A Study on an Intuitionistic Fuzzy Prime Γ- Hyperideals of Γ- Semihyperrings

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Abstract — A Γ- semihyperring is a generalization of a semiring, a generalization of a semihyperring and a generalization of a Γ- semiring. In this paper, we introduce intuitionistic fuzzy Γ- hyperideals of Γ- semihyperrings and intuitionistic fuzzy prime hyperideals of Γ- semihyperrings. Here we enumerate intuitionistic fuzzy Γ- hyperideals and intuitionistic fuzzy prime hyperideals and investigate their properties.

Index Terms — Intuitionistic Fuzzy Γ- Hyperideal, Intuitionistic Fuzzy Γ- ideal, The Sum, The Product and the Compostion of two Intuitionistic Fuzzy Γ- Hyperideals, Intuitionistic Fuzzy prime Γ- Hyperideal, Image and inverse image of homomorphisms.

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1 INTRODUCTION

In 1964, the notion of Γ- rings was introduced by N. Nobosawa in [2] and immediately after him in 1966, Barnes extended this notation and obtained more results. Barnes, Luh [9] and Kyuno investigated the new aspects of Γ- rings, left and right unites of Γ- rings. Then the notion of Γ- semirings was introduced by Rao. In recent years Ozturk, Y.B. Jun and C.Y. Lee [8] applied the concept of fuzzy sets to the theory of Γ- rings.

Hyper structure theory was born in 1934, when Marty [7] defined hypergroups and began to analysis their properties and applied them to groups and in rational algebraic functions. Now they are widely studied from the theoretical point of view and for their applications to many subjects of pure, applied properties and applied mathematics. In a classical algebraic structure, the composion of two elements is an element, while in an algebraic hyperstructures, the composion of two elements is a set. In [3], Davvaz et. al. studied the notion of a Γ- semihyperring as a generalization of semiring, a generalization of a semihyperring and a generalization of a Γ- semiring.

The concept of a fuzzy set, introduced by Zadeh in his classical paper [10], provides a natural framework for generalizing some of the notions of classical algebraic structures. The concept of a fuzzy ideal of a ring was introduced by Liu. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. In [5], Davvaz and Leoreanu studied the notion of a fuzzy Γ- hyperideal of Γ- semihypergroup.

In the year 1986, Atanassov [1] introduced intuitionistic fuzzy set as a generalization of fuzzy set. The study of Intuitionistic fuzzy hyper algebraic structures has started with the introduction of the concepts of intuitionistic fuzzy hypergroups by us. Now, in this paper, we define the notion of an intuitionistic fuzzy prime Γ- hyperideal of Γ- semihyperring as a generalization of fuzzy prime Γ- hyperideal [4].

2 PRELIMINARIES

In this section, we summarize the definitions and results on hyperstructures and intuitionistic fuzzy sets that are needed in sequel.

Definition 2.1. [1] An *Intuitionistic Fuzzy Set* (IFS)A in X is an object of the form $A=\{(x,\mu_A(x),\gamma_A(x))|x \in X\}$, where the functions $\mu_A:X\to[0,1]$ and $\gamma_A: X\to[0,1]$ denote the degree of membership (namely,$\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x\in X$ to the set A respectively, and $0 \le \mu_A(x)+ \gamma_A(x)\le 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B be two Intuitionistic Fuzzy Sets of the forms $A= \{(x,\mu_A(x), \gamma_A(x))|x \in X\}$ and $B= \{(x,\mu_B(x), \gamma_B(x))|x \in X\}$, then

(a) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$, $\gamma_A(x)\ge \gamma_B(x)$ for all $x \in X$,

(b) A= B if and only if $A\subseteq B$ and $B\subseteq A$,

(c) The complement of A is denoted by $\bar{A}$ and is defined by $\bar{A}=\{(x, \gamma_A(x),\mu_A(x))|x \in X\}$,

(d)$A\cap B= \{(x, \mu_A (x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x))|x \in X\}$,

(e)$A\cup B= \{(x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x))|x \in X\}$.

Definition 2.3. [4] Let R be a commutative semigroup and Γ be a commutative group. Then R is called a *Γ- semiring* if there exists a map $R \times \Gamma \times R \to R$ which satisfies the following condions,

(i) $a\alpha(b+c)= a\alpha b +a\alpha c$,

(ii) $(a+b)\alpha c =a\alpha c +b\alpha c$,

(iii) $a(\alpha + \beta)c=a \alpha c +a\beta c$,

(iv) $a\alpha(b\beta c)= (a\alpha b)\beta c$ for all a, b, c $\in$ R and $\alpha, \beta \in \Gamma$.

Definition 2.4. [4] Let R be a - semiring and μ be a fuzzy subset of R. Then,

(a) μ is called a *fuzzy left Γ- ideal* of R if
$\min \{\mu(x),\mu(y)\} \le \mu(x+y)$ for all $x,y\in R$,
$\mu(y) \le \mu(x\gamma y)$ for all $x,y\in R$ and $\gamma \in \Gamma$.

(b) μ is called a *fuzzy right Γ- ideal* of R if
$\min \{\mu(x),\mu(y)\}\le \mu(x+y)$ for all $x,y\in R$,
$\mu(x)\le \mu(x\gamma y)$ for all $x,y\in R$ and $\gamma \in \Gamma$.

(c) μ is called a *fuzzy Γ- ideal* of R if is both a fuzzy left Γ-ideal and a fuzzy right Γ- ideal of R.

Definition 2.5. [4] Let R be a commutative semihypergroup and Γ be a commutative group. Then R is called a *Γ-*

semihyperring if there exists a map $R \times \Gamma \times R \to P^*(R)$ (image to be denoted by $a\alpha b$ for $a, b \in R$ and $\alpha \in \Gamma$) and $P^*(R)$ is the set of all non-empty subsets of R satisfying the following condions,

(i) $a\alpha(b+c)= a\alpha\, b + a\alpha\, c$,
(ii) $(a+b)\alpha\, c = a\alpha\, c + b\alpha\, c$,
(iii) $a(\alpha+\beta)c = a\alpha c + a\beta c$,
(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$.

In the above definion, if R is a semigroup, then R is called a *multiplicative Γ- semihyperring*.

Definition. 2.6 [4] Let R be a Γ- semihyperring and μ be a fuzzy subset of R. Then,

(a) μ is called a *fuzzy left Γ- hyperideal* of R if
$\min\{\mu(x),\mu(y)\} \le \inf_{z\in x+y}\{\mu(z)\}$ for all $x,y \in R$
$\mu(y) \le \inf_{z\in x\gamma y}\{\mu(z)\}$ for all $x,y \in R$ and $\gamma \in \Gamma$.

(b) μ is called a *fuzzy right Γ- hyperideal* of R if
$\min\{\mu(x),\mu(y)\} \le \inf_{z\in x+y}\{\mu(z)\}$ for all $x,y \in R$
$\mu(x) \le \inf_{z\in x\gamma y}\{\mu(z)\}$ for all $x,y \in R$ and $\gamma \in \Gamma$.

(c) μ is called a *fuzzy Γ- hyperideal* of R if is both a fuzzy left Γ- hyperideal and a fuzzy right Γ- hyperideal of R.

Definition 2.7. [4] Let R be a Γ- semihyperring and θ, σ be two fuzzy subsets of R. Then, the *sum* θ + σ, the *product* θ ●σ and the *composion* θ ○σ are defined by,

$$(\theta+\sigma)(z) = \begin{cases} \sup\limits_{z\in x+y}\{\min\{\theta(x),\sigma(y)\}\} & \text{for } x,y\in R \\ 0 & \text{otherwise} \end{cases}$$

$$(\theta\bullet\sigma)(z) = \begin{cases} \sup\limits_{z\in x\gamma y}\{\min\{\theta(x),\sigma(y)\}\} & \text{for } x,y\in R \text{ and } \gamma\in R \\ 0 & \text{otherwise} \end{cases}$$

$$(\theta\circ\sigma)(z) = \begin{cases} \sup\{\min_i\{\min\{A(x_i),B(y_i)\}\}\} & z \in \sum_{i=1}^{n} x_i\gamma_i y_i,\ x_i,y_i \in R, \gamma_i \in \Gamma \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.8. [4] A non-constant fuzzy Γ- hyperideal μ of a Γ- semihypergroup S is called a *fuzzy prime Γ- hyperideal* of S if for any two fuzzy Γ- hyperideals θ, σ of S, $\theta \bullet \sigma \subseteq \mu$ implies $\theta \subseteq \mu$ or $\sigma \subseteq \mu$.

Definition 2.9. [4] Let R and R' be Γ and Γ'- semihyperrings, respectively $\psi: R \to R'$ and $f: \Gamma \to \Gamma'$ be two maps. Then, (ψ,f) is called a *(Γ,Γ') - homomorphims* if,

(a) $\psi(x+y)=\{\psi(t) \mid t \in x+y\} \subseteq \psi(x)+\psi(y)$,
(b) $\psi(x\alpha\, y)=\{\psi(t) \mid t \in x\alpha y\} \subseteq \psi(x)f(\alpha)\psi(y)$,
(c) $f(x+y)=f(x)+f(y)$.

In the above definion if $\psi(x+y) = \psi(x) + \psi(y)$ and $\psi(x\alpha\, y) = \psi(x)\, f(\alpha)\psi(y)$, then (ψ,f) is called a *strong (Γ, Γ') - homomorphism*. An ordered set (ψ,f) is called an epimorphism, if $\psi: R \to R'$ and $f:\Gamma \to \Gamma'$ are surjective and is called a (Γ,Γ') - isomorphism if $\psi: R \to R'$ and $f:\Gamma \to \Gamma'$ are bijective.

Definition 2.10. [4] Let ψ be a mapping from a set X to a set Y. Let μ be a fuzzy subset of X and λ be a fuzzy subset of Y. Then, The *inverse image* $\psi^{-1}(\lambda)$ of λ is the fuzzy subset of X defined by $\psi^{-1}(\lambda)= \lambda(\psi(x))$ for all $x \in X$.
The *image* ψ(μ) of μ is the fuzzy subset of Y defined by
$$\psi(\mu)(y) = \begin{cases} \sup\{\mu(t) \setminus t \in \psi^{-1}(y)\} & \text{if } \psi^{-1}(y) \neq \phi \\ 0 & \text{otherwise for all } y\in Y \end{cases}$$

3. Intuitionistic Fuzzy Γ-Hyperideals of Γ-Semihyperrings.

In this section, we introduce the notion of an intuitionistic fuzzy Γ- hyperideal of Γ- semihyperring and study some of its properties.

Definition 3.1. Let R be a Γ-semiring and $A=\{(x,\mu_A(x),\gamma_A(x)) \mid x \in R\}$ be an intuitionistic fuzzy subset of R. Then,

(i) A is called an *intuitionistic fuzzy left Γ- ideal* of R where $\min\{\mu_A(x),\mu_A(y)\} \le \mu_A(x+y)$ for all $x,y \in R$
$\mu_A(y) \le \mu_A(x\alpha\, y)$ for all $x,y \in R$, $\alpha \in \Gamma$.
and $\max\{\gamma_A(x), \gamma_A(y)\} \ge \gamma_A(x+y)$ for all $x,y \in R$
$\gamma_A(y) \ge \gamma_A(x\beta\, y)$ for all $x,y \in R$, $\beta \in \Gamma$.

(ii) A is called an *intuitionistic fuzzy right Γ- ideal* of R , where $\min\{\mu_A(x),\mu_A(y)\} \le \mu_A(x+y)$ for all $x,y \in R$
$\mu_A(x) \le \mu_A(x\alpha\, y)$ for all $x,y \in R$ $\alpha \in \Gamma$.
and $\max\{\gamma_A(x), \gamma_A(y)\} \ge \gamma_A(x+y)$ for all $x,y \in R$
$\gamma_A(x) \ge \gamma_A(x\beta y)$ for all $x,y \in R$, $\beta \in \Gamma$.

(iii) A is called an *intuitionistic fuzzy Γ- ideal* of R, if it is both an intuitionistic fuzzy left Γ- ideal and an intuitionistic fuzzy right Γ- ideal of R.

Definition 3.2. Let R be a Γ- semihyperring and $A=\{(x,\mu_A(x), \gamma_A(x)) \mid x \in R\}$ be an intuitionistic fuzzy subset of R. Then,

(i) A is called an *intuitionistic fuzzy left Γ- hyperideal* of R where $\min\{\mu_A(x),\mu_A(y)\} \le \inf_{z\in x+y}\{\mu_A(z)\}$ for all $x,y \in R$,
$\mu_A(y) \le \inf_{z\in x\alpha y}\{\mu_A(z)\}$ for all $x,y \in R$ and $\alpha \in \Gamma$
and $\max\{\gamma_A(x), \gamma_A(y)\} \ge \sup_{z\in x+y}\{\gamma_A(z)\}$ for all $x,y \in R$,
$\gamma_A(y) \le \sup_{z\in x\beta y}\{\gamma_A(z)\}$ for all $x,y \in R$ and $\beta \in \Gamma$.

(ii) A is called an *intuitionistic fuzzy right Γ- hyperideal* of R , where $\min\{\mu_A(x),\mu_A(y)\} \le \inf_{z\in x+y}\{\mu_A(z)\}$ for all $x,y \in R$
$\mu_A(x) \le \inf_{z\in x\alpha y}\{\mu_A(z)\}$ for all $x,y \in R$ and $\alpha \in \Gamma$.
and $\max\{\gamma_A(x), \gamma_A(y)\} \ge \sup_{z\in x+y}\{\gamma_A(z)\}$ for all $x,y \in R$
$\gamma_A(x) \ge \sup\{\gamma_A(z)\}$ for all $x,y \in R$ and $\beta \in \Gamma$.

$z \in x\beta y$

(iii) A is called an *intuitionistic fuzzy Γ- hyperideal* of R, if it is both an intuitionistic fuzzy left Γ- hyperideal and an intuitionistic fuzzy right Γ- hyperideal of R.

Example.3.3. Let R={a, b, c} and Γ={a, b}. Then, R is a multiplicative Γ- semihyperring wh the following hyperoperations $x\alpha y$={a, b}, where x, y ∈R and α ∈ Γ and the hyperoperations table are as follows,

*	a`	b	c
a	a	b	c
b	b	b	{b,c}
c	c	{b,c}	{a,b}

a	a	b	c
a	a	b	a
b	b	b	b
c	a	a	a

Now, we define an intuitionistic fuzzy subset A of R by,
A= {(x,μ_A(x), γ_A(x))|x ∈ X} where

$$\mu_A(x) = \frac{0.26}{a} + \frac{0.43}{b} + \frac{0.22}{c}$$

$$\gamma_A(x) = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.11}{c}$$

Then, A is an intuitionistic fuzzy Γ- hyperideal of R.

Lemma.3.4. Let R be a Γ- semihyperring, if $\{A_i\}_{i\in\Lambda}$ is a collection of intuitionistic fuzzy Γ-hyperideals of R, then $\cap_{i\in\Lambda} A_i$ and $\cup_{i\in\Lambda} A_i$ are intuitionistic fuzzy Γ- hyperideals of R.

Proof: Let $\{A_i\}_{i\in\Lambda}$ is a collection of intuitionistic fuzzy Γ- hyperideals of R.

Claim: $\cap_{i\in\Lambda} A_i$ and $\cup_{i\in\Lambda} A_i$ are also an intuitionistic fuzzy Γ- hyperideals of R, where

(i) $\cap_{i\in\Lambda} A_i$ ={(x,$\wedge_{i\in\Lambda}\mu_{A_i(x)}$,$\vee_{i\in\Lambda}\gamma_{A_i(x)}$)|x∈R}and

(ii) $\cup_{i\in\Lambda} A_i$={(x,$\vee_{i\in\Lambda}\mu_{A_i(x)}$,$\wedge_{i\in\Lambda}\gamma_{A_i(x)}$)|x∈R}

For all a,b ∈ R and Γ ∈ Γ, we have

(i). $inf_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \mu_{A_i}(x)\} = inf_{x\in a+b}\{inf_{i\in\Lambda}\mu_{A_i}(x)\}$

$= inf_{i\in\Lambda}\{inf_{x\in a+b}\mu_{A_i}(x)\}$

$\geq inf_{i\in\Lambda} \min\{\mu_{A_i}(a), \mu_{A_i}(b)\}$

$= \min\{inf_{i\in\Lambda}\mu_{A_i}(a), inf_{i\in\Lambda}\mu_{A_i}(b)\}$

$= \min\{(\wedge_{i\in\Lambda}\ \mu_{A_i})(a), \min\{(\wedge_{i\in\Lambda}\ \mu_{A_i})(b)\}$

$\min\{(\wedge_{i\in\Lambda}\ \mu_{A_i})(a), \min\{(\wedge_{i\in\Lambda}\ \mu_{A_i})(b)\} \leq inf_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \mu_{A_i}(x)\}$

$inf_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \mu_{A_i}(x)\} = inf_{x\in a\alpha b}\{inf_{i\in\Lambda}\mu_{A_i}(x)\}$

$\geq inf_{i\in\Lambda}\{\mu_{A_i}(b)\} = \wedge_{i\in\Lambda}\ \mu_{A_i}(b)$

Thus $\wedge_{i\in\Lambda}\ \mu_{A_i}(b) \leq inf_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \mu_{A_i}(x)\}$

$sup_{x\in a+b}\{(\vee_{i\in\Lambda}\ \gamma_{A_i}(x)\} = sup_{x\in a+b}\{sup_{i\in\Lambda}\gamma_{A_i}(x)\}$

$= sup_{i\in\Lambda}\{sup_{x\in a+b}\gamma_{A_i}(x)\}$

$\leq sup_{i\in\Lambda} \max\{\gamma_{A_i}(a), \gamma_{A_i}(b)\}$

$= \max\{sup_{i\in\Lambda}\gamma_{A_i}(a), sup_{i\in\Lambda}\gamma_{A_i}(b)\}$

$= \max\{(\vee_{i\in\Lambda}\ \gamma_{A_i})(a), \max\{(\vee_{i\in\Lambda}\ \gamma_{A_i})(b)\}$

$\max\{(\vee_{i\in\Lambda}\ \gamma_{A_i})(a), \max\{(\vee_{i\in\Lambda}\ \gamma_{A_i})(b)\} \geq sup_{x\in a+b}\{(\vee_{i\in\Lambda}\ \gamma_{A_i}(x)\}$

$sup_{x\in a+b}\{(\vee_{i\in\Lambda}\ \gamma_{A_i}(x)\} = sup_{x\in a\alpha b}\{sup_{i\in\Lambda}\gamma_{A_i}(x)\}$

$= sup_{i\in\Lambda}\{sup_{x\in a\alpha b}\gamma_{A_i}(x)\}$

$\leq sup_{i\in\Lambda}\{\gamma_{A_i}(b)\} = \vee_{i\in\Lambda}\ \gamma_{A_i}(b)$

$\vee_{i\in\Lambda}\ \gamma_{A_i}(b) \geq sup_{x\in a+b}\{(\vee_{i\in\Lambda}\ \gamma_{A_i}(x)\}$

Hence $\cap_{i\in\Lambda} A_i$ = {(x, $\wedge_{i\in\Lambda}\mu_{A_i(x)}$, $\vee_{i\in\Lambda}\gamma_{A_i(x)}$ |x ∈ R} is an intuitionistic Fuzzy left Γ- hyperideal of R.

(ii). $inf_{x\in a+b}\{(\vee_{i\in\Lambda}\ \mu_{A_i}(x)\} = inf_{x\in a+b}\{sup_{i\in\Lambda}\mu_{A_i}(x)\}$

$= sup_{i\in\Lambda}\{inf_{x\in a+b}\mu_{A_i}(x)\}$

$\geq sup_{i\in\Lambda} \min\{\mu_{A_i}(a), \mu_{A_i}(b)\}$

$= \min\{sup_{i\in\Lambda}\mu_{A_i}(a), sup_{i\in\Lambda}\mu_{A_i}(b)\}$

$= \min\{(\vee_{i\in\Lambda}\ \mu_{A_i})(a), \min\{(\vee_{i\in\Lambda}\ \mu_{A_i})(b)\}$

$\min\{(\vee_{i\in\Lambda}\ \mu_{A_i})(a), \min\{(\vee_{i\in\Lambda}\ \mu_{A_i})(b)\} \leq inf_{x\in a+b}\{(\vee_{i\in\Lambda}\ \mu_{A_i}(x)\}$

$inf_{x\in a+b}\{(\vee_{i\in\Lambda}\ \mu_{A_i}(x)\} = inf_{x\in a\alpha b}\{sup_{i\in\Lambda}\mu_{A_i}(x)\}$

$= sup_{i\in\Lambda}\{inf_{x\in a\alpha b}\mu_{A_i}(x)\}$

$\geq sup_{i\in\Lambda}\{\mu_{A_i}(a)\} = \vee_{i\in\Lambda}\ \mu_{A_i}(a)$

Thus $\vee_{i\in\Lambda}\ \mu_{A_i}(a) \leq inf_{x\in a+b}\{(\vee_{i\in\Lambda}\ \mu_{A_i}(x)\}$

$sup_{x\in a+b}\{(\vee_{i\in\Lambda}\ \gamma_{A_i}(x)\} = sup_{x\in a+b}\{inf_{i\in\Lambda}\gamma_{A_i}(x)\}$

$= inf_{i\in\Lambda}\{sup_{x\in a+b}\gamma_{A_i}(x)\}$

$\leq inf_{i\in\Lambda} \max\{\gamma_{A_i}(a), \gamma_{A_i}(b)\}$

$= \max\{inf_{i\in\Lambda}\gamma_{A_i}(a), inf_{i\in\Lambda}\gamma_{A_i}(b)\}$

$= \max\{(\wedge_{i\in\Lambda}\ \gamma_{A_i})(a), \max\{(\wedge_{i\in\Lambda}\ \gamma_{A_i})(b)\}$

$\max\{(\wedge_{i\in\Lambda}\ \gamma_{A_i})(a), \max\{(\wedge_{i\in\Lambda}\ \gamma_{A_i})(b)\} \geq sup_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \gamma_{A_i}(x)\}$

$sup_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \gamma_{A_i}(x)\} = sup_{x\in a\alpha b}\{inf_{i\in\Lambda}\gamma_{A_i}(x)\}$

$= inf_{i\in\Lambda}\{sup_{x\in a\alpha b}\gamma_{A_i}(x)\}$

$\leq inf_{i\in\Lambda}\{\gamma_{A_i}(a)\} = \wedge_{i\in\Lambda}\ \gamma_{A_i}(a)$

$\wedge_{i\in\Lambda}\ \gamma_{A_i}(a) \geq sup_{x\in a+b}\{(\wedge_{i\in\Lambda}\ \gamma_{A_i}(x)\}$

$\cup_{i\in\Lambda} A_i$= {(x,$\vee_{i\in\Lambda}\mu_{A_i(x)}$,$\wedge_{i\in\Lambda}\gamma_{A_i(x)}$|x∈R} is an intuitionistic fuzzy right Γ- hyperideal of R.

The following proposition characterizes intuitionistic fuzzy Γ- hyperideal.

Proposition 3.5. An intuitionistic fuzzy subset A of a Γ-semihyperring R is an intuitionistic fuzzy Γ- hyperideal of R if and only if for every t ∈ [0,1] the sets U(μ_A;t) and U(γ_A;t) are Γ- hyperideal of R when they are non-empty sets.

Proof: Let A be an intuitionistic fuzzy Γ- hyperideal of R.

Membership: For every x,y in U(μ_A;t), μ_A(x)≥ t and μ_A(y)≥ t.

Then, min {μ_A(x),μ_A(y)}≥ t and hence $inf_{z\in x+y}\{\mu_A(z)\}$≥ t.

Then, for every z ∈ x+y, μ_A(z)≥ t and hence z ∈ U(μ_A;t). Thus, x+y ⊆ U(μ_A;t).

Claim: U(μ_A;t)Γ R ⊆ U(μ_A;t).

Let us assume that x ∈ U(μ_A;t), λ ∈Γ and r ∈ R. Since,

x ∈U(μ_A;t) ⟹ μ_A(x)≥ t

⟹ t≤ μ_A(x)≤ $inf_{z\in x\lambda r}\{\mu_A(z)\}$

For every, z ∈ xλr , μ_A (z)≥ t, z ∈U(μ_A;t)

Thus, xλr ⊆ U(μ_A;t). Similarly, we can prove for R Γ U(μ_A;t) ⊆ U(μ_A;t).

Non-Membership: For every x,y in $U(\gamma_A;t)$, $\gamma_A(x) \leq t$ and $\gamma_A(y) \leq t$.
Then max $\{\gamma_A(x), \gamma_A(y)\} \leq t$ and hence $inf_{z\in x+y}\{\gamma_A(z)\} \leq t$.

Then, for every $z \in x+y$, $\gamma_A(z) \leq t$, $z \in U(\gamma_A;t)$.

Thus, $x+y \subseteq U(\gamma_A;t)$.

Claim: $U(\gamma_A;t)\Gamma R \subseteq U(\gamma_A;t)$.

Let us assume that $y \in U(\gamma_A;t)$, $\lambda' \in \Gamma$ and $r' \in R$.

Since, $y \in U(\gamma_A;t) \Longrightarrow \gamma_A(y) \leq t$

$\Longrightarrow t \geq \gamma_A(y) \geq sup_{z\in y\lambda' r'}\{\gamma_A(z)\}$

For every, $z \in x\lambda' r'$, $\gamma_A(z) \geq t \Longrightarrow z \in U(\gamma_A;t)$

Thus, $y\lambda' r' \subseteq U(\gamma_A;t)$.

Similarly, we can prove for $R \bullet U(\gamma_A;t) \subseteq U(\gamma_A;t)$.

Conversely, let $U(\mu_A;t)$ and $U(\gamma_A;t)$ are Γ- hyperideal of R for every $0 \leq t \leq 1$.

Claim: A is an intuitionistic fuzzy Γ-hyperideal of R.

Membership: For every x,y in R, we can write, $\mu_A(x) \geq t_0$ and $\mu_A(y) \geq t_0$, where $t_0=\min\{\mu_A(x), \mu_A(y)\}$. Then, $x \in U(\mu_A;t_0)$ and $y \in U(\mu_A;t_0)$, and hence $x+y \subseteq U(\mu_A;t_0)$ since $U(\mu_A;t_0)$ is a Γ-hyperideal. For every, $z \in x+y$, $\mu_A(z) \geq t_0$ and hence

$inf_{z\in x+y}\{\mu_A(z)\} \geq t_0$

Thus, $inf_{z\in x+y}\{\mu_A(z)\} \geq \min\{\mu_A(x),\mu_A(y)\}$.

Now, suppose that $x,y \in R$ and $\lambda \in \Gamma$, such that $\mu_A(x)=S_0$.

Then $x \in U(\mu_A;S_0)$, since $U(\mu_A;S_0)$ is a Γ- hyperideal, $x\lambda y \subseteq U(\mu_A;S_0)$.

For every, $z \in x\lambda y$, $\mu_A(z) \geq S_0$ and hence

$inf_{z\in x\lambda y}\{\mu_A(z)\} \geq S_0 = \mu_A(x)$.

Thus, $\mu_A(x) \leq inf_{z\in x\lambda y}\{\mu_A(z)\}$.

Similarly, we can prove $\mu_A(y) \leq inf_{z\in x\lambda y}\{\mu_A(z)\}$

Non-Membership: For every $x',y' \in R$, we can write, $\gamma_A(x') \leq t_1$ and $\gamma_A(y') \leq t_1$, where $t_1=\max\{\gamma_A(x'), \gamma_A(y')\}$. Then, $x' \in U(\gamma_A;t_1)$ and $y' \in U(\gamma_A;t_1)$ and hence $x'+y' \subseteq U(\gamma_A;t_1)$, Since $U(\gamma_A;t_1)$ is a Γ- hyperideal. For every, $z \in x'+y'$, $\gamma_A(z) \leq t_1$

$\sup_{z\in x'+y'}\{\gamma_A(z)\} \leq t_1$

Thus, $\sup_{z\in x'+y'}\{\gamma_A(z)\} \leq \max\{\gamma_A(x'), \gamma_A(y')\}$

Now, suppose that $x',y' \in R$ and $\lambda' \in \Gamma$, such that $\gamma_A(x')=S_1$ and $x' \in U(\gamma_A;S_1)$, Since $U(\gamma_A;S_1)$ is a Γ- hyperideal thus, $x'\lambda' y' \subseteq U(\gamma_A;S_1)$ For every, $z' \in x'\lambda' y'$, $\gamma_A(z') \leq S_1$

$\sup_{z' \in x'\lambda' y'}\{\gamma_A(z')\} \leq S_1 = \gamma_A(x')$.

$\gamma_A(x') \geq \sup_{z' \in x'\lambda' y'}\{\gamma_A(z')\}$

Similarly, we can prove $\gamma_A(y') \geq \sup_{z' \in x'\lambda' y'}\{\gamma_A(z')\}$

Hence, A is an intuitionistic fuzzy Γ- hyperideal of R.

4. Intuitionistic Fuzzy prime Γ- hyperideal of Γ- semihyperring.

In this section, we introduce the notion of an intuitionistic fuzzy prime Γ- hyperideal of Γ- semihyperring.

Definition 4.1: Let R be a Γ- semihyperring and A, B be two intuitionistic fuzzy subsets of R. Then, the *sum* A+B, the *product* A● B and the *composion* A○ B are defined by,

1). $A+B = \{z, \mu_{A+B}(z),\gamma_{A+B}(z)\}$, $z \in R$, where

$\mu_{A+B}(z)= \sup_{z\in x+y}\{\min\{\mu_A(x), \mu_B(y)\}\}$

and $\gamma_{A+B}(z)= \inf_{z\in x+y}\{\max\{\mu_A(x), \mu_B(y)\}$ for all $x, y \in R$.

2). $A\bullet B = \{z,\mu_{A\bullet B}(z), \gamma_{A\bullet B}(z)\}$, $z \in R$. where

$\mu_{A\bullet B}(z)= \sup_{z\in x\alpha y}\{\min\{\mu_A(x), \mu_B(y)\}\}$

and $\gamma_{A\bullet B}(z)= \inf_{z\in x+y}\{\max\{\mu_A(x), \mu_B(y)\}$ for all $x, y \in R$, $\alpha \in \Gamma$.

3). $A\circ B =\{z,\mu_{A\circ B}(z), \gamma_{A\circ B}(z)\}$, $z \in \sum_{i=1}^{n} x_i\alpha_i y_i$

where $\mu_{A\circ B}(z)= \sup\{\min\{\min\{\mu_A(x_i), \mu_B(y_i)\}\}\}$

and $\gamma_{A\circ B}(z)= \inf\{\max\{\max\{\mu_A(x), \mu_B(y)\}\}\}$ for all $x_i, y_i \in R$,

$\alpha_i \in \Gamma$.

Lemma 4.2: Let R be a Γ- semihyperring and A be an intuitionistic fuzzy Γ- hyperideal of R. Then,

$\min\{\mu_A(x_1),\mu_A(x_2),..., \mu_A(x_n)\} \leq \inf_{z \in x_1+x_2+...+x_n}\{\mu_A(z)\}$

and $\max\{\gamma_A(x_1), \gamma_A(x_2),..., \gamma_A(x_n)\} \geq \sup_{z \in x_1+x_2+...+x_n}\{\gamma_A(z)\}$

Proposition 4.3: Let R be a Γ- semihyperring and A,B be two intuitionistic fuzzy Γ-hyperideals of R.

Then, $A\bullet B \subseteq A\circ B \subseteq A\cap B$.

Proof Let R be a Γ- semihyperring and let A,B be two intuitionistic fuzzy Γ-hyperideals of R.

Claim: $A\bullet B \subseteq A\circ B \subseteq A \cap B$.

By definion 4.1 it is enough to prove that $A\circ B \subseteq A\cap B$.

Now, suppose that, $x \in \sum_{i=1}^{n} x_i\alpha_i y_i$ where $x_i,y_i \in R$ and $\alpha_i \in \Gamma$.

Then, there exists $a_i \in x_i\alpha_i y_i$ for $1 \leq i \leq n$ such that $x \in \sum_{i=1}^{n} a_i$

By lemma 4.2 $\mu_A(x) \geq \min\{\mu_A(a_1), \mu_A(a_2),..., \mu_A(a_n)\}$

$\mu_A(x) \geq \min\{\inf\{\mu_A(z_1)\}, \sup\{\mu_A(z_2)\},..., \sup\{\mu_A(z_n)\}$
$z_1 \in x_1\alpha_1 y_1$ $\quad z_2' \in x_2\alpha_2 y_2$ $\quad z_n \in x_n\alpha_n y_n$

$\geq \min\{\mu_A(x_1), \mu_A(x_2),..., \mu_A(x_n)\}$.

Similarly, $\mu_B(x) \geq \min\{\mu_B(y_1),\mu_B(y_2),...,\mu_B(y_n)\}$. Now,

$\mu_A(x) \wedge \mu_B(x)= \min\{\mu_A(x),\mu_B(x)\}$
$\geq \min\{\min\{\mu_A(x_1),\mu_A(x_2),...,\mu_A(x)\},$
$\text{Min}\{\mu_B(y_1),\mu_B(y_2),...,\mu_B(y_n)\}\}$.
$=\min_i\{\min\{\mu_A(x_i),\mu_B(y_i)\}\}$

$\mu_A(x) \wedge \mu_B(x) \geq \sup\{\min\{\min\{\mu_A(x_i),\mu_B(y_i)\}\}\}$ ------- (i)

Next, suppose that $x \in \sum_{j=1}^{n} x_j'\beta_j' y_j'$ for $1 \leq j \leq n$ where $x_j',y_j' \in R$ and $\beta_j \in \Gamma$.

Then, there exists $b_j \in x_j'\beta_j' y_j'$ for $1 \leq j \leq n$ such that $x \in \sum_{j=1}^{n} b_j$

By lemma 4.2 $\gamma_A(x) \leq \max\{\gamma_A(b_1), \gamma_A(b_2),..., \gamma_A(b_n)\}$

$\gamma_A(x) \leq \max\{\sup\{\gamma_A(z_1)\}, \sup\{\gamma_A(z_2)\},..., \sup\{\gamma_A(z_n)\}$
$z_1' \in x_1'\beta_1' y_1'$ $\quad z_2' \in x_2'\beta_1' y_2'$ $\quad z_n' \in x_n'\beta_n' y_n'$

$\leq \max\{\gamma_A(x_1'), \gamma_A(x_2'),..., \gamma_A(x_n')\}$.

Similarly, $\gamma_B(x) \leq \max\{\gamma_B(y_1'), \gamma_B(y_2'),..., \gamma_B(y_n')\}$.

Now, $\gamma_A(x) \vee \gamma_B(x)= \max\{\gamma_A(x), \gamma_B(x)\}$
$\leq \max\{\max\{\gamma_A(x_1'), \gamma_A(x_2'),..., \gamma_A(x_n')\}$
$\max\{\gamma_B(y_1'), \gamma_B(y_2'),..., \gamma_B(y_n')\}$.

$=\max_{j}\{\max\{\gamma_A(x_j'), \gamma_B(y_j')\}$

$\gamma_A(x)\vee\gamma_B(x)\leq \inf\{\max\{\max\{\gamma_A(x_j'), \gamma_B(y_j')\}\}\}$ ------ (ii)

Using (i) and (ii)

$A\cap B=\{(x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\vee\gamma_B(x))/x\in X\}$

By definion 4.1, $A\circ B\subseteq A\cap B$.

Definition 4.4: A non-constant intuitionistic fuzzy Γ- hyperideal A of a Γ- semihyperring S is called an *intuitionistic fuzzy prime Γ- hyperideal* of S if for any two intuitionistic fuzzy Γ- hyperideal B, C of S, $B\bullet C\subseteq A$ implies $B\subseteq A$ or $C\subseteq A$.

Example 4.5: Let R= {a, b, c} and Γ={a, b}. Then, R is a multiplicative Γ- semihyperring with the following hyperoperations xα y= {a, b}, where x, y ∈ R and α ∈ Γ and the hyperoperations table is as follows,

*	a	b	C
A	a	b	C
B	b	b	{b,c}
C	c	{b,c}	{a,b}

a	A	b	c
a	A	b	a
b	B	b	b
c	A	a	a

Now, we define an intuitionistic fuzzy subset A of R by,

$A=\{(x,\mu_A(x),\gamma_A(x))/x\in X\}$

Where $\mu_A(x)=\frac{0.26}{a}+\frac{0.43}{b}+\frac{0.22}{c}$ $\gamma_A(x)=\frac{0.7}{a}+\frac{0.5}{b}+\frac{0.11}{c}$ is an intuitionistic fuzzy prime Γ-hyperideal if there exists two intuitionistic fuzzy Γ- hyperideals B,C of R such that $B\bullet C\subseteq A$ implies $B\subseteq A$ or $C\subseteq A$, where

$B=\{(x,\mu_B(x),\gamma_B(x))/x\in X\}$

Where $\mu_B(x)=\frac{0.12}{a}+\frac{0.3}{b}+\frac{0.1}{c}$ $\gamma_B(x)=\frac{0.55}{a}+\frac{0.6}{b}+\frac{0.3}{c}$

$C=\{(x,\mu_C(x),\gamma_C(x))/x\in X\}$

$\mu_C(x)=\frac{0.05}{a}+\frac{0.22}{b}+\frac{0.15}{c}$ $\gamma_C(x)=\frac{0.2}{a}+\frac{0.3}{b}+\frac{0.06}{c}$

Proposition 4.6 Let I be a Γ- hyperideal of a Γ- semihyperring R, t ∈ [0, 1) and A be an intuitionistic fuzzy subset of R defined by,

$A=\{(x,\mu_A(x),\gamma_A(x))/x\in I\}$

$\mu_A(x)=\begin{cases}1 & if\ x\in I\\ t & if\ x\notin I\end{cases}$ $\gamma_A(x)=\begin{cases}0 & if\ x\in I\\ t & if\ x\notin I\end{cases}$

Then, A is an intuitionistic fuzzy prime Γ- hyperideal of R if and only if I is a prime Γ- hyperideal of R.

Proof: Let I be a Γ- hyperideal of a Γ- semihyperring R, t ∈ [0, 1) and A be an intuitionistic fuzzy subset of R. Suppose that, I is a prime Γ- hyperideal of R.

To Prove: A is an intuitionistic fuzzy prime Γ- hyperideal of R. Clearly, A is an intuitionistic fuzzy Γ- hyperideal of R. Let B, C be two intuitionistic fuzzy Γ- hyperideals of R such that $B\bullet C\subseteq A$ implies $B\nsubseteq A$ and $C\nsubseteq A$. Then, there exists x,y ∈ R such that $\mu_A(x)<\mu_B(x)$, $\gamma_A(x)>\gamma_B(x)$ and $\mu_A(y)<\mu_C(y)$, $\gamma_A(y)>\gamma_C(y)$. Hence, $x\notin I$ and $y\notin I$. Since, I is prime, $x\bullet R\bullet y\nsubseteq I$ and there exists r ∈ R and $\alpha_1, \alpha_2\in\Gamma$ such that $x\alpha_1 r\alpha_2 y\nsubseteq I$, hence there exists $z\in x\alpha_1 r\alpha_2 y$ such that $\mu_A(z)=t$ and $\gamma_A(z)=t$.

Membership: $t=\mu_A(z)$

$\geq\min\{\mu_B(x),\mu_C(y)\}$, since $z\in x\,\Gamma\,y$

$t>\min\{\mu_A(x),\mu_A(y)\}=t$

$t>t$ which is a contradiction.

Thus, A is an intuitionistic fuzzy prime Γ- hyperideal of R.

Conversely, let A be an intuitionistic fuzzy prime Γ- hyperideal of R. Suppose, I is not a prime Γ- hyperideal of R. Then , there exists two Γ- hyperideals of R B,C such that $B\bullet C\subseteq I$ implies $B\nsubseteq I$ and $C\nsubseteq I$. So there exists a ∈ A\ I, b ∈B\I.

Define two intuitionistic fuzzy subsets θ, σ of R as,

$\mu_\theta(x)=\begin{cases}1 & if\ x\in B\\ t & if\ x\notin B\end{cases}$ $\gamma_\theta(x)=\begin{cases}0 & if\ x\in B\\ t & if\ x\notin B\end{cases}$

$\mu_\sigma(x)=\begin{cases}1 & if\ x\in C\\ t & if\ x\notin C\end{cases}$ $\gamma_\sigma(x)=\begin{cases}0 & if\ x\in C\\ t & if\ x\notin C\end{cases}$

Clearly θ, σ are two intuitionistic fuzzy Γ- hyperideals of R such that $\theta\bullet\sigma\subseteq A$.

Membership: $\mu_\theta(a)=1>t=\mu_A(a)$

$\mu_\theta(a)>\mu_A(a)\Longrightarrow\theta\not\subset A$ Similarly, can be proved that, $\sigma\nsubseteq A$, which is a contradiction. Hence, I is a prime Γ- hyperideal of R.

5. Image and inverse image of homomorphisms

In this section, we define the notion of an image and inverse image of homomorphisms.

Definition 5.1. Let ψ be a mapping from a set X to a set Y. Let A be an intuitionistic fuzzy subset of X and B be an intuitionistic fuzzy subset of Y. Then, the *inverse image* $\psi^{-1}(B)$ of B is an intuitionistic fuzzy subset of X defined by,

$\psi^{-1}(B)=\{(x,\mu_{\psi-1(B)}(x),\gamma_{\psi-1(B)}(x)|\ x\in X\}$ where

$\mu_{\psi-1}(B)(x)=\mu_B\psi(x)$ and $\gamma_{\psi-1}(B)(x)=\gamma_B\psi(x)$.

The *image* ψ(A) of A is an intuitionistic fuzzy subset of Y defined by

$\psi(A)=\{(x,\mu_{\psi_A}(x),\gamma_{\psi_A}(x))/x\in X\}$

$$\mu_{\psi_A}(x)=\begin{cases}\sup\limits_{t_1\in\psi^{-1}(x)}\mu_A(t_1) & if\ \psi^{-1}(x)\neq\phi\\ 0 & otherwise\end{cases}$$

$$\gamma_{\psi_A}(x)=\begin{cases}\inf\limits_{t_2\in\psi^{-1}(x)}\gamma_A(t_2) & if\ \psi^{-1}(x)\neq\phi\\ 0 & otherwise\end{cases}$$

Proposition 5.2. Let R be a Γ- semihyperring and R' be a Γ'- semihyperring. Let (ψ, f) be a *strong (Γ, Γ') - homomo rphism* from R to R'. Then,

(i). If A is an intuitionistic fuzzy Γ- hyperideal of R', then $\psi^{-1}(A)$ is an intuitionistic fuzzy Γ- hyperideal of R.

(ii). If (ψ, f) is an epimorphism and A is an intuitionistic fuzzy Γ- hyperideal of R, then ψ(A) is an intuitionistic fuzzy Γ- hyperideal of R'.

Proof: Let R be a Γ- semihyperring and R' be a Γ'- semihyperring. Let (ψ, f) be a strong (Γ, Γ') - homomo rphism from R to R^'.

(i). Let A be an intuitionistic fuzzy Γ- hyperideal of R'. Suppose that x', y'∈ R and α', β' ∈ Γ. Then, we have

Membership: $\inf_{z\in x+y}\{\mu_{\psi-1}(A)(z)\}=\inf_{z\in x+y}\{\mu_A(z)\}$

$\geq\inf\{\mu_A\psi(z)\}$

$\psi(z)\in \psi(x+y)$

$\geq \inf \{ \mu_A\psi(z)\}$

$\psi(z)\in \psi(x)+ \psi(y)$

$\geq \min \{\mu_A\psi(x), \mu_A\psi(y)\}$

$= \min \{\mu_{\psi\text{-}1}A(x), \mu_{\psi\text{-}1}A(y)\}$

Hence, $\inf_{z\in x+y} \{ \mu_{\psi\text{-}1(A)}(z)\}\geq \min \{\mu_{\psi\text{-}1}A(x), \mu_{\psi\text{-}1}A(y)\}$ and

$\inf_{z\in x\alpha y}\{ \mu_{\psi\text{-}1(A)}(z)\} = \inf_{z\in x\alpha y}\{ \mu_A\psi(z)\}$

$\geq \inf_{\psi(z)\in \psi(x\alpha y)} \{ \mu_A\psi(z)\}$

$\geq \inf \{ \mu_A\psi^{-1}(z')\}$

$\psi(z)\in \psi(x)f(\alpha)\ \psi(y)$

$\geq \mu_A\psi(y)= \mu_{\psi(A)}(y')$

Hence, $\inf_{z\in x\alpha y}\{\mu_{\psi\text{-}1}(A)(z) \geq \mu_{\psi(A)}(y)$

Non-Membership: $\sup_{z\in x+y} \{\gamma_{\psi\text{-}1}(A)(z)\} = \sup_{z\in x+y}\{ \gamma_A\psi(z)\}$

$\leq \sup \{ \gamma_A\psi(z)\}$

$\psi(z)\in \psi(x+y)$

$\leq \sup \{ \gamma_A\psi(z)\}$

$\psi(z)\in \psi(x)+ \psi(y)$

$\leq \max \{\gamma_A\psi(x), \gamma_A\psi(y)\}$

$= \max \{\gamma_A\psi(x), \gamma_A\psi(y)\}$

Hence, $\sup_{z\in x+y} \{\gamma_{\psi(A)}(z)\} \leq \max \{\gamma_{\psi(A)}(x), \gamma_{\psi(A)}(y) \}$ and

$\sup_{z\in x\beta y} \{\gamma_\psi(A)(z)\} = \sup_{z\in x\beta y}\{\inf \gamma_A(z)\}$

$\leq \sup \{ \gamma_A\psi(z)\}$

$\psi(z)\in \psi(x\beta y)$

$\leq \sup\{ \gamma_A\psi(z)\}$

$\psi(z)\in \psi(x)f(\beta)\ \psi(y)$

$\leq \gamma_A\psi(y)= \gamma_{\psi\text{-}1(A)}(y)$

Hence, $\sup \{\gamma_{\psi\text{-}1}(A)(z) \geq \gamma_{\psi\text{-}1(A)}(y)$

Hence, ψ-1(A) is an intuitionistic fuzzy Γ- hyperideal of R'.

(ii). Let A be an intuitionistic fuzzy Γ- hyperideal of R. Suppose that x', y'∈ R and α', β' ∈ Γ. Then, there exists x,y ∈ R and α, β ∈Γ such that ψ(x)=x',ψ(y)=y' and f(α)=α' , f(β)= β'.

Membership: $\inf_{z\in x+y}\{\mu_\psi(A)(z')\} = \inf_{z'\in x'+y'}\{\sup_{t_1\in \psi^{-1}(z')} \mu_A(t_1)\}$

$\geq \inf \{ \mu_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x'+y')$

$\geq \inf \{ \mu_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x')+ \psi^{-1}(y')$

$\geq \min \{\mu_A\psi^{-1}(x'), \mu_A\psi^{-1}(y')\}$

$= \min \{\mu_A\psi(x'), \mu_A\psi(y')\}$

Hence, $\inf_{z'\in x'+y'} \mu_{\psi(A)}(z')\geq \min \{\mu_{\psi(A)}(x'), \mu_{\psi(A)}(y') \}$ and

$\inf_{z\in x'\alpha'y'}\{\mu_\psi(A)(z')\} = \inf_{z'\in x'\alpha'y'}\{\sup_{t_1\in \psi^{-1}(z')} \mu_A(t_1)\}$

$\geq \inf \{ \mu_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x'\alpha'y')$

$\geq \inf \{ \mu_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x')f^{-1}(\alpha')\ \psi^{-1}(y')$

$\geq \mu_A\psi^{-1}(y')= \mu_{\psi(A)}(y')$

Hence, $\inf_{z\in x'\alpha'y'}\{\mu_\psi(A)(z') \geq \mu_{\psi(A)}(y')$

Non-Membership: $\sup_{z\in x+y}\{\gamma_\psi(A)(z')\} = \sup_{z'\in x'+y'}\{\inf_{t_1\in \psi^{-1}(z')} \gamma_A(t_1)\}$

$\leq \sup\{ \gamma_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x'+y')$

$\leq \sup\{ \gamma_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x')+ \psi^{-1}(y')$

$\leq \max \{\gamma_A\psi^{-1}(x'), \gamma_A\psi^{-1}(y')\}$

$= \max \{\gamma_A\psi(x'), \gamma_A\psi(y')\}$

Hence, $\sup_{z'\in x'+y'} \{\gamma_{\psi(A)}(z')\} \leq \max \{\gamma_{\psi(A)}(x'), \gamma_{\psi(A)}(y') \}$ and

$\sup_{z\in x'\alpha'y'}\{\gamma_\psi(A)(z')\} = \sup_{z'\in x'\alpha'y'}\{\inf_{t_1\in \psi^{-1}(z')} \gamma_A(t_1)\}$

$\geq \inf \{ \gamma_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x'\alpha'y')$

$\geq \inf \{ \gamma_A\psi^{-1}(z')\}$

$\psi^{-1}(z')\in \psi^{-1}(x')f^{-1}(\alpha')\ \psi^{-1}(y')$

$\geq \gamma_A\psi^{-1}(y')= \gamma_{\psi(A)}(y')$

Hence, $\sup\{\gamma_\psi(A)(z') \geq \gamma_{\psi(A)}(y')$

Hence, ψ(A) is an intuitionistic fuzzy Γ- hyperideal of R'.

Proposition 5.3. Let R be a Γ- semihyperring and R' be a Γ'- semihyperring. Let (ψ, f) be a strong (Γ, Γ') - homomor phism from R onto R'. If A is an intuitionistic fuzzy prime Γ- hyperideal of R', then $\psi^{-1}(A)$ is an intuitionistic fuzzy prime Γ- hyperideal of R.

Proof: Let R be a Γ- semihyperring and R' be a Γ'- semihyperring. Let (ψ, f) be a strong (Γ, Γ') - homomorphism from R onto R'. Let A be an intuitionistic fuzzy prime Γ- hyperideal of R'.

Claim: $\psi^{-1}(A)$ is an intuitionistic fuzzy prime Γ- hyperideal of R. Using theorem 4.6, I is a prime Γ- hyperideal of R. Clearly, $\psi^{-1}(A)$ is an intuitionistic fuzzy Γ- hyperideal of R'. Let B, C be an intuitionistic fuzzy Γ- hyperideal of R' such that $B\bullet C \subseteq \psi^{-1}(A)$ implies $B\nsubseteq \psi^{-1}(A)$ and $C\nsubseteq \psi^{-1}(A)$. Then, there exists x,y ∈ R' such that $\psi^{-1}(A)(x)<B(x) \Longrightarrow \mu_{\psi\text{-}1}(A)(x)<\mu_B(x),\gamma_{\psi\text{-}1}(A)(x)> \gamma_B(x)$ also $\psi^{-1}(A)(y)<C(y) \Longrightarrow \mu_{\psi\text{-}1}(A)(y)<\mu_C(y),\ \gamma_{\psi\text{-}1}(A)(y)> \gamma_C(y)$. Hence, x ∉ I and y ∉ I. Since, I is prime, $x\bullet R \bullet y \nsubseteq I$ and there exists r ∈ R and $\alpha_1, \alpha_2\in \Gamma$ such that $x\alpha_1 r\alpha_2 y \nsubseteq I$, therefore there exists $z \in x\alpha_1 r\alpha_2 y$ such that $\mu_{\psi\text{-}1}(A)(z)=t$ and $\gamma_{\psi\text{-}1}(A)(z)=t$.

Membership: $t= \mu^{\psi\text{-}1}(A)(z) \geq \min \{\mu_B(x), \mu_C(y)\}$,
since $z \in x\Gamma y$, $t \geq \min \{\mu_{\psi\text{-}1}(A)(x), \mu_{\psi\text{-}1}(A)(y)\} =t\Longrightarrow t > t$ which is a contradiction. Thus, $\psi^{-1}(A)$ is an intuitionistic fuzzy prime Γ-hyperideal of R.

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