A Study on an Intuitionistic Fuzzy Prime Γ-Hyperideals of Γ-Semihyperrings

S. Abirami 1, S. Murugesan 2

Abstract — A Γ- semihyperring is a generalization of a semiring, a generalization of a semihyperring and a generalization of a Γ- semiring. In this paper, we introduce intuitionistic fuzzy Γ- hyperideals of Γ- semihyperrings and intuitionistic fuzzy prime hyperideals of Γ- semihyperrings. Here we enumerate intuitionistic fuzzy Γ- hyperideals and intuitionistic fuzzy prime hyperideals and investigate their properties.


1 INTRODUCTION

In 1964, the notion of Γ- rings was introduced by N. Nobosawa in [2] and immediately after him in 1966, Barnes extended this notion and obtained more results. Barnes, Luh [9] and Kyuno investigated the new aspects of Γ- rings, left and right unites of Γ- rings. Then the notion of Γ- semirings was introduced by Rao. In recent years Ozturk, Y.B. Jun and C.Y. Lee [8] applied the concept of fuzzy sets to the theory of Γ- rings.

Hyper structure theory was born in 1934, when Marty [7] defined hypergroups and began to analysis their properties and applied them to groups and in rational algebraic functions. Now they are widely studied from the theoretical point of view and for their applications to many subjects of pure, applied properties and applied mathematics. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructures, the composition of two elements is a set. In [3], Davvaz et. al. studied the notion of a Γ- semihyperring as a generalization of semiring, a generalization of a semihyperring and a generalization of a Γ- semiring.

The concept of a fuzzy set, introduced by Zadeh in his classical paper [10], provides a natural framework for generalizing some of the notions of classical algebraic structures. The concept of a fuzzy ideal of a ring was introduced by Liu. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. In [5], Davvaz and Leoreanu studied the notion of a fuzzy Γ- hyperideal of Γ- semihypergroup.

In the year 1986, Atanassov [1] introduced intuitionistic fuzzy set as a generalization of fuzzy set. The study of intuitionistic fuzzy hyper algebraic structures has started with the introduction of the concepts of intuitionistic fuzzy hypergroups by us. Now, in this paper, we define the notion of an intuitionistic fuzzy prime Γ- hyperideal of Γ- semihyperring as a generalization of fuzzy prime Γ- hyperideal [4].

2 PRELIMINARIES

In this section, we summarize the definitions and results on hyperstructures and intuitionistic fuzzy sets that are needed in sequel.

Definition 2.1. [1] An Intuitionistic Fuzzy Set (IFS)A in X is an object of the form A=\{(x,μA(x),γA(x))|x∈X\}, where the functions μA:X→[0,1] and γA:X→[0,1] denote the degree of membership (namely, μA(x)) and the degree of non-membership (namely, γA(x)) of each element x∈X to the set A respectively, and 0 ≤ μA(x)+γA(x) ≤ 1 for each x∈X.

Definition 2.2. [1] Let A and B be two Intuitionistic Fuzzy Sets of the forms A=\{(x,μA(x),γA(x))|x∈X\} and B=\{(x,μB(x),γB(x))|x∈X\}, then

(a) A⊆ B if and only if μA(x)≤μB(x), γA(x)≥γB(x) for all x∈X,
(b) A= B if and only if A⊆ B and B⊆ A,
(c) The complement of A is denoted by A=\{(x,1−μA(x),1−γA(x))|x∈X\},
(d) A∩B=\{(x,μA(x)∧μB(x),γA(x)∨γB(x))|x∈X\},
(e) A∪B=\{(x,μA(x)∨μB(x),γA(x)∧γB(x))|x∈X\}.

Definition 2.3. [4] Let R be a commutative semigroup and Γ be a commutative group. Then R is called a Γ- semiring if there exists a map R×Γ→ R which satisfies the following conditions,

(i) aα(b+c)= aα b +aα c,
(ii) (a+b)c =aα c +bα c,
(iii) a(α + β)c= aα c +aβ c,
(iv) aα(bβc)= (aα b)c for all a, b, c ∈ R and α, β ∈ Γ.

Definition 2.4. [4] Let R be a - semiring and μ be a fuzzy subset of R. Then,

(a) μ is called a fuzzy left Γ- ideal of R if min\{μ(x),μ(y)\} ≤ μ(x+y) for all x,y∈ R,
μ(y) ≤ μ(x+y) for all x,y∈ R and γ ∈ Γ.
(b) μ is called a fuzzy right Γ- ideal of R if min\{μ(x),μ(y)\} ≤ μ(x+y) for all x,y∈ R,
μ(x) ≤ μ(x+y) for all x,y∈ R and γ ∈ Γ.
(c) μ is called a fuzzy Γ- ideal of R if is both a fuzzy left Γ-ideal and a fuzzy right Γ- ideal of R.

Definition 2.5. [4] Let R be a commutative semihypergroup and Γ be a commutative group. Then R is called a Γ-
semihypergroup if there exists a map $R \times \Gamma \times R \to \mathcal{P}^*(R)$ (image to be denoted by $a:b=c$ for $a, b \in R$ and $c \in \Gamma$) and $\mathcal{P}^*(R)$ is the set of all non-empty subsets of $R$ satisfying the following conditions,

(i) $\psi(b+c) = \psi(b) + \psi(c),$
(ii) $\psi(a\cdot b) = \psi(a) \cdot \psi(b),$
(iii) $\psi(ab) = \psi(a) \cdot \psi(b).$

In the above definition, if $R$ is a semigroup, then $R$ is called a multiplicative $\Gamma$-semihypergroup.

**Definition 2.6** [4] Let $R$ be a $\Gamma$-semihypergroup and $\mu$ be a fuzzy subset of $R$. Then,

(a) $\mu$ is called a fuzzy left $\Gamma$-hyperideal of $R$ if

$$
\sup_{x, y \in R} \min \{\mu(x), \mu(y)\} \leq \inf \{\mu(z)\} \text{ for all } x, y, z \in R \text{ and } \gamma \in \Gamma.
$$

(b) $\mu$ is called a fuzzy right $\Gamma$-hyperideal of $R$ if

$$
\sup_{x, y \in R} \min \{\mu(x), \mu(y)\} \leq \inf \{\mu(z)\} \text{ for all } x, y, z \in R \text{ and } \gamma \in \Gamma.
$$

(c) $\mu$ is called a fuzzy $\Gamma$-hyperideal of $R$ if it is both a fuzzy left $\Gamma$-hyperideal and a fuzzy right $\Gamma$-hyperideal of $R$.

**Definition 2.7** [4] Let $R$ be a $\Gamma$-semihypergroup and $\theta, \sigma$ be two fuzzy subsets of $R$. Then, the sum $\theta + \sigma$, the product $\theta \circ \sigma$ and the composition $\theta \circ \sigma$ are defined by,

$$(\theta + \sigma)(z) = \begin{cases} 
\sup_{x, y \in R \text{ and } \gamma \in \Gamma} \min \{\theta(x), \sigma(y)\} & \text{if } z \neq x+y \\
0 & \text{otherwise}
\end{cases}
$$

$$(\theta \circ \sigma)(z) = \begin{cases} 
\sup_{x, y \in R} \min \{\theta(x), \sigma(y)\} & \text{if } z \neq x\sigma y \\
0 & \text{otherwise}
\end{cases}
$$

$$(\theta \sigma)(z) = \begin{cases} 
\sup \{\min \{A(x), B(y)\}\} & \text{if } z \neq x\theta\sigma y \\
0 & \text{otherwise}
\end{cases}
$$

**Definition 2.8** [4] A non-constant fuzzy $\Gamma$-hyperideal $\mu$ of a $\Gamma$-semihypergroup $S$ is called a fuzzy prime $\Gamma$-hyperideal of $S$ if for any two fuzzy $\Gamma$-hyperideals $\sigma$, $\theta$ of $S$, the following holds: $\theta \subseteq \sigma \lor \sigma \subseteq \mu$.

**Definition 2.9** [4] Let $R$ and $R'$ be $\Gamma$ and $\Gamma'$-semihypergroups, respectively $\psi: R \to R'$ and $f: \Gamma \to \Gamma'$ be two maps. Then, $(\psi, f)$ is called a $\Gamma$-homomorphism if,

(a) $\psi(x+y) = \psi(x) + \psi(y)$
(b) $\psi(ax) = \psi(a) \psi(x)$
(c) $f(x+y) = f(x) + f(y)$.

In the above definition if $\psi(x+y) = \psi(x) + \psi(y)$ and $\psi(xa) = \psi(x) \psi(a)$, then $(\psi, f)$ is called a strong $\Gamma$-$\Gamma'$-homomorphism. An ordered set $(\psi, f)$ is called an epimorphism, if $\psi: R \to R'$ and $f: \Gamma \to \Gamma'$ are surjective and is called a $\Gamma$-$\Gamma'$-isomorphism if $\psi: R \to R'$ and $f: \Gamma \to \Gamma'$ are bijective.

**Definition 2.10** [4] Let $\psi$ be a mapping from a set $X$ to a set $Y$. Let $\mu$ be a fuzzy subset of $X$ and $\lambda$ be a fuzzy subset of $Y$. Then, the inverse image $\psi^{-1}(\lambda)$ of $\lambda$ is the fuzzy subset of $X$ defined by $\psi^{-1}(\lambda)(x) = \lambda(\psi(x))$ for all $x \in X$.

The image $\psi(\mu)$ of $\mu$ is the fuzzy subset of $Y$ defined by $\psi(\mu)(y) = \sup \{\psi^{-1}(\lambda)(y)\} \text{ if } \psi^{-1}(\lambda)(y) \neq \emptyset \text{ otherwise for all } y \in Y$.

### 3. Intuitionistic Fuzzy $\Gamma$-Hyperideals of $\Gamma$-Semihypergroups.

In this section, we introduce the notion of an intuitionistic fuzzy $\Gamma$-hyperideal of $\Gamma$-semihypergroup and study some of its properties.

**Definition 3.1.** Let $R$ be a $\Gamma$-semiring and $A = \{(x, \mu_{A}(x), \gamma_{A}(x))| x \in R\}$ be an intuitionistic fuzzy subset of $R$. Then,

(i) $A$ is called an intuitionistic fuzzy left $\Gamma$-ideal of $R$ if

$$
\min \{\mu_{A}(x), \mu_{A}(y)\} \leq \inf \{\mu_{A}(z)\} \text{ for all } x, y, z \in R \text{ and } \gamma \in \Gamma.
$$

and $\max \{\gamma_{A}(x), \gamma_{A}(y)\} \geq \gamma_{A}(x+y)$ for all $x, y \in R, \gamma \in \Gamma.$

(ii) $A$ is called an intuitionistic fuzzy right $\Gamma$-ideal of $R$ if

$$
\min \{\mu_{A}(x), \mu_{A}(y)\} \leq \inf \{\mu_{A}(z)\} \text{ for all } x, y, z \in R \text{ and } \gamma \in \Gamma.
$$

and $\max \{\gamma_{A}(x), \gamma_{A}(y)\} \geq \gamma_{A}(x+y)$ for all $x, y \in R, \gamma \in \Gamma.$

(iii) $A$ is called an intuitionistic fuzzy left $\Gamma$-ideal and an intuitionistic fuzzy right $\Gamma$-ideal of $R$.

**Definition 3.2.** Let $R$ be a $\Gamma$-semihypergroup and $A = \{(x, \mu_{A}(x), \gamma_{A}(x))| x \in R\}$ be an intuitionistic fuzzy subset of $R$. Then,

(i) $A$ is called an intuitionistic fuzzy left $\Gamma$-hyperideal of $R$ if

$$
\min \{\mu_{A}(x), \mu_{A}(y)\} \leq \inf \{\mu_{A}(z)\} \text{ for all } x, y, z \in R, \alpha \in \Gamma
$$

and $\max \{\gamma_{A}(x), \gamma_{A}(y)\} \geq \gamma_{A}(x+y)$ for all $x, y \in R, \gamma \in \Gamma.$

and $\max \{\gamma_{A}(x), \gamma_{A}(y)\} \geq \gamma_{A}(x+y)$ for all $x, y \in R, \gamma \in \Gamma.$

(ii) $A$ is called an intuitionistic fuzzy right $\Gamma$-hyperideal of $R$, if it is both an intuitionistic fuzzy left $\Gamma$-ideal and an intuitionistic fuzzy right $\Gamma$-ideal of $R$.
(iii) A is called an intuitionistic fuzzy $\Gamma$-hyperideal of $R$, if it is both an intuitionistic fuzzy left $\Gamma$-hyperideal and an intuitionistic fuzzy right $\Gamma$-hyperideal of $R$.

**Example 3.3.** Let $R=\{a, b, c\}$ and $\Gamma=\{a, b\}$. Then, $R$ is a multiplicative $\Gamma$-semihyperring with the following hyperoperations $xa=\{a, b\}$, where $x, y \in R$ and $a \in \Gamma$ and the hyperoperations table are as follows,

<table>
<thead>
<tr>
<th>$\ast$</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>{b,c}</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>{b,c}</td>
<td>{a,b}</td>
</tr>
</tbody>
</table>

Now, we define an intuitionistic fuzzy subset $A$ of $R$ by,

$$A = \{(x, \mu_A(x), \gamma_A(x))|x \in X\}$$

where $\mu_A(x) \geq 0.26 + 0.43 \times 0.22 = 0.61$ and $\gamma_A(x) \geq 0.7 + 0.5 + 0.11 = 1.31$.

Then, $A$ is an intuitionistic fuzzy $\Gamma$-hyperideal of $R$.

**Lemma 3.4.** Let $R$ be a $\Gamma$-semihyperring, if $\{A_i\}_{i \in I}$ is a collection of intuitionistic fuzzy $\Gamma$-hyperideals of $R$, then $\bigcap_{i \in I} A_i$ and $\bigcup_{i \in I} A_i$ are intuitionistic fuzzy $\Gamma$-hyperideals of $R$.

**Proof:** Let $\{A_i\}_{i \in I}$ be a collection of intuitionistic fuzzy $\Gamma$-hyperideals of $R$.

**Claim:** $\bigcap_{i \in I} A_i$ and $\bigcup_{i \in I} A_i$ are also intuitionistic fuzzy $\Gamma$-hyperideals of $R$.

(i) $\bigcap_{i \in I} A_i = \{(x, \mu_{\bigcap_{i \in I} A_i}(x), \gamma_{\bigcap_{i \in I} A_i}(x))|x \in R\}$ and

(ii) $\bigcup_{i \in I} A_i = \{(x, \mu_{\bigcup_{i \in I} A_i}(x), \gamma_{\bigcup_{i \in I} A_i}(x))|x \in R\}$

For all $a, b \in R$ and $\alpha \in \Gamma$, we have

(i). $\inf_{x \in R} \mu_{\bigcap_{i \in I} A_i}(x) = \inf_{x \in R} \mu_{\bigcap_{i \in I} A_i}(x)$

$$\geq \inf_{x \in R} \mu_{A_i}(x) \geq \inf_{x \in R} \mu_{\bigcup_{i \in I} A_i}(x)$$

Thus, $\bigcap_{i \in I} A_i \subseteq \bigcup_{i \in I} A_i$. Similarly, we can prove for $\mu_{\bigcup_{i \in I} A_i}(x)$.
Non-Membership: For every x, y in U(γA,t), γA(x) ≤ t and γA(y) ≤ t. Then max {γA(x), γA(y)} ≤ t and hence $inf_{x \in x+y} \{γA(z)\} ≤ t$.

Thus, for every z ∈ x+y, γA(z) ≤ t, z ∈ U(γA,t).

Claim: U(γA,t) ⊆ U(γA,t).

Let us assume that $y \in U(γA,t)$, $z \in Γ$ and $t' \in R$.

Since, $y \in U(γA,t)$, $γA(y) ≤ t$.

Then $γA(z) ≥ t$ and hence $z \in U(γA,t)$.

$γA(z) ≥ t$.

Similarly, we can prove for $R \cdot U(γA,t) \subseteq U(γA,t)$.

Conversely, let U(μA,t) and U(γA,t) be Γ-hyper-ideals of R for every 0 ≤ t ≤ 1.

Claim: A is an intuitionistic fuzzy Γ-hyperideal of R.

Membership: For every x, y in R, we can write, $μA(x) \geq x_0$ and $μA(y) ≥ y_0$, where $t = min \{μA(x), μA(y)\}$. Then, $x \in U(μA,t)$ and $y \in U(μA,t)$, and hence $x + y \subseteq U(μA,t)$ since $U(μA,t)$ is a Γ-hyperideal. For every $z \in x + y$, $μA(z) ≥ x_0$.

Thus, $inf_{x \in x+y} \{μA(z)\} ≥ x_0$.

Now, suppose that $x \in Ψ R$ and $λ \in Γ$, such that $μA(x) = S_0$.

Then $x \in U(μA,S_0)$, since $U(μA,S_0)$ is a Γ-hyperideal, $xλy \subseteq U(μA,S_0)$.

For every $z \in xλy$, $μA(z) ≥ S_0$.

Thus, $μA(x) ≤ inf_{x \in x+y} \{μA(z)\}$.

Non-Membership: For every $x', y' \in R$, we can write, $γA(x') \leq t_1$ and $γA(y') \leq t_1$, where $t_1 = max \{γA(x), γA(y')\}$. Then, $x' \in U(γA,t_1)$ and $y' \in U(γA,t_1)$ and hence $x'y' \subseteq U(γA,t_1)$. Since $U(γA,t_1)$ is a Γ-hyperideal. For every $z \in x'y'$, $γA(z) ≤ S_1$.

Thus, $sup_{x \in x+y} \{γA(z)\} ≤ S_1$.

$γA(x') \leq γA(x)$.

$γA(y') \leq γA(y')$.

Similarly, we can prove $γA(x') \geq sup\{γA(z')\}$.

$z' \in x'y'$.

Hence, A is an intuitionistic fuzzy Γ-hyperideal of R.

4. Intuitionistic Fuzzy prime Γ-hyperideal of R.

Γ-semihyperring.

In this section, we introduce the notion of an intuitionistic fuzzy prime Γ-hyperideal of Γ-semihyperring.

Definition 4.1: Let R be a Γ-semihyperring and A, B be two intuitionistic fuzzy subsets of R. Then, the sum A+B, the product A•B and the composition A◦B are defined by, 1. $A+B = \{z, μA+B(z), γA+B(z)\}, z \in R$, where $μA+B(z) = sup \{min \{μA(x), μB(y)\} : x \in z \in + y$ and $γA+B(z) = inf \{max \{μA(x), μB(y)\} : x \in z \in + y$.

2. $A○B = \{z, μA○B(z), γA○B(z)\}, z \in R$. where $μA○B(z) = sup \{min \{μA(x), μB(y)\} : x \in z \in + y$ and $γA○B(z) = inf \{max \{μA(x), μB(y)\} : x \in z \in + y$.

3. $A ◦ B = \{z, μA ◦ B(z), γA ◦ B(z)\}, z \in R$. where $μA ◦ B(z) = sup \{min \{μA(x), μB(y)\} : x \in z \in + y$ and $γA ◦ B(z) = inf \{max \{μA(x), μB(y)\} : x \in z \in + y$.

Lemma 4.2: Let R be a Γ-semihyperring and A be an intuitionistic fuzzy Γ-hyperideal of R. Then, $min \{μA(x), μA(x), ..., μA(x)\} \leq \inf \{μA(z)\}$.

Max $\{γA(x), ..., γA(x)\} \geq γA(z)$.

Proposition 4.3: Let R be a Γ-semihyperring and A, B be two intuitionistic fuzzy Γ-hyperideals of R.

Then, $A \odot B \subseteq A◦B \subseteq A \odot B$.

Proof: Let R be a Γ-semihyperring and A, B be two intuitionistic fuzzy Γ-hyperideals of R. Then, there exists $a \in xαy, γA(x) \leq AαB(y)$.

$\alpha \in Γ$.

By lemma 4.2, it is enough to prove that $A ◦ B \subseteq A \odot B$.

Now, suppose that $x \in \sum_{i=1}^{n} x_0 α y_i$, where $x_0 α y_i$ ∈ R and $α_i \in Γ$.

Then, there exists $a \in xαy, γA(x) \leq AαB(y)$.

By lemma 4.2, $μA(x) ≥ sup \{min \{μA(x), μB(y)\} : x \in z \in + y$ and $γA(z) = inf \{max \{μA(x), μB(y)\} : x \in z \in + y$.

$z_i \in xαy_i$.

$z_i \in xαy_i$.

$z_i \in xαy_i$.

Similarly, $μA(x) ≥ sup \{min \{μA(x), μB(y)\} : x \in z \in + y$ and $γA(z) = inf \{max \{μA(x), μB(y)\} : x \in z \in + y$.

$z_i \in xαy_i$.

$z_i \in xαy_i$.

Now, $μA(x) / μB(x) \geq min \{μA(x), μB(x)\} \geq min \{μA(x), μB(x)\}$.\}

Next, suppose that $x \in \sum_{i=1}^{n} x_i β y_i$ for $1 \leq i \leq n$ where $x_i β y_i \in R$ and $β_i \in Γ$.

Then, there exists $b \in x_i β y_i$ for $1 \leq j \leq n$ such that $x \in \sum_{i=1}^{n} x_i β y_i$.

By lemma 4.2, $γA(x) \leq sup \{γA(x_i), γA(y_i)\}$.

Similarly, $γA(x) \leq sup \{γA(x_i), γA(y_i)\}$.

$γA(x) \leq sup \{γA(x_i), γA(y_i)\}$.

Now, $γA(x) V γA(y) = max \{γA(x), γA(y)\}$.

$γA(x) V γA(y) = max \{γA(x), γA(y)\}$.

$γA(x) V γA(y) = max \{γA(x), γA(y)\}$.
Since, I is prime, \( x \bullet R \bullet y \)

Using (i) and (ii)

\[ A \bowtie B = \{ (x, \mu(x), \gamma(x)) \mid \mu(x) \leq \min \{ \mu(x') \mid x' \in X \} \} \]

By definition 4.1, \( A \bowtie B \subseteq A \bowtie B \).

**Definition 4.4:** A non-constant intuitionistic fuzzy \( \Gamma \)-hyperideal \( A \) of a \( \Gamma \)-semihyperring \( S \) is called an *intuitionistic fuzzy prime \( \Gamma \)-hyperideal* of \( S \) if for any two intuitionistic fuzzy \( \Gamma \)-hyperideal \( B, C \) of \( S \), \( B \bowtie C \) implies \( A \bowtie \Gamma \).

**Example 4.5:** Let \( R = \{ a, b, c \} \) and \( \Gamma = \{ a, b \} \). Then, \( R \) is a multiplicative \( \Gamma \)-semihyperring with the following hyperoperations:

<table>
<thead>
<tr>
<th>*</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>{b, c}</td>
</tr>
<tr>
<td>c</td>
<td>{b, c}</td>
<td>{a, b}</td>
<td>a</td>
</tr>
</tbody>
</table>

Now, we define an intuitionistic fuzzy subset \( A \) of \( R \) by:

\[ A = \{ (x, \mu_1(x), \gamma_1(x)) \mid x \in X \} \]

Where \( \mu_1(x) = \frac{0.26}{a} + \frac{0.41}{x} + \frac{0.22}{c} \) and \( \gamma_1(x) = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.11}{c} \) is an intuitionistic fuzzy prime \( \Gamma \)-hyperideal if there exists two intuitionistic fuzzy \( \Gamma \)-hyperideals \( B, C \) of \( R \) such that \( B \bowtie C \subseteq A \).

**Proposition 4.6** Let \( I \bowtie A \) be a \( \Gamma \)-hyperideal of a \( \Gamma \)-semihyperring \( R \), \( t \in \{0, 1\} \), and \( A \) be an intuitionistic fuzzy subset of \( R \) defined by:

\[ A = \{ (x, \mu(x), \gamma(x)) \mid x \in I \} \]

Where \( \mu(x) = \frac{0.26}{a} + \frac{0.41}{x} + \frac{0.22}{c} \) and \( \gamma(x) = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.11}{c} \),

\[ C = \{ (x, \mu(x), \gamma(x)) \mid x \in I \} \]

Where \( \mu(x) = \frac{0.26}{a} + \frac{0.41}{x} + \frac{0.22}{c} \) and \( \gamma(x) = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.11}{c} \),

<table>
<thead>
<tr>
<th>*</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>{b, c}</td>
</tr>
<tr>
<td>c</td>
<td>{b, c}</td>
<td>{a, b}</td>
<td>a</td>
</tr>
</tbody>
</table>

Then, \( A \) is an intuitionistic fuzzy prime \( \Gamma \)-hyperideal of \( R \) if and only if \( I \bowtie A \) is a \( \Gamma \)-hyperideal of \( R \).

**Proof:** Let \( I \bowtie A \) be a \( \Gamma \)-hyperideal of a \( \Gamma \)-semihyperring \( R \), \( t \in \{0, 1\} \), and \( A \) be an intuitionistic fuzzy subset of \( R \). Suppose that, \( I \bowtie A \) is a \( \Gamma \)-hyperideal of \( R \).

**To Prove:** \( A \) is an intuitionistic fuzzy prime \( \Gamma \)-hyperideal of \( R \). Clearly, \( A \) is an intuitionistic fuzzy \( \Gamma \)-hyperideal of \( R \). Let \( B, C \) be two intuitionistic fuzzy \( \Gamma \)-hyperideals of \( R \) such that \( B \bowtie C \subseteq A \). Then, there exists \( x, y \in R \) such that \( \mu(x) = \mu(y) \), \( \gamma(x) > \gamma(y) \) and \( \gamma(y) > \gamma(y) \). Hence, \( x \in I \) and \( y \in I \). Since, \( I \bowtie A \), there exists \( r \in R \) and \( a, u \in G \) such that \( x = r \alpha(y) \).

**Membership:**

\[ t = \frac{\min \{ \mu(x), \mu(y) \}}{t} \]

Thus, \( A \) is an intuitionistic fuzzy prime \( \Gamma \)-hyperideal of \( R \).

**Conversely,** let \( A \) be an intuitionistic fuzzy prime \( \Gamma \)-hyperideal of \( R \). Suppose, \( I \bowtie A \) is not a \( \Gamma \)-hyperideal of \( R \). Then, there exists two \( \Gamma \)-hyperideals \( B, C \) of \( R \).

**Definition 4.4:** Let \( R \) be an intuitionistic fuzzy subset of \( X \) and \( B \) be an intuitionistic fuzzy subset of \( Y \). Then, the inverse image of homomorphisms.

**Definition 5.1:** Let \( \psi \) be a mapping from a set \( X \) to a set \( Y \). Let \( A \) be an intuitionistic fuzzy subset of \( X \) and \( B \) be an intuitionistic fuzzy subset of \( Y \). Then, the inverse image \( \psi^{-1}(B) \) of \( B \) is an intuitionistic fuzzy subset of \( X \) defined by,

\[ \psi^{-1}(B) = \{ (x, \mu(x), \gamma(x)) \mid x \in X \} \]

Where \( \mu(x) = \frac{0.26}{a} + \frac{0.41}{x} + \frac{0.22}{c} \) and \( \gamma(x) = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.11}{c} \),

<table>
<thead>
<tr>
<th>*</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>{b, c}</td>
</tr>
<tr>
<td>c</td>
<td>{b, c}</td>
<td>{a, b}</td>
<td>a</td>
</tr>
</tbody>
</table>

Then, there exists \( x, y \in R \) such that \( \mu(x) = \mu(y) \), \( \gamma(x) > \gamma(y) \) and \( \gamma(y) > \gamma(y) \). Hence, \( x \in I \) and \( y \in I \). Since, \( I \) is prime, \( x \bullet R \bullet y \) and \( x \bullet R \bullet y \).

**Membership:**

\[ t = \frac{\min \{ \mu(x), \mu(y) \}}{t} \]

Clearly, \( \theta \bowtie A \) are two intuitionistic fuzzy \( \Gamma \)-hyperideals of \( R \) such that \( \theta \bowtie \Gamma \).

**Proposition 5.2:** Let \( R \) be a \( \Gamma \)-semihyperring and \( R' \) be a \( \Gamma' \)-semihyperring. Let \( (p, \theta) \) be a strong \( (\Gamma, \Gamma') \) - homomorphic from \( R \) to \( R' \).

**Proof:** Let \( R \) be a \( \Gamma \)-semihyperring and \( R' \) be a \( \Gamma' \)-semihyperring. Let \( (p, \theta) \) be a strong \( (\Gamma, \Gamma') \) - homomorphism from \( R \) to \( R' \). Then, we have

\[ \text{Membership:} \]

\[ \text{If } x \bowtie y \text{ in } R \]

\[ \text{Then, } z \bowtie x+y \text{ in } R' \]

\[ \geq \inf \{ \mu_A(z) \} \]
ψ(z) ∈ ψ(x + y)
≥ inf \{ μ, ψ(z) \}
ψ(z) ∈ ψ(x) + ψ(y)
≥ min \{ μ, ψ(x), μ, ψ(y) \}
= min \{ μ, ψ(A(x), μ, ψ(A(y)) \}

Hence, \( \inf \{ μ, ψ(A(z)) \} ≥ min \{ μ, ψ(A(x), μ, ψ(A(y)) \} \) and
\( z ∈ x + y \)
\( \inf \{ μ, ψ(A(z)) \} = \inf \{ μ, ψ(A(z)) \} \)
\( z ∈ x + y \)
\( ≥ inf \{ μ, ψ(z) \} \)
\( ψ(z) ∈ ψ(x) + ψ(y) \)
\( ≥ μ, ψ(z) = μ, ψ(A(y)) \)

Hence, \( \inf \{ μ, ψ(A(z)) \} ≥ min \{ μ, ψ(A(x), μ, ψ(A(y)) \} \) and
\( z ∈ x + y \)

Non-Membership: \( sup \{ γ, ψ(A(z)) \} = sup \{ γ, ψ(A(z)) \} \)
\( z ∈ x + y \)
\( ≤ sup \{ γ, ψ(A(z)) \} \)
\( ψ(z) ∈ ψ(x) + ψ(y) \)
\( ≤ sup \{ γ, ψ(A(z)) \} \)
\( ψ(z) ∈ ψ(x) + ψ(y) \)
\( ≤ max \{ γ, ψ(A(z)), γ, ψ(A(y)) \} \)
\( = max \{ γ, ψ(A(z)), γ, ψ(A(y)) \} \)

Hence, \( sup \{ γ, ψ(A(z)) \} ≤ max \{ γ, ψ(A(z)), γ, ψ(A(y)) \} \) and
\( z ∈ x + y \)

Claim: \( ψ(A) \) is an intuitionistic fuzzy prime \( Γ \)-hyperideal of \( R \).
Using theorem 4.6, \( I \) is a prime \( Γ \)-hyperideal of \( R \). Clearly, \( ψ(A) \) is an intuitionistic fuzzy \( Γ \)-hyperideal of \( R \). Let \( B, C \) be an intuitionistic fuzzy \( Γ \)-hyperideal of \( R \) such that
\( B \subseteq C \subseteq ψ(A) \) implies \( B \subseteq ψ(A) \) and \( C \subseteq ψ(A) \). Then, there exists \( x, y \in R \) such that
\( ψ(A(x)) \leq B(x) \Rightarrow ψ(A(x)) \leq ψ(A(x)) \)
\( = γ, ψ(A(x)) \)
\( ≤ γ, ψ(A(x)) \)
\( = γ, ψ(A(x)) \)

Hence, \( ψ(A) \) is an intuitionistic fuzzy \( Γ \)-hyperideal of \( R \).

Acknowledgement: The authors are highly grateful to the referees for their valuable comments and suggestions for improving the paper.

References


