(2, (c₁, c₂))-Regular Intuitionistic Fuzzy Graphs

S. Ravi Narayanan, N.R. Santhi Maheswari

Abstract—In this paper \( d_2 \) - degree of a vertex and total \( d_2 \) -degree of a vertex in an intuitionistic fuzzy graphs are defined. Also \( (2, (c_1, c_2)) \)-regularity and totally \( (2, (c_1, c_2)) \)-regularity of an intuitionistic fuzzy graphs are defined. A relation between \( (2, (c_1, c_2)) \)-regularity and totally \( (2, (c_1, c_2)) \)-regularity on intuitionistic fuzzy graph is studied. \( (2, (c_1, c_2)) \)-regularity on a path on four vertices, a Barbell graph \( B_{m,n} \) \((n>1)\) and a cycle \( C_n \) are studied with some specific membership functions.

Index Terms— degree of a vertex in an intuitionistic fuzzy graph, regular intuitionistic fuzzy graph, intuitionistic fuzzy graph, total degree, totally regular intuitionistic fuzzy graph, \( d_2 \)-degree of a vertex in fuzzy graph, semiregular graphs.

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INTRODUCTION

In 1965, Lofti A. Zadeh [15] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T.Attanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T.Attanassov added a new component( which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [3, 4].


N.R.Santhy Maheswari and C.Sekar introduced \( d_2 \) -degree of vertex in fuzzy graphs and introduced \( (2, k) \)-regular fuzzy graphs and totally \( (2, k) \)-regular fuzzy graphs[13]. Also, they introduced \( d_m \)-degree, total \( d_m \)-degree of a vertex in fuzzy graphs and introduced an \( m \)-neighbourly irregular fuzzy graphs [14]. S. Ravinarayanan and N.R. Santhy Maheswari introduced \( (2, (c_1, c_2)) \)-regular bipolar fuzzy graphs[10].These motivates us to introduce \( d_2 \) -degree, total \( d_2 \)-degree of a vertex in an intuitionistic fuzzy graph and discussed some properties.

2 Preliminaries

We present some known definitions related to fuzzy graphs and intuitionistic fuzzy graphs for ready reference to go through the work presented in this paper.

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S. Ravi Narayanan, Department of Mathematics, Sri S. Ramaassamy Naidu Memorial College, Sattur, Tamil Nadu India

N.R. Santhi Maheswari, Department of Mathematics, G. Venkataswamy Naidu College, Sattur, Tamil Nadu, India

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Definition 2.1. A fuzzy graph \( G : (\sigma, \mu) \) is a pair of functions \((\sigma, \mu)\), where \(\sigma : V \rightarrow [0, 1] \) is a fuzzy subset of a non-empty set \(V\) and \(\mu : V \times V \rightarrow [0, 1] \) is a symmetric fuzzy relation on \(\sigma\) such that for all \(u, v \in V\), the relation \(\mu(u, v) \leq \sigma(u) \land \sigma(v)\) is satisfied. A fuzzy graph \(G\) is called complete fuzzy graph if the relation \(\mu(u, v) = \sigma(u) \land \sigma(v)\) is satisfied[6].

Definition 2.2. Let \( G : (\sigma, \mu) \) be a fuzzy graph. The \(d_m\)-degree of a vertex \(u\) in \(G\) is \(d_m(u) = \sum \mu^m(uv), \) where \(\mu^m(uv) = \sup \{\mu(uu_1) \land \mu(u_1u_2) \land \ldots \} \land \mu(u_{m-1}v) : u, u_1, u_2, \ldots, u_{m-1}, v \) is the shortest path connecting \(u\) and \(v\) of length \(m\). Also, \(\mu(uv) = 0, \) for \(uv\) not in \(E[14].\)

Definition 2.3. Let \( G : (\sigma, \mu) \) be a fuzzy graph on \(G^* : (V,E)\). The total \(d_m\)-degree of a vertex \(u \in V\) is defined as \(t_{d_m}(u) = \sum \mu^m(uv) + \sigma(u) = d_m(u) + \sigma(u)/[14].\)

Definition 2.4. If each vertex of \(G\) has the same \(d_m\)-degree \(k\), then \(G\) is said to be an \((m,k)\)-regular fuzzy graph[14].

Definition 2.5. If each vertex of \(G\) has the same total \(d_m\)-degree \(k\), then \(G\) is said to be totally \((m,k)\)-regular fuzzy graph[14].

Definition 2.6. An intuitionistic fuzzy graph with underlying set \(V\) is defined to be a pair \(G = (V,E)\) where
(i) \(V = \{v_1, v_2, v_3 \ldots v_n\}\) such that \(\mu_1 : V \rightarrow [0, 1]\) and \(\gamma_1 : V \rightarrow [0, 1]\) denote the degree of membership and non-membership of the element \(v_i \in V\).
(ii) \(E \subseteq V\) where \(\mu_2 : V \times V \rightarrow [0, 1]\) and \(\gamma_2 : V \times V \rightarrow [0, 1]\) are such that \(\mu_2(v_i, v_j) \leq \min \{\mu_1(v_i), \mu_1(v_j)\}\) and \(\gamma_2(v_i, v_j) \leq \max \{\gamma_1(v_i), \gamma_1(v_j)\}\) and \(0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1\), for every \((v_i, v_j) \in E, (i,j = 1,2, \ldots, n)[8].\)

Definition 2.7. If \(v_i, v_j \in V \subseteq G\), the \(\mu\)-strength of connectedness between two vertices \(v_i\) and \(v_j\) is defined as \(\mu^k(v_i, v_j) = \sup \{\mu^k(v_i, v_j) : k = 1, 2, \ldots, n\}\) and \(\gamma\)-strength of connectedness between two vertices \(v_i\) and \(v_j\) is defined as \(\gamma^k(v_i, v_j) = \inf \{\gamma^k(v_i, v_j) : k = 1, 2, \ldots, n\}\).

If \(u\) and \(v\) are connected by means of paths of length \(k\) then \(\mu^k(u, v)\) is defined as \(\sup \{\mu^k(u, v) \land \mu^k(v, v_1) \land \ldots \land \mu^k(v_{k-1}, v) : (u, v_1, v_2, \ldots, v_{k-1}, v) \in V\}\)
and \(\gamma^k(u, v)\) is defined as \(\inf \{\gamma^k(u, v_1) \land \gamma^k(v_2, v) \land \gamma^k(v_3, v_4) \land \ldots \land \gamma^k(v_{k-1}, v) : (u, v_1, v_2, \ldots, v_{k-1}, v) \in V\}\).

Definition 2.8. Let \(G = (V,E)\) be an intuitionistic fuzzy graph on \(G^* : (V,E)\). Then the degree of a vertex \(v_i \in G\) is defined by \(d(v_i) = (d_0(v_i), d_1(v_i)),\) where \(d_0(v_i) = \sum \mu_2(v_i, v_j)\) and \(d_1(v_i) = \sum \gamma_2(v_i, v_j),\) for \(v_i, v_j \in E\) and \(d_0(v_i) = 0\) and \(d_1(v_i) = 0\) for \(v_i, v_j \in E[8].\)

Definition 2.9. Let \(G = (V,E)\) be an intuitionistic fuzzy graph on \(G^* : (V,E)\). Then the total degree of a vertex \(v_i \in G\) is defined by \(t_{d_2}(v_i) = td_0(v_i) + \mu_1(v_i)\) and \(td_1(v_i) = d_1(v_i)\) [8].

3 \(d_2\) - degree of vertex in Intuitionistic Fuzzy Graph

In this section, \(d_2\) - degree of a vertex in an intuitionistic fuzzy graph is introduced.

Definition 3.1. Let \(G = (A,B)\) be an intuitionistic fuzzy graph. The \(\mu\)-degree of a vertex \(u \in G\) is defined as \(d_{\mu_2}(u) = \sum \mu_2^{(2)}(u,v)\) where \(\mu_2^{(2)}(u,v) = \sup \{\mu_2(u,u_1) \land \mu_2(u_1,v) : u, u_1, v\) is the shortest path connecting \(u\) and \(v\) of length 2\}. The \(\gamma\)-degree of a vertex \(v \in G\) is defined as \(d_{\gamma_2}(v) = \sum \gamma_2^{(2)}(u,v)\) where \(\gamma_2^{(2)}(u,v) = \inf \{\gamma_2^{(2)}(u_1,v) \land \gamma_2^{(2)}(u_2,v) : u, u_1, v\) is the shortest path connecting \(u\) and \(v\) of length 2\}. The \(d_2\) - degree of a vertex \(u\) is defined as \(d_2(u) = (d_{\mu_2}(u), d_{\gamma_2}(u)).\)

The minimum \(d_2\) - degree of \(G\) is \(d_{\mu_2}(G) = A\{d_{\mu_2}(v) : v \in V\}\).

The maximum \(d_2\) - degree of \(G\) is \(d_{\gamma_2}(G) = A\{d_{\gamma_2}(v) : v \in V\}\).

Example 3.2. Consider an intuitionistic fuzzy graph \(G = (A,B)\) on \(G^* : (V,E)\)

![Figure.1](image-url)
Example 4.2. Consider an intuitionistic fuzzy graph $G = (A, B)$ on $G^* : (V,E)$

\[ d_{2}(u) = (0.3 \land 0.4, 0.4 \lor 0.5) + (0.3 \land 0.5, 0.4 \lor 0.5) = (0.3, 0.5) + (0.3, 0.5) = (0.6, 1.0) \]

\[ d_{2}(v) = (0.4 \land 0.5, 0.5 \lor 0.5) + (0.4 \land 0.4, 0.5 \lor 0.5) = (0.4, 0.5) + (0.4, 0.5) = (0.8, 1.0) \]

\[ d_{2}(w) = (0.5 \land 0.2, 0.5 \lor 0.6) = (0.2, 0.6) \]

\[ d_{2}(x) = (0.5 \land 0.3, 0.5 \lor 0.4) + (0.5 \land 0.4, 0.5 \lor 0.5) = (0.3, 0.5) + (0.4, 0.5) = (0.7, 1.0) \]

\[ d_{2}(y) = (0.4 \land 0.3, 0.5 \lor 0.4) + (0.4 \land 0.4, 0.5 \lor 0.5) = (0.3, 0.5) + (0.4, 0.5) = (0.7, 1.0) \]

\[ d_{2}(z) = (0.2 \land 0.3, 0.6 \lor 0.5) + (0.2 \land 0.5, 0.6 \lor 0.5) = (0.2, 0.6) + (0.2, 0.6) = (0.4, 1.2) \]

Note that $d_{2}(u) = (0.4, 1.0)$, $d_{2}(v) = (0.4, 1.0)$, $d_{2}(w) = (0.4, 1.0)$ and $d_{2}(x) = (0.4, 1.0)$. This graph is $(2, (0.4, 1.0))$-regular intuitionistic fuzzy graph.

Definition 4.3. Let $G = (A, B)$ be an intuitionistic fuzzy graph. Then the total $d_{2}$-degree of a vertex $u \in V$ is defined as $\gamma_{2}(u) = \gamma_{2}(u) + \gamma_{2}(u)$ where $\gamma_{2}(u) = d_{2}(u) + \mu_{2}(u)$ and $\gamma_{2}(u) = d_{2}(u) + \gamma_{2}(u)$. Also it can be defined as $d_{2}(u) = d_{2}(u) + A(u)$ where $A(u) = (\mu_{2}(u), \gamma_{2}(u))$.

Definition 4.4. Let $G = (A, B)$ be an intuitionistic fuzzy graph. If each vertex of $G$ has same total $d_{2}$-degree, then $G$ is said to be totally $(2, (c_1, c_2))$-regular intuitionistic fuzzy graph.

Example 4.5. Consider an intuitionistic fuzzy graph $G = (A, B)$ on $G^* : (V,E)$

\[ d_{2}(u) = \{\text{sup}(0.1 \land 0.2, 0.1 \land 0.3), \text{inf}(0.2 \lor 0.3, 0.2 \lor 0.4)\} \]

\[ d_{2}(v) = \{\text{sup}(0.1 \land 0.1, 0.2 \land 0.3), \text{inf}(0.2 \lor 0.2, 0.3 \lor 0.4)\} \]

\[ d_{2}(w) = \{\text{sup}(0.1 \land 0.2, 0.3 \land 0.1), \text{inf}(0.3 \lor 0.2, 0.4 \lor 0.2)\} \]

\[ d_{2}(x) = \{\text{sup}(0.1 \land 0.1, 0.2 \land 0.3), \text{inf}(0.2 \lor 0.2, 0.3 \lor 0.4)\} \]

\[ d_{2}(y) = \{\text{sup}(0.1 \land 0.2, 0.3 \land 0.4), \text{inf}(0.2 \lor 0.2, 0.4 \lor 0.4)\} \]

\[ d_{2}(z) = \{\text{sup}(0.1 \land 0.2, 0.3 \land 0.4), \text{inf}(0.2 \lor 0.2, 0.4 \lor 0.4)\} \]

4 $(2, (c_1, c_2))$ - Regular and Totally $(2, (c_1, c_2))$ - Regular Intuitionistic Fuzzy Graph

In this section $(2, (c_1, c_2))$-regular and totally $(2, (c_1, c_2))$-regular intuitionistic fuzzy graphs are introduced

Definition 4.1. Let $G = (A, B)$ be an intuitionistic fuzzy graph. If $d_{2}(u) = (c_1, c_2)$, for all $u \in V$, then $G$ is said to be $(2, (c_1, c_2))$-regular intuitionistic fuzzy graph.

Example 4.2. Consider an intuitionistic fuzzy graph $G = (A, B)$ on $G^* : (V,E)$

Here $td_{2}(u) = (0.2, 0.4) + (0.5, 0.5) = (0.7, 0.9)$

$td_{2}(v) = (0.3, 0.3) + (0.4, 0.6) = (0.7, 0.9)$

$td_{2}(w) = (0.2, 0.4) + (0.5, 0.5) = (0.7, 0.9)$

$td_{2}(x) = (0.3, 0.3) + (0.4, 0.6) = (0.7, 0.9)$

This graph is totally $(2, (0.7, 0.9))$-regular intuitionistic fuzzy graph.

Example 4.6. A totally $(2, (c_1, c_2))$-regular intuitionistic fuzzy graph need not be $(2, (c_1, c_2))$-regular intuitionistic fuzzy graph. Consider an intuitionistic fuzzy graph $G = (A, B)$ on $G^* : (V,E)$
Here, \( t_{d_2}(u) = (1.1, 1.1) \). So \( G \) is totally \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph.

But \( d_{\frac{1}{2}}(u) \neq d_{\frac{1}{2}}(w) \). Hence \( G \) is not \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph

### Example 4.7

A \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph need not be totally \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph. Consider intuitionistic fuzzy graph \( G = (A, B) \) on \( G^2 : (V,E) \)

Then \( d_{\frac{1}{2}}(u) = (0.4, 0.6) \), for all \( u \in V \) and \( t_{d_2}(u) = (0.8, 1.1) \), for all \( u \in V \). Hence \( G \) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph and totally \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph.

### Theorem 4.9

Let \( G = (A, B) \) be an intuitionistic fuzzy graph on \( G^2(V,E) \). Then \( A(u) = (k_1, k_2) \), for all \( u \in V \) if and only if the following conditions are equivalent.

1. \( G = (A, B) \) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph.
2. \( G = (A, B) \) is totally \((2, (c_1 + k_1, c_2 + k_2))\) - regular intuitionistic fuzzy graph.

Proof. Suppose \( A(u) = (k_1, k_2) \), for all \( u \in V \). Assume that \( G \) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph. Then \( d_{\frac{1}{2}}(u) = (c_1, c_2) \), for all \( u \in V \)

So \( t_{d_2}(u) = d_{\frac{1}{2}}(u) + A(u) = (c_1, c_2) + (k_1, k_2) = (c_1 + k_1, c_2 + k_2) \)

Hence \( G \) is a totally \((2, (c_1 + k_1, c_2 + k_2))\) - regular intuitionistic fuzzy graph.

Thus (i) \( \Rightarrow \) (ii) is proved.

Now suppose \( G \) is totally \((2, (c_1 + k_1, c_2 + k_2))\) - regular intuitionistic fuzzy graph.

\( \Rightarrow \) \( t_{d_2}(u) = (c_1 + k_1, c_2 + k_2), \) for all \( u \in V \)

\( \Rightarrow \) \( d_{\frac{1}{2}}(u) + A(u) = (c_1 + k_1, c_2 + k_2), \) for all \( u \in V \)

\( \Rightarrow \) \( d_{\frac{1}{2}}(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2), \) for all \( u \in V \)

\( \Rightarrow \) \( d_{\frac{1}{2}}(u) = (c_1, c_2), \) for all \( u \in V \)

Hence \( G \) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph. Thus (i) and (ii) are equivalent.

Conversely assume (i) and (ii) are equivalent. Let \( G \) be a \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph and totally \((2, (c_1 + k_1, c_2 + k_2))\) - regular intuitionistic fuzzy graph.

\( \Rightarrow \) \( t_{d_2}(u) = (c_1 + k_1, c_2 + k_2) \) and \( d_{\frac{1}{2}}(u) = (c_1, c_2), \) for all \( u \in V \)

\( \Rightarrow \) \( d_{\frac{1}{2}}(u) + A(u) = (c_1 + k_1, c_2 + k_2) \) and \( d_{\frac{1}{2}}(u) = (c_1, c_2), \) for all \( u \in V \)

\( \Rightarrow \) \( d_{\frac{1}{2}}(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2) \) and \( d_{\frac{1}{2}}(u) = (c_1, c_2), \) for all \( u \in V \)

\( \Rightarrow \) \( A(u) = (k_1, k_2), \) for all \( u \in V \). Hence \( A(u) = (k_1, k_2) \).

### 5 (2, (c_1, c_2)) - regularity on path on four vertices with specific membership function

In this section \((2, (c_1, c_2))\) - regularity on path on four vertices is discussed with some specific membership function.
Theorem 5.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices. If $B$ is constant function then $G$ is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Proof. Suppose that $B$ is constant function, say $B(\{u\}) = (k_1, k_2)$, for all $u \in E$. Then $d_{G^*}(u) = (k_1, k_2)$. Hence $G$ is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Remark 5.2. The converse of Theorem 5.1 need not be true. For example consider $G = (A,B)$ an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices.

![Figure 8](image)

Note that $d_{G^*}(u) = (0.2, 0.4)$, for all $u \in V$. So, $G$ is a $(2, (0.2, 0.4))$ - regular intuitionistic fuzzy graph. But $B$ is not a constant function.

Theorem 5.3. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices. If alternate edges have same membership values then $G$ is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph where $c_1 = \min\{\mu^2_{G^*}\}$ and $c_2 = \max\{\nu^2_{G^*}\}$.

Theorem 5.4. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices. If the middle edge have membership value less than membership value of remaining edges and non-membership value greater than non-membership value of remaining edges, then $G$ is a $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph where $c_1$ and $c_2$ are membership values of the middle edge.

Example 5.5. Consider an intuitionistic fuzzy graph $G = (A,B)$ on $G^* : (V,E)$

![Figure 9](image)

Note that $d_{G^*}(u) = (0.1, 0.4), d_{G^*}(v) = (0.1, 0.4), d_{G^*}(w) = (0.1, 0.4)$ and $d_{G^*}(x) = (0.1, 0.4)$. Hence $G$ is a $(2, (0.1, 0.4))$ - regular intuitionistic fuzzy graph.

Remark 5.6. If $A$ is constant function, then Theorems 5.1, 5.3 and 5.4 hold good for totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

6 $(2, (c_1, c_2))$ - regularity on Barbell graph $B_{n,n}(n > 1)$ with some specific membership function

In this section $(2, (c_1, c_2))$ - regularity on Barbell graph is discussed with some specific membership function.

Theorem 6.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is a barbell graph $B_{n,n}$ of order $2n$. If $B$ is a constant function, then $G$ is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph where $(c_1, c_2) = nB(\{uv\})$ where $uv \in E$.

Remark 6.2. The converse of theorem 6.1 need not be true. For example consider $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is barbell graph $B_{2,2}$ of order 6.

![Figure 10](image)

Note that $d_{G^*}(u) = (0.4, 0.6)$, for all $u \in V$. So, $G$ is a $(2, (0.4, 0.6))$ - regular intuitionistic fuzzy graph. But $B$ is not a constant function.

Theorem 6.3. Let $G = (A,B)$ be an intuitionistic fuzzy graph on $G^*(V,E)$, a barbell graph $B_{n,n}(n > 1)$ . If the pendant edge has membership value less than the membership value of middle edge and non-membership value greater than non-membership value of middle edge then $G$ is $(2, (n(c_1, c_2))$ - regular intuitionistic fuzzy graph where $(c_1, c_2)$ is the membership value of pendant edge.

Remark 6.4. If $A$ is constant function, then the theorem 6.1 and 6.3 hold good for totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
7. (c₁, c₂) - regularity on a cycle with some specific membership functions

In this section (c₁, c₂)- regularity on a cycle is discussed with some specific membership functions.

**Theorem 7.1.** Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G'(V,E)$ is the cycle of length ≥ 5. If $\mu_2^{(2)}$ and $\nu_2^{(2)}$ are constant functions, then $G$ is a (2, (c₁, c₂)) - regular intuitionistic fuzzy graph where $(c₁, c₂) = \{(\mu_2^{(2)}, \nu_2^{(2)})\}$.

**Remark 7.2.** The converse of the theorem 7.1 need not be true. For example consider $G = (A,B)$ be an intuitionistic fuzzy graph such that $G'(V,E)$ is an odd cycle of length five.

![Figure 11](image1)

Note that $d_{(2)}(u) = (0.4, 0.8)$, for all $u \in V$.
So $G$ is a (2, (0.4, 0.8)) - regular intuitionistic fuzzy graph. But $\mu_2^{(2)}$ and $\nu_2^{(2)}$ are not constant functions.

**Theorem 7.3.** Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G'(V,E)$ is an even cycle of length ≥ 6. If alternate edges have same positive and negative membership values then $G$ is a (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

Proof. If alternate edges have same positive and negative membership values then

$$\mu_2(e_i) = \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 & \text{if } i \text{ is even} \end{cases}$$

$$\nu_2(e_i) = \begin{cases} c_3 & \text{if } i \text{ is odd} \\ c_4 & \text{if } i \text{ is even} \end{cases}$$

Here we have 4 possible cases

1. $c_1 > c_2$ and $c_3 > c_4$
2. $c_1 > c_2$ and $c_3 < c_4$
3. $c_1 < c_2$ and $c_3 > c_4$
4. $c_1 < c_2$ and $c_3 < c_4$

In all cases $d_{(2)}(u)$ is constant for all $u \in V$.

Hence $G$ is a (2, (c₁, c₂)) - regular intuitionistic fuzzy graph where $d_{(2)}(u) = (c₁, c₂)$.

![Figure 12](image2)

Note that $d_{(2)}(u) \neq d_{(2)}(v)$. Hence $G$ is not a (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

**Remark 7.4.** If all the vertices take same positive and negative membership values then the Theorem 7.3 holds good for totally (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

**Remark 7.5.** Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G'(V,E)$ is an odd cycle of length > 5. Even if the alternate edges have same positive and same negative membership values, then $G$ need not be a (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

**Theorem 7.6.** Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G'(V,E)$ is any cycle of length > 4. Let $k_2 \geq k_1$ and $k_4 \geq k_3$. Let

$$\mu_2(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases}$$

$$\nu_2(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

Then $G$ is a (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

Proof. Case (i) $G'$ be an even cycle.

$$d_{(2)}(e_{i_1}) = (k_1 \land k_2, k_3 \lor k_4) + (k_1 \land k_2, k_3 \lor k_4)$$

$$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4)$$

$$d_{(2)}(e_{i_2}) = (c_1, c_2)$$

Hence $G$ is (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

case(ii) $G'$ be an odd cycle. Let $e_{i_1}, e_{i_2}, \ldots, e_{2n+1}$ be edges of $G'$

$$d_{(2)}(e_{i_1}) = \mu_2^{(2)}(c_1, c_2) \land \mu_2^{(2)}(e_{i_2}) \lor \nu_2^{(2)}(e_{i_2})$$

$$= (k_1 \land k_2, k_3 \lor k_4) + (k_1 \land k_2, k_3 \lor k_4)$$

$$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4)$$

$$d_{(2)}(e_{i_2}) = (c_1, c_2)$$

Hence $G$ is (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.

$$d_{(2)}(e_{i_2}) = \mu_2^{(2)}(c_1, c_2) \land \mu_2^{(2)}(e_{i_2}) \lor \nu_2^{(2)}(e_{i_2})$$

$$= (k_1 \land k_2, k_3 \lor k_4) + (k_1 \land k_2, k_3 \lor k_4)$$

$$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4)$$

$$d_{(2)}(e_{i_2}) = (c_1, c_2)$$

Hence $G$ is (2, (c₁, c₂)) - regular intuitionistic fuzzy graph.
Remark 7.7. The above theorem 7.6 holds good for totally \((2, (c_1, c_2))\) – regular intuitionistic fuzzy graph if all the vertices take same positive and same negative membership values.

Theorem 7.8. Let \(G = (A,B)\) be intuitionistic fuzzy graph such that \(G^* (V,E)\) is any cycle of length \(>4\).

\[
\mu_2(e_i) = \begin{cases} 
  k_1 & \text{if } i \text{ is odd} \\
  k_2 & \text{if } i \text{ is even}
\end{cases}
\]

where \(k_2 \leq k_1\) and \(k_3 \leq k_4\) are not constants, then \(G\) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph.

Proof. case (i) \(G^*\) be an even cycle.

\[
d_{G^*}(v_i) = (k_1 \land k_2, k_3 \lor k_4) + (k_1 \land k_2, k_3 \lor k_4) = (k_2, k_3) + (k_2, k_3) = (2k_2, 2k_3)
\]

Hence \(G^*\) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph.

case (ii) \(G^*\) be an odd cycle.

Let \(e_1, e_2, \ldots, e_{2n+1}\) be edges of \(G^*\).

\[
d_{G^*}(v_i) = \left(\mu_2^{(2)}(e_1) \land \mu_2^{(2)}(e_2), \gamma_2^{(2)}(e_1) \lor \gamma_2^{(2)}(e_2)\right) + \left(\mu_2^{(2)}(e_{2n}) \land \mu_2^{(2)}(e_{2n+1}), \gamma_2^{(2)}(e_{2n}) \lor \gamma_2^{(2)}(e_{2n+1})\right)
\]

where \(c_1 \leq k_1\) and \(c_2 \leq k_2\) are not constants, then \(G\) is \((2, (c_1, c_2))\) - regular intuitionistic fuzzy graph.

Remark 7.9. The above theorem 7.8 holds good for totally \((2, (c_1, c_2))\) – regular intuitionistic fuzzy graph if all the vertices take same positive and same negative membership values.

REFERENCES


