International Journal of Advanced Research in Computer Engineering & Technology (IJARCET)
Volume 4, Issue 12, December 2015

(2, (c_1, c_2))-Regular Intuitionistic Fuzzy Graphs

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Abstract— In this paper d_2 - degree of a vertex and total d_2 -degree of a vertex in an intuitionistic fuzzy graphs are defined. Also $(2, (c_1, c_2))$-regularity and totally $(2, (c_1, c_2))$-regularity of an intuitionistic fuzzy graphs are defined. A relation between $(2, (c_1, c_2))$-regularity and totally $(2, (c_1, c_2))$-regularity on intuitionistic fuzzy graph is studied. $(2, (c_1, c_2))$-regularity on a path on four vertices, a Barbell graph $B_{m,n}$ $(n>1)$ and a cycle C_n are studied with some specific membership functions.

Index Terms— degree of a vertex in an intuitionistic fuzzy graph, regular intuitionistic fuzzy graph, intuitionistic fuzzy graph, total degree, totally regular intuitionistic fuzzy graph, d_2-degree of a vertex in fuzzy graph, semiregular graphs.

AMS Subject Classification: 05C12, 03E72, 05C72.

Introduction

In 1965, Lofti A. Zadeh [15] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T.Attanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T.Atanassov added a new component(which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set(and the non- membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [3, 4].

Azriel Rosenfeld introduced the concept fuzzy graphs in 1975 [12]. It has been growing fast and has numerous application in various fields. Bhattacharya [5] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Morderson and Peng[6]. Krassimir T Atanassov [4] introduced the intuitionistic fuzzy graph theory. R.Parvathi and M.G.Karunambigai[9] introduced intuitionistic fuzzy graphs as a special case of Atanassov's IFG and discussed some properties of regular intuitionistic fuzzy graphs[7]. M.G. Karunambigai, R.Parvathi and R.Buvaneswari introduced constant intuitionistic fuzzy graphs[8]. M. Akram, W. Dudek [1] introduced the regular intuitionistic fuzzy graphs. M.Akram and Bijan Davvaz [2] introduced the notion of strong intuitionistic fuzzy graphs and discussed some of their properties.
N.R.Santhi Maheswari and C.Sekar introduced d_2 -degree of vertex in fuzzy graphs and introduced $(2, k)$ -regular fuzzy graphs and totally $(2, k)$ - regular fuzzy graphs[13]. Also, they introduced d_m -degree, total d_m –degree of a vertex in fuzzy graphs and introduced an m-neighbourly irregular fuzzy graphs [14]. S. Ravinarayanan and N.R. Santhi Maheswari introduced $(2, (c_1, c_2))$ -regular bipolar fuzzy graphs[10].These motivates us to introduce d_2 -degree, total d_2 -degree of a vertex in an intuitionistic fuzzy graph and discussed some properties.

2 Preliminaries

We present some known definitions related to fuzzy graphs and intuitionistic fuzzy graphs for ready reference to go through the work presented in this paper.

Manuscript received Dec, 2015

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ISSN: 2278 – 1323

Definition 2.1. A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ) , where $\sigma : V \rightarrow [0,1]$ is a fuzzy subset of a non empty set V and $\mu : V \times V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V , the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied[6].

Definition 2.2. Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_m -degree of a vertex u in G is $d_m(u) = \sum \mu^m(uv)$, where $\mu^m(uv) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge . . . \wedge \mu(u_{m-1}v) : u, u_1, u_2, . . . , u_{m-1}, v$ is the shortest path connecting u and v of length $m\}$. Also, $\mu(uv) = 0$, for uv not in E[14].

Definition 2.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*: (V,E)$. The total d_m -degree of a vertex $u \in V$ is defined as $td_m(u) = \sum \mu^m(uv) + \sigma(u) = d_m(u) + \sigma(u)$[14].

Definition 2.4. If each vertex of G has the same d_m - degree k, then G is said to be an (m, k) -regular fuzzy graph[14].

Definition 2.5. If each vertex of G has the same total d_m - degree k, then G is said to be totally (m, k) -regular fuzzy graph [14].

Definition 2.6. An intuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (V,E)$ where
(i) $V = \{v_1, v_2, v_3...v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\gamma_1 : V \rightarrow [0, 1]$ denote the degree of membership and non membership of the element $v_i \in V$, $(i=1,2,3,...n)$, such that $0 \leq \mu_1(v_i)+ \gamma_1(v_i) \leq 1$
(ii) $E \subseteq V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\gamma_2 : V \times V \rightarrow [0, 1]$ are such that $\mu_2(v_i, v_j) \leq min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\gamma_2(v_i, v_j) \leq max\{ \gamma_1(v_i) , \gamma_1(v_j) \}$ and $0 \leq \mu_2(v_i, v_j)+ \gamma_2(v_i, v_j) \leq 1$,for every $(v_i, v_j) \in E$,$(i,j=1,2,....n)$[8].

Definition 2.7. If $v_i, v_j \in V \subseteq G$, the μ -strength of connectedness between two vertices v_i and v_j is defined as $\mu_2^\infty (v_i, v_j) = sup\{\mu_2^k(v_i, v_j) : k = 1, 2, ...n \}$ and γ -strength of connectedness between two vertices v_i and v_j is defined as $\gamma_2^\infty(v_i, v_j) = inf\{ \gamma_2^k(v_i, v_j) : k = 1, 2, ...n \}$.
If u and v are connected by means of paths of length k then $\mu_2^k(u, v)$ is defined as sup $\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge .... \wedge \mu_2(v_{k-1}, v) : (u, v_1, v_2, ...v_{k-1}, v) \in V \}$ and $\gamma_2^k(u, v)$ is defined as inf $\{ \gamma_2(u, v_1) \wedge \gamma_2(v_1, v_2) \wedge .... \wedge \gamma_2(v_{k-1}, v) :(u, v_1, v_2, ...v_{k-1}, v) \in V \}$[8].

Definition 2.8. Let $G = (V,E)$ be an intuitionistic fuzzy graph on $G^*(V,E)$. Then the degree of a vertex $v_i \in G$ is defined by $d(v_i) = (d_{\mu 1}(v_i), d_{\gamma 1}(v_i))$, where $d_{\mu 1}(v_i) =\sum \mu_2(v_i, v_j)$ and $d_{\gamma 1}(v_i) =\sum \gamma_2(v_i, v_j)$, for $v_i, v_j \in E$ and $\mu_2(v_i, v_j) =0$ and $\gamma_2(v_i, v_j)= 0$ for $v_i, v_j \notin E$ [8].

Definition 2.9. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V,E)$. Then the total degree of a vertex $v_i \in G$ is defined by $td(v_i) = (td_{\mu 1}(v_i), td_{\gamma 1}(v_i))$, where $td_{\mu 1}(v_i) = d_{\mu 1}(v_i) + \mu_1(v_i)$ and $td_{\gamma 1}(v_i) = d_{\gamma 1}(v_i) + \gamma_1(v_i)$ [8].

3 d_2 - degree of vertex in Intuitionistic Fuzzy Graph

In this section, d_2 - degree of a vertex in an intuitionistic fuzzy graph is introduced

Definition 3.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph. The μ d_2 - degree of a vertex $u \in G$ is defined as $d_{(2)\mu 1}(u) =\sum \mu_2^{(2)}(u, v)$ where $\mu_2^{(2)}(u, v) = sup\{\mu_2(u, u_1) \wedge \mu_2(u_1, v)$: u, u_1, v is the shortest path connecting u and v of length 2)}. The γ d_2 - degree of a vertex $u \in G$ is defined as $d_{(2)\gamma 1}(u) =\sum \gamma_2^{(2)}(u, v)$ where $\gamma_2^{(2)}(u, v) = inf\{ \gamma_2(u, u_1) \vee \gamma_2 (u_1, v)$: u, u_1, v is the shortest path connecting u and v of length 2)}. The d_2 - degree of a vertex u is defined as $d_{(2)}(u) = (d_{(2)\mu 1} (u), d_{(2)\gamma 1}(u))$.
The minimum $d_{(2)}$ - degree of G is $\delta_{(2)}(G) = \wedge\{d_{(2)}(v) : v \in V \}$.
The maximum $d_{(2)}$ - degree of G is $\Delta_{(2)}(G) = \vee\{d_{(2)}(v) : v \in V \}$.

Example 3.2. Consider an intuitionistic fuzzy graph $G= (A,B)$ on $G^* : (V,E)$

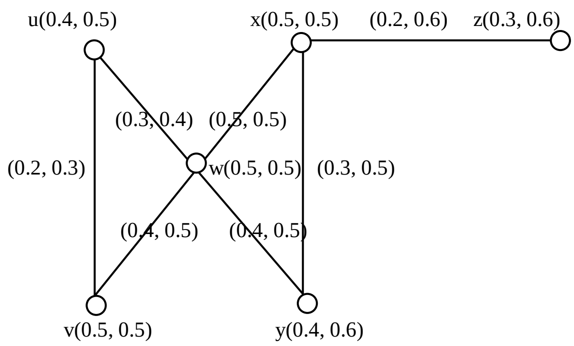

Figure.1

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$d_{(2)}(u) = (0.3 \wedge 0.4, 0.4 \vee 0.5) + (0.3 \wedge 0.5, 0.4 \vee 0.5) = (0.3, 0.5) + (0.3, 0.5) = (0.6, 1.0)$
$d_{(2)}(v) = (0.4 \wedge 0.5, 0.5 \vee 0.5) + (0.4 \wedge 0.4, 0.5 \vee 0.5) = (0.4, 0.5) + (0.4, 0.5) = (0.8, 1.0)$
$d_{(2)}(w) = (0.5 \wedge 0.2, 0.5 \vee 0.6) = (0.2, 0.6)$
$d_{(2)}(x) = (0.5 \wedge 0.3, 0.5 \vee 0.4) + (0.5 \wedge 0.4, 0.5 \vee 0.5) = (0.3, 0.5) + (0.4, 0.5) = (0.7, 1.0)$
$d_{(2)}(y) = (0.4 \wedge 0.3, 0.5 \vee 0.4) + (0.4 \wedge 0.4, 0.5 \vee 0.5) + (0.3 \wedge 0.2, 0.6 \vee 0.5)$
$= (0.3, 0.5) + (0.4, 0.5) + (0.2, 0.6) = (0.9, 1.6)$
$d_{(2)}(z) = (0.2 \wedge 0.3, 0.6 \vee 0.5) + (0.2 \wedge 0.5, 0.6 \vee 0.5) = (0.2, 0.6) + (0.2, 0.6) = (0.4, 1.2)$

Example 3.3. Consider an intuitionistic fuzzy graph $G = (A,B)$ on $G^* : (V,E)$

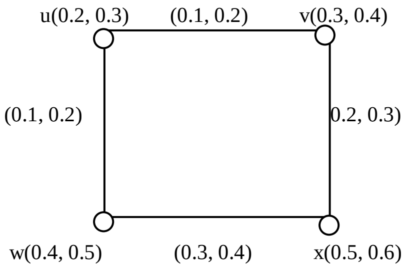

Figure.2

$d_{(2)}(u) = \{sup(0.1 \wedge 0.2, 0.1 \wedge 0.3), inf(0.2 \vee 0.3, 0.2 \vee 0.4)\}$
$= \{sup(0.1, 0.1), inf(0.3, 0.4)\} = (0.1, 0.3)$
$d_{(2)}(v) = \{sup(0.1 \wedge 0.1, 0.2 \wedge 0.3), inf(0.2 \vee 0.2, 0.3 \vee 0.4)\}$
$= \{sup(0.1, 0.2), inf(0.2, 0.4)\} = (0.2, 0.2)$
$d_{(2)}(w) = \{sup(0.2 \wedge 0.1, 0.3 \wedge 0.1), inf(0.3 \vee 0.2, 0.4 \vee 0.2)\}$
$= \{sup(0.1, 0.1), inf(0.3, 0.4)\} = (0.1, 0.3)$
$d_{(2)}(x) = \{sup(0.1 \wedge 0.1, 0.2 \wedge 0.3), inf(0.2 \vee 0.2, 0.3 \vee 0.4)\}$
$= \{sup(0.1, 0.2), inf(0.2, 0.4)\} = (0.2, 0.2)$

4 (2,(c_1,c_2)) - Regular and Totally (2,(c_1,c_2)) - Regular Intuitionistic Fuzzy Graph

In this section, $(2,(c_1, c_2))$- regular and totally $(2, (c_1, c_2))$- regular intuitionistic fuzzy graphs are introduced

Definition 4.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph. If $d_{(2)}(u) = (c_1, c_2)$, for all $u \in V$, then G is said to be $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Example 4.2. Consider an intuitionistic fuzzy graph $G = (A,B)$ on $G^* : (V,E)$

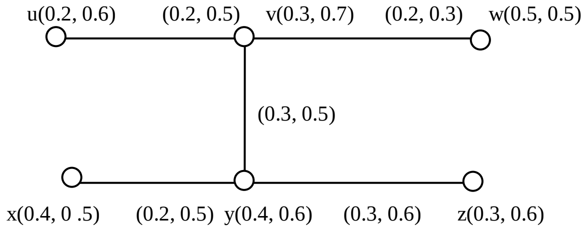

Figure.3

Note that $d_{(2)}(u) = (0.4, 1.0)$, $d_{(2)}(v) = (0.4, 1.0)$, $d_{(2)}(w) = (0.4, 1.0)$ and $d_{(2)}(x) = (0.4, 1.0)$. This graph is $(2, (0.4, 1.0))$ - regular intuitionistic fuzzy graph.

Definition 4.3. Let $G = (A,B)$ be an intuitionistic fuzzy graph. Then the total d_2 - degree of a vertex $u \in V$ is defined as $td_{(2)}(u) = (td_{(2)\mu 1}(u), td_{(2)\gamma 1}(u))$ where $td_{(2)\mu 1}(u) = d_{(2)\mu 1}(u) + \mu_1(u)$ and $td_{(2)\gamma 1}(u) = d_{(2)\gamma 1}(u) + \gamma_1(u)$. Also it can be defined as $td_{(2)}(u) = d_{(2)}(u) + A(u)$ where $A(u) = (\mu_1(u), \gamma_1(u))$.

Definition 4.4. Let $G = (A,B)$ be an intuitionistic fuzzy graph. If each vertex of G has same total d_2 - degree, then G is said to be totally $(2,(c_1, c_2))$ -regular intuitionistic fuzzy graph.

Example 4.5. Consider an intuitionistic fuzzy graph $G = (A,B)$ on $G^*: (V,E)$

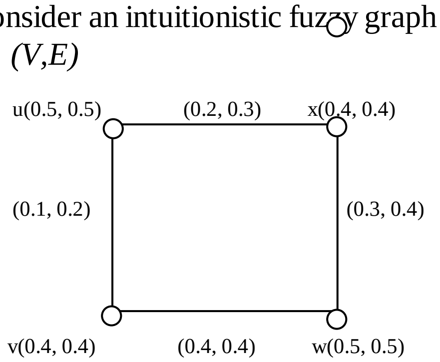

Figure.4

Here $td_{(2)}(u) = (0.2, 0.4) + (0.5, 0.5) = (0.7, 0.9)$
$td_{(2)}(v) = (0.3, 0.3) + (0.4, 0.6) = (0.7, 0.9)$
$td_{(2)}(w) = (0.2, 0.4) + (0.5, 0.5) = (0.7, 0.9)$
$td_{(2)}(x) = (0.3, 0.3) + (0.4, 0.6) = (0.7, 0.9)$
This graph is totally $(2, (0.7, 0.9))$ - regular intuitionistic fuzzy graph.

Example 4.6. A totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph need not be $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph. Consider an intuitionistic fuzzy graph $G = (A,B)$ on $G^* : (V,E)$

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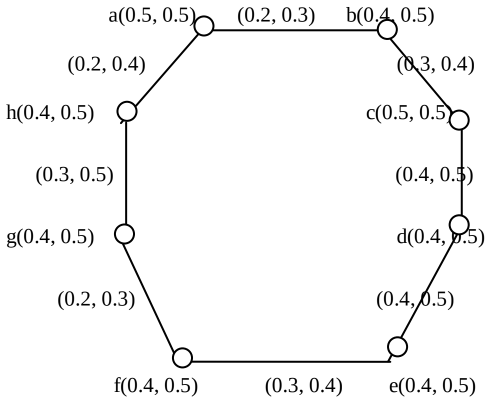

Figure.5

Here, $td_{(2)}(u) = (1.1, 1.1)$ .So G is totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
But $d_{(2)}(u) \neq d_{(2)}(w)$. Hence G is not $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph

Example 4.7. A $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph need not be totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph. Consider intuitionistic fuzzy graph $G= (A,B)$ on $G^*: (V,E)$

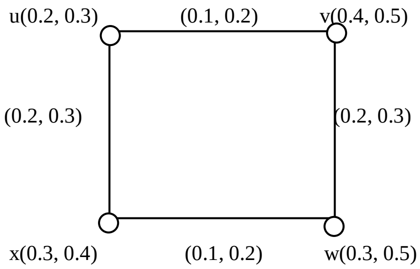

Figure.6

Note that $d_{(2)}(u) = (0.1, 0.3)$, for all $u \in V$. Hence G is $(2, (0.1, 0.3))$ – regular intuitionistic fuzzy graph. But $td_{(2)}(u) \neq td_{(2)}(v)$. Hence G is not totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Example 4.8. A $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph which is totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph. Consider an intuitionistic fuzzy graph $G= (A,B)$ on $G^* : (V,E)$

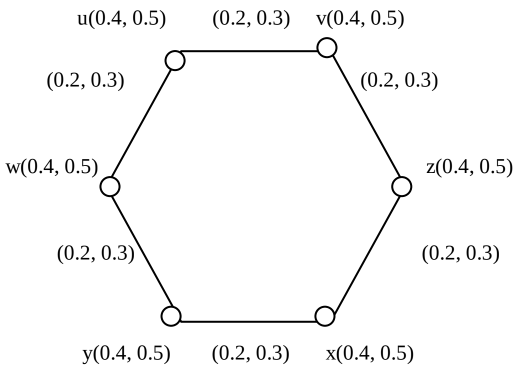

Figure.7

Then $d_{(2)}(u) = (0.4, 0.6)$, for all $u \in V$ and $td_{(2)}(u) = (0.8, 1.1)$, for all $u \in V$. Hence G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph and totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Theorem 4.9. Let $G = (A,B)$ be an intuitionistic fuzzy graph on $G^*(V,E)$. Then $A(u) = (k_1, k_2)$, for all $u \in V$ if and only if the following conditions are equivalent.
1. $G = (A,B)$ is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
2. $G = (A,B)$ is totally $(2, (c_1 + k_1, c_2 + k_2)$ - regular intuitionistic fuzzy graph.

Proof. Suppose $A(u) = (k_1, k_2)$, for all $u \in V$.
Assume that G is a $(2, (c_1, c_2))$ – regular intuitionistic fuzzy graph . Then $d_{(2)}(u) = (c_1, c_2)$, for all $u \in V$
So $td_{(2)}(u) = d_{(2)}(u) + A(u) = (c_1, c_2) + (k_1, k_2) = (c_1 + k_1 , c_2 + k_2)$
Hence G is a totally $(2, (c_1 + k_1, c_2 + k_2))$ - regular intuitionistic fuzzy graph.
Thus (i) $\Longrightarrow$ (ii) is proved.
Now suppose G is totally $(2, (c_1 + k_1, c_2 + k_2))$ - regular intuitionistic fuzzy graph.
$\Longrightarrow$) $td_{(2)}(u) = (c_1 + k_1, c_2 + k_2)$, for all $u \in V$
$\Longrightarrow d_{(2)}(u) + A(u) = (c_1 + k_1, c_2 + k_2)$, for all $u \in V$
$\Longrightarrow d_{(2)}(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2)$, for all $u \in V$
$\Longrightarrow d_{(2)}(u) = (c_1, c_2)$, for all $u \in V$
Hence G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph. Thus (i) and (ii) are equivalent.
Conversely assume (i) and (ii) are equivalent. Let G be a $(2,(c_1 , c_2))$ – regular intuitionistic fuzzy graph and totally $(2,(c_1+k_1 , c_2+k_2))$ - regular intuitionistic fuzzy graph.
$\Longrightarrow td_{(2)}(u) = (c_1+k_1 , c_2+k_2)$ and $d_{(2)}(u) = (c_1, c_2)$, for all $u \in V$
$\Longrightarrow d_{(2)}(u) + A(u) = (c_1+k_1 , c_2+k_2)$ and $d_{(2)}(u) = (c_1, c_2)$, for all $u \in V$
$\Longrightarrow d_{(2)}(u) + A(u) = (c_1, c_2) + (k_1, k_2)$ and $d_{(2)}(u) = (c_1, c_2)$, for all $u \in V$
$\Longrightarrow A(u) = (k_1, k_2)$, for all $u \in V$. Hence $A(u) =(k_1, k_2)$.

5 $(2, (c_1, c_2))$ - regularity on path on four vertices with specific membership function

In this section $(2,(c_1 , c_2))$- regularity on path on four vertices is discussed with some specific membership function

ISSN: 2278 – 1323

Theorem 5.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices. If B is constant function then G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
Proof. Suppose that B is constant function, say $B(uv) = (k_1, k_2)$, for all $uv \in E$.
Then $d_{(2)}(u) = (k_1, k_2)$. Hence G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Remark 5.2. The converse of Theorem 5.1 need not be true. For example consider $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices.

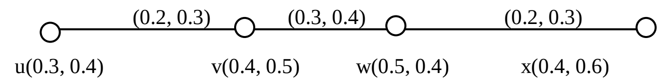

Figure.8

Note that $d_{(2)}(u) = (0.2, 0.4)$, for all $u \in V$. So, G is a $(2, (0.2, 0.4))$ – regular intuitionistic fuzzy graph. But B is not a constant function.

Theorem 5.3. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices. If alternate edges have same membership values then G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph where $c_1 = min\{\mu_2^{(2)}\}$ and $c_2 = max\{\gamma_2^{(2)}\}$.

Theorem 5.4. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is path on four vertices. If the middle edge have membership value less than membership value of remaining edges and non-membership value greater than non-membership value of remaining edges, then G is a $(2, (c_1, c_2))$ – regular intuitionistic fuzzy graph where c_1 and c_2 are membership values of the middle edge.

Example 5.5. Consider an intuitionistic fuzzy graph $G= (A,B)$ on $G^* : (V,E)$

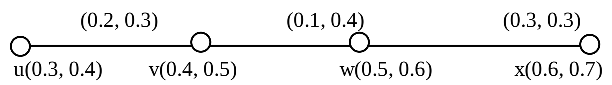

Figure.9

Note that $d_{(2)}(u) = (0.1, 0.4)$, $d_{(2)}(v) = (0.1, 0.4)$, $d_{(2)}(w) = (0.1, 0.4)$ and $d_{(2)}(x) =(0.1, 0.4)$. Hence G is a $(2, (0.1, 0.4))$ -regular intuitionistic fuzzy graph.

Remark 5.6. If A is constant function, then Theorems 5.1, 5.3 and 5.4 hold good for totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

6 (2, (c_1, c_2)) - regularity on Barbell graph $B_{n,n}$(n >1) with some specific membership function

In this section $(2, (c_1 , c_2))$- regularity on Barbell graph is discussed with some specific membership function

Theorem 6.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is a barbell graph $B_{n,n}$ of order $2n$. If B is a constant function, then G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph where $(c_1, c_2) = nB(uv)$ where $uv \in E$.

Remark 6.2. The converse of theorem 6.1 need not be true. For example consider $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is barbell graph $B_{2,2}$ of order 6.

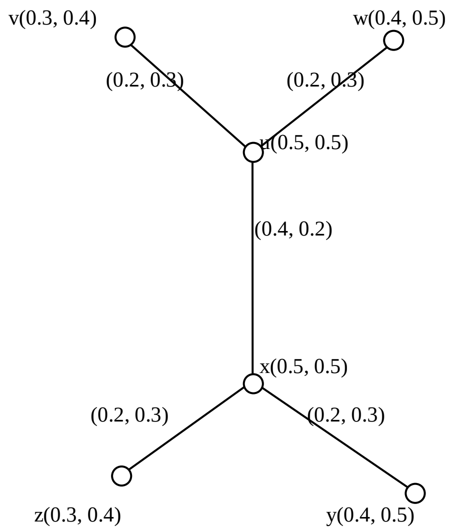

Figure.10

Note that $d_{(2)}(u) = (0.4, 0.6)$, for all $u \in V$. So, G is a $(2, (0.4, 0.6))$ – regular intuitionistic fuzzy graph. But B is not a constant function.

Theorem 6.3. Let $G = (A,B)$ be an intuitionistic fuzzy graph on $G^*(V,E)$, a barbell graph $B_{n,n}(n > 1)$. If the pendant edge has membership value less than the membership value of middle edge and non-membership value greater than the non-membership value of middle edge then G is $(2, n(c_1, c_2))$ - regular intuitionistic fuzzy graph where (c_1, c_2) is the membership value of pendant edge.

Remark 6.4. If A is constant function, then the theorem 6.1 and 6.3 hold good for totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

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7 (2, (c_1, c_2)) - regularity on a cycle with some specific membership functions

In this section $(2, (c_1, c_2))$- regularity on a cycle is discussed with some specific membership function

Theorem 7.1. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is the cycle of length ≥ 5 . If $\mu_2^{(2)}$ and $\gamma_2^{(2)}$ are constant functions, then G is a $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph where $(c_1, c_2) = 2(\mu_2^{(2)}, \gamma_2^{(2)})$.

Remark 7.2. The converse of the theorem 7.1 need not be true. For example consider $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is an odd cycle of length five.

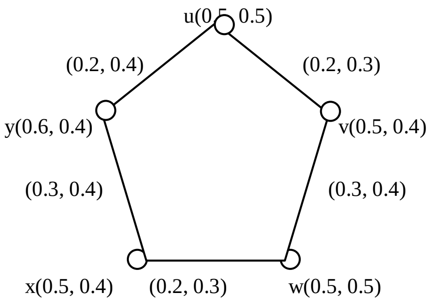

Figure.11

Note that $d_{(2)}(u) = (0.4, 0.8)$, for all $u \in V$
So G is a $(2, (0.4, 0.8))$ - regular intuitionistic fuzzy graph. But $\mu_2^{(2)}$ and $\gamma_2^{(2)}$ are not constant functions.

Theorem 7.3. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is an even cycle of length ≥ 6. If alternate edges have same positive and negative membership values then G is a $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
Proof. If alternate edges have same positive and negative membership values then

$$\mu_2(e_i) = \begin{cases} c_1 \text{ if } i \text{ is odd} \\ c_2 \text{ if } i \text{ is even} \end{cases} \qquad \gamma_2(e_i) = \begin{cases} c_3 \text{ if } i \text{ is odd} \\ c_4 \text{ if } i \text{ is even} \end{cases}$$

Here we have 4 possible cases
1. $c_1 > c_2$ and $c_3 > c_4$
2. $c_1 > c_2$ and $c_3 < c_4$
3. $c_1 < c_2$ and $c_3 > c_4$
4. $c_1 < c_2$ and $c_3 < c_4$

In all cases $d_{(2)}(u)$ is constant for all $u \in V$.
Hence G is a $(2,(c_1 , c_2))$ - regular intuitionistic fuzzy graph where $d_{(2)}(u) = (c_1, c_2)$.

Remark 7.4. If all the vertices take same positive and negative membership values then the Theorem 7.3 holds good for totally $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Remark 7.5. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is an odd cycle of length >5. Even if the alternate edges have same positive and same negative membership values, then G need not be a $(2, (c_1, c_2))$ – regular intuitionistic fuzzy graph.

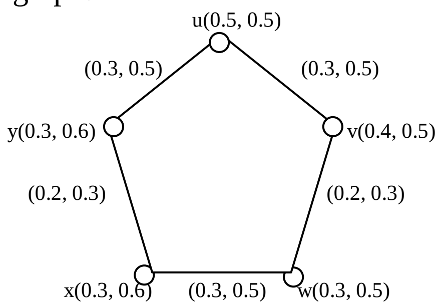

Figure.12

Note that $d_{(2)}(u) \neq d_{(2)}(v)$. Hence G is not a $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Theorem 7.6. Let $G = (A,B)$ be an intuitionistic fuzzy graph such that $G^*(V,E)$ is any cycle of length >4. Let $k_2 \geq k_1$ and $k_4 \geq k_3$. Let

$$\mu_2(e_i) = \begin{cases} k_1 \text{ if } i \text{ is odd} \\ k_2 \text{ if } i \text{ is even} \end{cases} \qquad \gamma_2(e_i) = \begin{cases} k_3 \text{ if } i \text{ is odd} \\ k_4 \text{ if } i \text{ is even} \end{cases}$$

then G is a $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
Proof. case (i) G^* be an even cycle.

$$d_{(2)}(v_i) = (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$$
$$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4)$$

$d_{(2)}(v_i) = (c_1, c_2)$ where $c_1 = 2k_1$, $c_2 = 2k_4$
Hence G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.
case(ii) G^* be an odd cycle. Let $e_1, e_2, \ldots, e_{2n+1}$ be edges of G^*

$$d_{(2)}(v_1) = (\mu_2^{(2)}(e_1) \wedge \mu_2^{(2)}(e_2), \gamma_2^{(2)}(e_1) \vee \gamma_2^{(2)}(e_2)) + (\mu_2^{(2)}(e_{2n}) \wedge \mu_2^{(2)}(e_{2n+1}), \gamma_2^{(2)}(e_{2n}) \vee \gamma_2^{(2)}(e_{2n+1}))$$
$$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$$
$$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4)$$

$d_{(2)}(v_1) = (c_1, c_2)$ where $c_1 = 2k_1$, $c_2 = 2k_4$

$$d_{(2)}(v_2) = (\mu_2^{(2)}(e_2) \wedge \mu_2^{(2)}(e_3), \gamma_2^{(2)}(e_2) \vee \gamma_2^{(2)}(e_3)) + (\mu_2^{(2)}(e_1) \wedge \mu_2^{(2)}(e_{2n+1}),$$

ISSN: 2278 – 1323

$\gamma_2^{(2)}(e_1) \vee \gamma_2^{(2)}(e_{2n+1}))$

$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$

$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4)$

$d_{(2)}(v_2) = (c_1, c_2)$ where $c_1 = 2k_1$, $c_2 = 2k_4$

Proceeding like this we get $d_{(2)}(v_n) = (c_1, c_2)$ where $c_1 = 2k_1$, $c_2 = 2k_4$

Hence $d_{(2)}(v_i) = (c_1, c_2)$. So G is $(2, (c_1 , c_2))$ - regular intuitionistic fuzzy graph.

Remark 7.7. The above theorem 7.6 holds good for totally $(2, (c_1, c_2))$ – regular intuitionistic fuzzy graph if all the vertices take same positive and same negative membership values.

Theorem 7.8. Let $G = (A,B)$ be intuitionistic fuzzy graph such that $G^*(V,E)$ is any cycle of length >4.

$$\mu_2(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases} \quad \gamma_2(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

where $k_2 \leq k_1$ and $k_4 \leq k_3$ are not constants, then G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Proof. case (i) G^* be an even cycle.

$d_{(2)}(v_i) = (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$

$= (k_2, k_3) + (k_2, k_3) = (2k_2, 2k_3)$

$d_{(2)}(v_i) = (c_1, c_2)$ where $c_1 = 2k_2$, $c_2 = 2k_3$

Hence G is $(2, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

case(ii) G^* be an odd cycle.

Let $e_1, e_2, . . . , e_{2n+1}$ be edges of G^*.

$d_{(2)}(v_1) = (\mu_2^{(2)}(e_1) \wedge \mu_2^{(2)}(e_2), \gamma_2^{(2)}(e_1) \vee \gamma_2^{(2)}(e_2)) + (\mu_2^{(2)}(e_{2n}) \wedge \mu_2^{(2)}(e_{2n+1}),$

$\gamma_2^{(2)}(e_{2n}) \vee \gamma_2^{(2)}(e_{2n+1}))$

$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$

$= (k_2, k_3) + (k_2, k_3) = (2k_2, 2k_3)$

$d_{(2)}(v_1) = (c_1, c_2)$ where $c_1 = 2k_1$, $c_2 = 2k_3$

$d_{(2)}(v_2) = (\mu_2^{(2)}(e_2) \wedge \mu_2^{(2)}(e_3), \gamma_2^{(2)}(e_2) \vee \gamma_2^{(2)}(e_3)) + (\mu_2^{(2)}(e_1) \wedge \mu_2^{(2)}(e_{2n+1}),$

$\gamma_2^{(2)}(e_1) \vee \gamma_2^{(2)}(e_{2n+1}))$

$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$

$= (k_2, k_3) + (k_2, k_3) = (2k_2, 2k_3)$

$d_{(2)}(v_2) = (c_1, c_2)$ where $c_1 = 2k_2$, $c_2 = 2k_3$

Proceeding like this we get $d_{(2)}(v_n) = (c_1, c_2)$ where $c_1 = 2k_2$, $c_2 = 2k_3$

Hence $d_{(2)}(v_i) = (c_1, c_2)$. So, G is $(2,(c_1 , c_2))$ - regular intuitionistic fuzzy graph.

Remark 7.9. The above theorem 7.8 holds good for totally $(2, (c_1, c_2))$ – regular intuitionistic fuzzy graph if all the vertices take same positive and same negative membership values.

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ISSN: 2278 – 1323