traffic according to MRE criterion

Forecasting methods	Kinds of passenger traffic		
	Railway	River	Automobile
Step of the forecast $\tau = 1$			
Autoregression method	0.0041	0.0148	0.0115
The method of least squares with weights	0.015	0.7975	0.1680
Brown's linear model	0.0358	0.0917	0.1478
Brown'squadratic model	0.0159	0.5516	0.086
Forecasting scheme	0.0039	0.0148	0.0115
Step of the forecast $\tau = 5$			
Autoregression method	0.0045	0.0111	0.0233
The method of least squares with weights	0.0048	0.0683	0.0595
Brown's linear model	0.0585	0.0757	0.1482
Brown'squadratic model	0.0317	0.2295	0.0797
Forecasting scheme	0.0031	0.0108	0.0225

Having analyzed the data in Table 4, we see that the least average relative error occurs in the constructed forecasting scheme. In the two cases (for $\tau = 1$), the error of the scheme coincides with the error of autoregression method. Thus, in general, the scheme developed in this work is the most effective among the methods on which it is based. To obtain the average error (%) of the prediction methods for the given time series in percentage, one should multiply by 100% the corresponding values of quality from Table 4. The quality of the prediction methods of passenger traffic for the forecast period (2014-2018) with the steps of the forecast $\tau = 1$ and $\tau = 5$ is shown in the following charts

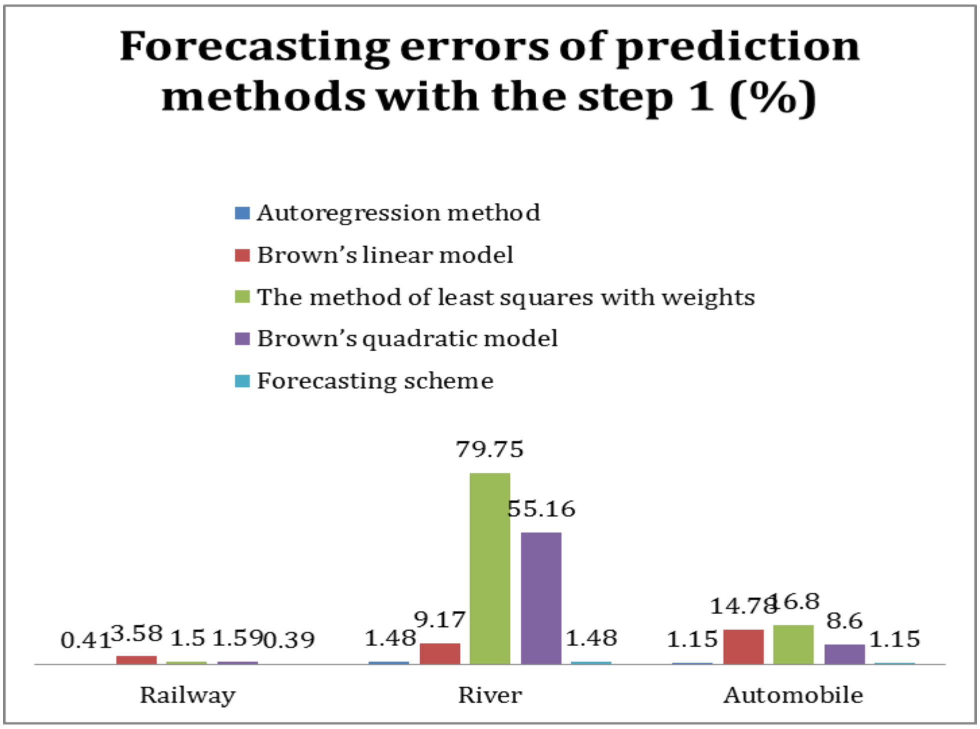

Fig. 3. Forecasting errors of prediction methods with the step 1

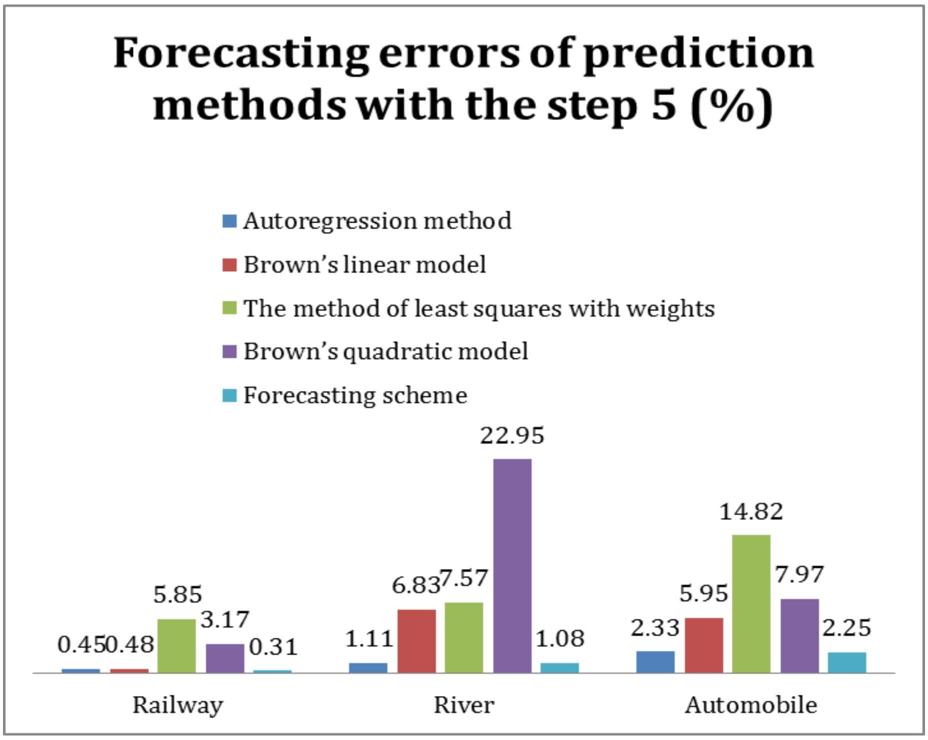

Fig. 4. Forecasting errors of prediction methods with the step 5

Note. The constructed forecasting scheme is flexible. This means that a new model can be added to or excluded from basic models (on basis of which the predictive scheme is constructed) at any time. It should be noted that the method of synthesis of the very predictive scheme does not change.

IV. FORECASTING MEDICAL DATA.

The object of study is the task of forecasting quantitative characteristics of officially registered HIV-infected persons in the region.

For prognosing quantitative characteristics of HIV-infected person, we were include in our scheme these method: autoregression, least square method with weight, 1-st order Braun's method, 2-nd order Braun's method and Winter's method, which not been included in scheme of foresting economical data.

Winter's method apply to account seasonal components of prognosing time series. Winter's model is a three-parametrical model of exponential smoothing.

The system of equations of Winter's method:

$$\begin{cases} z_t = \alpha \dfrac{v_t}{s_{t-k}} + (1-\alpha)(z_{t-1} - \omega_{t-1}) \\ \omega_t = \beta(z_t - z_{t-1}) + (1-\beta)\omega_{t-1} \\ s_t = \gamma \dfrac{v_t}{z_t} + (1-\gamma)s_{t-k} \\ v_{t+\tau} = (z_t + \tau\omega_t)s_{t-s+\tau} \end{cases}$$

where $\alpha, \beta, \gamma \in [0,1]$ **Error! Reference source not found.**.

First equation, in the system above, include seasonality with parameter v_t **Error! Reference source not found.**.

In the case of forecasting medical data, we were prognosed the number of HIV-infected person and number of AIDS patient in the Transcarpatian region of Ukraine from 1987 till 2014 years. Forecasting data we can see in the Table 5.

Table 5. Number of HIV-infected and AIDS person in region.

Transcarpation region (year)	Number of HIV-infected person by the year	Number of AIDS patient by the year
1987-1995	6	1
1996	8	2
1997	35	2
1998	51	4
1999	63	4
2000	77	4
2001	94	4
2002	91	4
2003	77	7
2004	73	16
2005	100	23
2006	109	18
2007	136	16
2008	154	17
2009	185	16
2010	221	16
2011	251	22
2012	288	40
2013	322	57
2014	370	87

Table 6. Forecasting number of HIV-infected person

Number of HIV-infected person						
Step 1(2015)	**Autoregression method (M4)**	**Winters method (M1)**	**Least square method with weight (M2)**	**1-st order Braun's method (M3)**	**2-nd order Braun's method (M5)**	**Schema**
Forecasting values	417,2785	326,8238	417,8323	324,6751	182,3688	417,5316
Root Mean Square Error	3,7217	5,8654	2,0990	34,1946	72,1486	2,0878
Mean Relative Error	0,0092	0,0134	0,0044	0,0817	0,1350	0,0043
$\tilde{v}_{2014+1} = 0*\tilde{v}^{(1)}_{2014+1} + 0.9*\tilde{v}^{(2)}_{2014+1} + 0*\tilde{v}^{(3)}_{2014+1} + 0.1*\tilde{v}^{(4)}_{2014+1} + 0*\tilde{v}^{(5)}_{2014+1}$						
Step 3(2017)	**Autoregression method (M4)**	**Winters method (M1)**	**Least square method with weight (M2)**	**1-st order Braun's method (M3)**	**2-nd order Braun's method (M5)**	**Schema**
Forecasting values	550,7062	387,4223	504,4810	262,2166	198,6956	524,9736
Root Mean Square Error	9,3124	7,0439	5,2190	55,8452	67,4809	3,4319
Mean Relative Error	0,0171	0,0137	0,0102	0,1014	0,1142	0,0051
$\tilde{v}_{2014+3} = 0.16*\tilde{v}^{(1)}_{2014+3} + 0.65*\tilde{v}^{(2)}_{2014+3} + 0*\tilde{v}^{(3)}_{2014+3} + 0.19*\tilde{v}^{(4)}_{2014+3} + 0*\tilde{v}^{(5)}_{2014+3}$						
Step 5(2019)	**Autoregression method (M4)**	**Winters method (M1)**	**Least square method with weight (M2)**	**1-st order Braun's method (M3)**	**2-nd order Braun's method (M5)**	**Schema**
Forecasting values	731,4491	451,2812	642,9561	457,7021	255,4746	648,6653
Root Mean Square Error	12,4666	6,8865	1,9769	35,8431	54,4761	1,4689
Mean Relative Error	0,0180	0,0117	0,0032	0,0629	0,0814	0,0024
$\tilde{v}_{2014+5} = 0*\tilde{v}^{(1)}_{2014+5} + 0.94*\tilde{v}^{(2)}_{2014+5} + 0*\tilde{v}^{(3)}_{2014+5} + 0.06*\tilde{v}^{(4)}_{2014+5} + 0*\tilde{v}^{(5)}_{2014+5}$						

Table 7. Forecasting number of AIDS patient

Number of AIDS patient						
Step 1(2015)	**Autoregression method (M4)**	**Winters method (M1)**	**Least square method with weight (M2)**	**1-st order Braun's method (M3)**	**2-nd order Braun's method (M5)**	**Schema**
Forecasting values	119,1014	62,8473	77,5556	28,5667	22,6173	73,1632
Root Mean Square Error	2,2723	2,6672	9,0178	15,6766	17,2257	1,3099
Mean Relative Error	0,0210	0,0201	0,2076	0,1046	0,1251	0,0139
$\tilde{v}_{2014+1} = 0.62 * \tilde{v}^{(1)}_{2014+1} + 0 * \tilde{v}^{(2)}_{2014+1} + 0 * \tilde{v}^{(3)}_{2014+1} + 0.38 * \tilde{v}^{(4)}_{2014+1} + 0 * \tilde{v}^{(5)}_{2014+1}$						
Step 3(2017)	**Autoregression method (M4)**	**Winters method (M1)**	**Least square method with weight (M2)**	**1-st order Braun's method (M3)**	**2-nd order Braun's method (M5)**	**Schema**
Forecasting values	100,9326	97,2076	81,5659	28,2007	22,5858	70,9085
Root Mean Square Error	5,9290	2,6620	8,8002	16,5761	17,5152	2,6620
Mean Relative Error	0,0665	0,0182	0,1441	0,1060	0,1207	0,0182
$\tilde{v}_{2014+3} = 1 * \tilde{v}^{(1)}_{2014+3} + 0 * \tilde{v}^{(2)}_{2014+3} + 0 * \tilde{v}^{(3)}_{2014+3} + 0 * \tilde{v}^{(4)}_{2014+3} + 0 * \tilde{v}^{(5)}_{2014+3}$						
Step 5(2019)	**Autoregression method (M4)**	**Winters method (M1)**	**Least square method with weight (M2)**	**1-st order Braun's method (M3)**	**2-nd order Braun's method (M5)**	**Schema**
Forecasting values	91,5008	132,6023	39,4722	26,0741	21,1784	91,5415
Root Mean Square Error	0,8784	2,6620	14,9432	17,5899	17,9610	0,8785
Mean Relative Error	0,0145	0,0180	0,0989	0,1087	0,1136	0,0145
$\tilde{v}_{2014+5} = 0.0009 * \tilde{v}^{(1)}_{2014+5} + 0 * \tilde{v}^{(2)}_{2014+5} + 0 * \tilde{v}^{(3)}_{2014+5} + 0.9991 * \tilde{v}^{(4)}_{2014+5} + 0 * \tilde{v}^{(5)}_{2014+5}$						

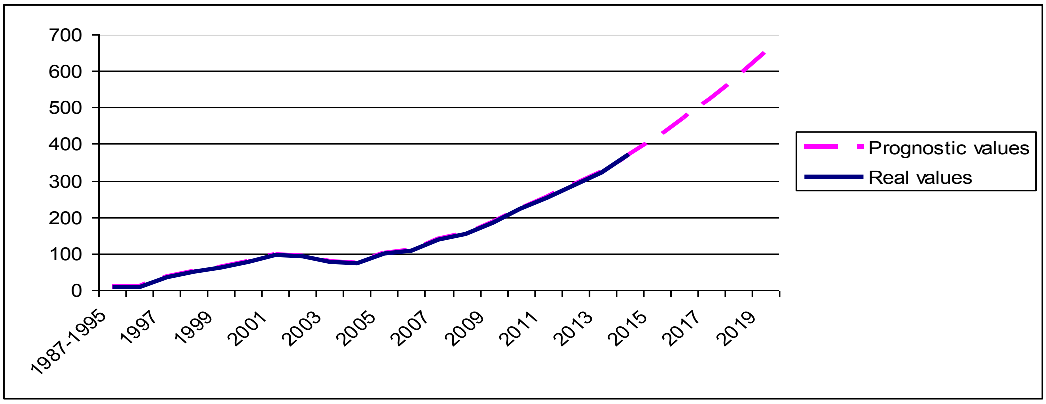

Fig. 5. Number of HIV-infected person 1987-2019 years

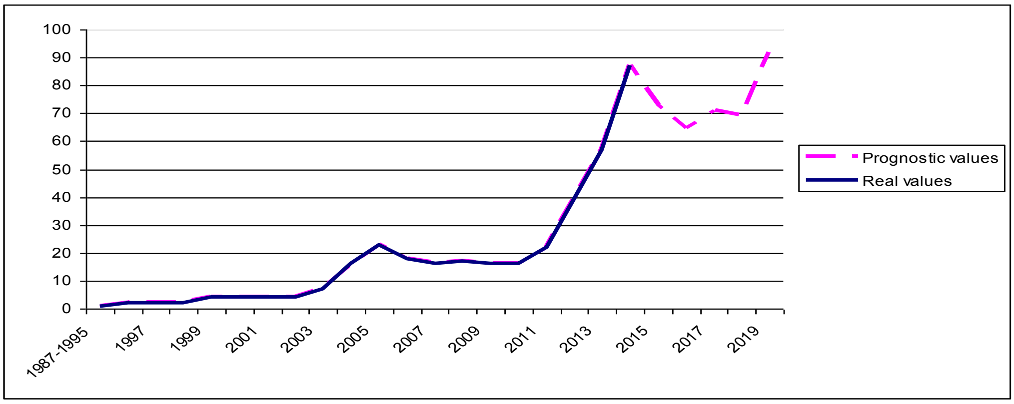

Fig. 6. Number of AIDS patient 1987-2019 years

Having analyzed the data in Table 5, we can see that in each case of step (1, 3, 5) “won” the Least square method with weight. But in the Table 6 we see, that in each case preference received Winters method.

In the process of forecasting medical data we obtained results, which shown on the fig. 5, 6.

V. CONCLUSIONS

A flexible scheme for forecasting of economic, social, environmental, engineering and technological indicators that can be successfully used in the development of reasonable strategic plans and decisions in the corresponding fields of human activity is worked out.

This forecasting scheme allows us to include new forecasting models of time series or to exclude a model or groups of models from it at any instant of time.

As for the models which remain in the scheme, the competition between them is made over a given period of time, and the final forecasting scheme represents a convex linear combination of models -winners with corresponding weighting coefficients.

The forecasting methods can be dynamically included to the prognosing scheme, and also can be excluded of the scheme in the process of forecasting. That makes our algorithm very flexible, it can adapt to different situations, so we can use it to forecasts many kinds of data: economical, medical, social, technical and so on.

REFERENCES

1. Blank, I.A. Strategy and Tactics of Financial Management. The Item LTD, Kyiv (1996) (in Ukrainian)
2. Heyets, V.M. Iinstability and Economic Growth. Institute of Economic Forecasting of National Academy of Sciences of Ukraine, Kyiv (2002) (in Ukrainian)
3. Zaychenko, Y.P., Moamed, M., Shapovalenko, N.V. Fuzzy Neural Networks and Genetic Algorithms in Problems of Macroeconomic Forecasting. Science news of "Kyiv Polytechnic Institute", 4, 20-30. Kyiv (2002) (in Ukrainian)
4. Ivakhnenko, V. Course of Economic Analysis. Znannya Press, Kyiv (2000). (in Ukrainian)
5. Ivakhnenko, O.H., Lapa, V.G. Prediction of Rrandom Processes. Naukova Dumka, Kyiv (1969) (in Ukrainian)
6. Yarkina, N.M. Econometric Modeling in the Management of Business Risks. Finance of Ukraine, 11, 77-80. Kyiv (2003) (in Ukrainian)
7. Timashova, L., Stepanenko O. Economic-mathematical Evaluation Model of Enterprise in Market Economy. Journal of the Academy of Labour and Social Affairs Federation of Trade Unions of Ukraine, 3 (27), 79-90. Kyiv (2004) (in Ukrainian)
8. Stepanenko, A.P. Modern Computer Tools and Technologies for the Information of the Financial System. New Computer Tools, Computers and Networks, Vol.2, 25-31. Kyiv, Institute of Cybernetics by V.Glushkov of National Academy of Sciences of Ukraine (2001) (in Ukrainian)
9. Matviichuk, A.V. Modeling of Economic Processes Using Fuzzy Logic Methods. Kyiv National Economic University, Kyiv (2007) (in Ukrainian)
10. Kukharev, V.N., Sally V.N., Erpert A.M. Economic-mathematical Methods and Models in the Planning and Management. Vyshcha shcola, Kyiv (1991) (in Russian)
11. Transport and Communication in Ukraine - 2013 [Text] / State Statistics Service. Statistical Yearbook, Kyiv(2013) (in Ukrainian)

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