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An improved two-warehouse partial backlogging inventory model fordeteriorating items under conditionally permissible delay in payment

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Abstract
Permissible delay in payment basically termed as "Trade credit financing" and in the present business scenario it has become a popular trend to attract the retailers by offering some interest free period by suppliers/wholesalers to increase the sales. The phenomenon of trade credit has drawn attention of many researches for last decay.In this work, for the development of model we have considered this phenomenon under some conditions with combination of constantdemand and holding cost. Shortages are permitted in the OW and partially backlogged at the next replenishmentcycle. Theobjective of modelling is to derive the retailer's optimal replenishment policy that minimizes the total relevant inventory cost per unit of time. This model dealt with single item. Numerical example is presented to illustrate and validate the model applicability and reliability.

Keywords: **Two warehouses, Deterioration, ConstantDemand and holding cost,Permissible delay in payment andpartially backlogged shortages**.

1.0 INTRODUCTION

In the research field, according to existing literature, Haley and Higgins[1] developed inventory policy and trade financing. Then Goyal[2] developed an economic order quantity (EOQ) model under the condition of permissible delay in payments with a constant demand rate. It was assumed that the costs, selling and purchase were same. TheGoyal's model was further extended by the Aggrawal and Jaggi [3] for deteriorating items. Thereafter several models were developed by researches considering trade financing e.g., [4-11].Sincelasttwo decay, the trade financing has become necessity in the present competitive business environment. Therefore to attract retailers, suppliers/wholesalers offered permissible delay period under some condition to increase their sales revenue in the competitive business environment.To get benefit of the permissible delay period, retailer invested his earned revenue from the sells during the permissible delay period and earns interest on it, which reducestotal inventory cost.

In the past, researchers have established a lot of research in the field of Inventory management and Inventory control system. Inventory management and control system basically deals with demand and supply chain problems and for this, production units(Producer of finished goods), venders, suppliers and retailers need to store the raw materials, finished goods for future demand and supply in the market and to the customers. Many models have been developed considering various time dependent demand with shortages and without shortage. Hartely[12]discussed an inventory model with two storage facilities**.** Ghare and Schrader[13] initially worked in this field and they extended Harris[3],EOQ model with deterioration and shortages**.**

In many literature researchers have developed inventory models considering deterioration. In general deterioration is defined as the spoilage, leakage, dryness, vaporization or expiry of self-life time of certain items during the storage period which results in decrease of usefulness of the original one. Assuming the deterioration in both warehouses**,** Sarma,[14] extended his earlier model to the case of infinite replenishment rate with shortages. Bhunia and Maitie [15] extended the model of Goswami and Chaudhary [16] in that model they were not consider the deterioration and shortages were allowed and backlogged. The storage is major problem of the present competitive businesstransaction due to unavailability of the space in the busy market palaces. Another case, of inadequate storage area, can occur when a procurement of a large amount of items is decided. That could be due to, an attractive price discount for bulk purchase which is available or, when the cost of procuring goods is higher than the other inventory related costs or, when demand for items is very high or, when the item under consideration is a seasonal product such as the yield of a harvest or, when there are some problems in frequent procurement. Since the retailer has limited storage space therefore he needs more spaces to store the product purchased and hence required another storage house. Hence retailer may rent a warehouse for a short period.

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In the present business scenario of the market economy, there is an intense competition among the wholesalers/suppliers to promote theirbusiness. As a result they offer different type of facilities to their retailers as discussed earlier. Some of such facilities are discount on bulk purchase; offer to sell a large volume of goods on credit. In case trade credit period, basicallyretailer offered a permissible delay period, during this period, no interest is charged by the supplier. However beyond this period, a higher rate of interest is charged by the supplier under certain terms and conditions by an agreement between retailer and supplier. Jamal et al.[17]furthergeneralisedthe model by allowing the completely backlogged shortages. Thereafter a lot of work has been done by severalresearchers. In this connection, the work of Jaggi and Khanna,[18]Jaggi and Kausar[19], Jaggiand Mittal [20]and others are worth mentioning. However they have developed the model for a single ware-house under the assumption that the available ware-house has unlimited capacity. This assumption is not realistic as a ware-house is of limited capacity.

In this paper a deterministic inventory model for deteriorating items with two level of storage and constant demand with partially backlogged shortages is developed. Stock is transferred RW to OW under continuous release pattern and the transportation cost isnot incurred on the goods transferred. It is assumed that the items are starts deteriorating as they entered in the storages premises.We have also discussed different cases depending upon the permissible delay period offered by supplier and compared theresults. Thenumerical example is presented to illustratethe model development and sensitivity analysis is performed to study the change in the model on change of parameter's value.

2. ASSUMPTION AND NOTATION

The mathematical model of two warehouse inventory model for instantaneousdeteriorating items is based on the following assumptions and notations:

2.1 Assumption

1. Goods of OW are consumed only after the goods in RW consumed. This is due to the more holding cost in RW than in OW.
2. Holding cost is constant in both warehouses.
3. Replenishment rate is infinite and lead time is negligible i.e. zero.
4. The OW has the limited capacity of storage and RW has unlimited capacity.
5. The time horizon of the inventory system is infinite.
6. Demand is deterministic at constant rate of D unit per unit of time and given by $f(t) = D$.
7. Shortages are permitted and demand is partially backlogged at the beginning of next replenishment. The fraction of backordered is decreasing function of waiting time denoted by $B(t) = e^{-\delta t}$ where backlogging parameter $\delta > 0$and $0 < b(t) < 1$.
8. The rented warehouse equipped with better preservation facilities than the OW and therefore the unit inventory cost (Holding cost + deterioration cost) in RW>OW.
9. Retailer pays his purchase cost to supplier at the time of ordering for next cycle.
10. Supplier/Wholesaler charges interest on the positive stock withheld by the retailer.
11. The deteriorated items are neither repaired nor replaced during the replenishment cycle.
12. Transportation cost is not incurred on the items transported from RW to OW on the basis of continuous release pattern.

Apart from the above assumptions the following notations are used throughout the paper:

2.2Notations:

A_c: Cost of Ordering per Order.
W:Capacity of OW.
T : The length of replenishment cycle.
M: Permissible delay period.
t_1:Point oftime up to which inventory level vanishes in RW.
t_2: Point of time at which inventory level vanishes in OW and shortages begins.
h_r: The holding cost per unit time in OW.
h_w: The holding cost per unit time in RW and$(h_r - h_w) > 0$.
b_c: The backlogging cost per unit per unit time.
l_c: Opportunity lost sale cost.
P_c: Purchase cost per unit of item.
S_p: Selling price per unit of item
I_p : Interest charges per unit of time.
I_e : Interest earned per unit of time.
N :Number of inventory kept in RW at t=0.
$Q_r(t)$: The level of inventory in RW at time epoch t.
$Q_{w,i}(t)$: The level of inventory in OW at time epoch t for $i = 1,2$.

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$S(t)$: Inventory shortages level at time epoch t.

N_{max} : Maximum number of inventory ordered at$t = T$.

$\Gamma^{i}(t_w, \text{T})$: The present worth oftotal relevant inventory costper unit of time for cases $i = 1,2,3,4$

$\Gamma^{m}(t_w, \text{T})$: The present worth of total optimalrelevant inventory cost per unit of time.

3.0 MATHEMATICAL MODEL

In the beginning of the cycle at t=0 a lot size of N_{max} units of inventory enters into the system in which backlogged shortages, if backlogged $(N_{max} - (N + W)$ units are cleared and the remaining unitsN is kept into RW and W units in OW.(See Figure-1)

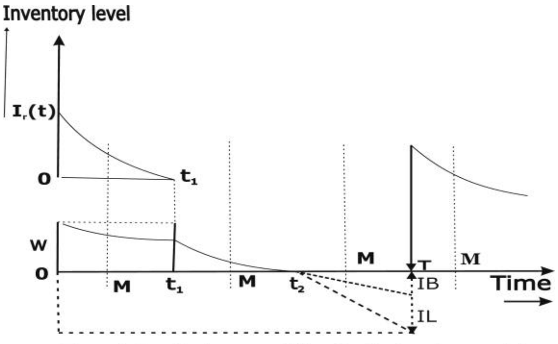

Figure 1.Graphical representation for the inventory model

Initially, during the time interval [0 t_1] the inventory level in RW depleted due toconstant and deterioration demand and in this period inventory level in the OW depleted due to deterioration only. Thesituation describing the inventory level is governed by the following differential equations:

$\frac{dQ_r(t)}{dt} = -D(t) - \beta Q_r(t); \ 0 \le t \le t_1 (1) \frac{dQ_{w,1}(t)}{dt} = -\alpha\, Q_{w,1}(t); 0 \le t \le t_1 (2)$

At the point of time$t = t_1$,when stock out in RW, the demand is met from OW until it vanishes i.e.during the time interval [$t_1 t_2$] in OW, inventory leveldepleted due to combined effect of constant demand and deterioration which governed by differential equation

$\frac{dQ_{w,2}(t)}{dt} = -D(t) - \alpha\, Q_{w,2}(t)\ ;\ t_1 \le t \le t_2 (3)$

Now at the point of time $t = t_2$, when stock out in OW and products are continuously on demand,shortages begins and is backlogged at the end of cycle length and supplied to the customers in the beginning of cycle length. Differential equation describing the situation is given as

$\frac{d\,S(t)}{dt} = -B(t)D(t); t_2 \le t \le T \qquad (4)$

Solving equations (1) to (4) with boundary conditions$Q_r(t) = 0 \ at\ t = t_1$,$I_{w,1}(t) = W \ at\ t = 0$, $I_{w,2}(t) = 0 \ at\ t = t_2$and $S(t) = 0 \ at\ t = T$,we get the following results:

$Q_r(t) = \frac{D}{\beta}(e^{\beta(t_w - t)} - 1); 0 \le t \le t_1 (5)$

$Q_{w,1}(t) = We^{-\alpha t}; 0 \le t \le t_1 (6)$

$Q_{w,2}(t) = \frac{D}{\alpha}(e^{\alpha(t_o - t)} - 1); t_1 \le t \le t_2 (7)$

$S(t) = \frac{D}{\delta}(e^{-\delta(T - t_o)} - e^{-\delta(T - t)}); t_2 \le t \le T (8)$Maximum numberof inventory backlogged at the end of the cycle length is

$IB(t = T) = \frac{D}{\delta}(e^{-\delta(T - t_o)} - 1) \qquad (9)$

Considering continuity of $Q_{w,i}(t) \ at\ t = t_1$, we have $Q_{w,1}(t_1) = Q_{w,2}(t_1)$ we have

$$t_2 = t_1 + \alpha^{-1}\log[1 + \frac{\alpha W e^{-\alpha t_w}}{D}]$$

$= f(t_1) \qquad (10)$

Maximum number of inventory purchased is

$$N_{max} = Q_r(0) + W + IB(t = T)$$
$$= \frac{D}{\beta}(e^{\beta t_w} - 1) + W + \frac{D}{\delta}(e^{-\delta(T - t_o)} - 1) (11)$$

Total inventory deteriorated in RW and OW are

$Q_r(0) - \int_0^{t_1} D(t)dt$ And $\alpha\left(\int_0^{t_1} Q_{w,1}(t)dt + \int_{t_1}^{t_2} Q_{w,2}(t)dt\right)$ respectively

Now present worth of total inventory cost consist of the following components:

1. Present worth ordering cost A_o
2. Present worth of holding cost in RW,$h_r \left(\int_0^{t_1} Q_r(t)dt\right)$=$h_r \left(\frac{D}{\beta^2}(e^{\beta t_1} - \beta t_1 - 1)\right)$
3. Present worth of holding cost in OW is $h_w \left(\int_0^{t_1} Q_{w,1}(t)dt + \int_{t_1}^{t_2} Q_{w,2}(t)dt\right)$

$$= h_w \left(\frac{W}{\alpha}(1 - e^{\alpha t_1}) + \left(\frac{D}{\alpha^2}(e^{\alpha(t_2 - t_1)} - \alpha(t_2 - t_1) - 1)\right)\right)$$

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4. Present worth of inventory deteriorating cost is

$$P_c \left\{ \begin{array}{c} \left(Q_r(0) - \int_0^{t_1} D(t)dt\right) \\ +\alpha\left(\int_0^{t_1} Q_{w,1}(t)dt + \int_{t_1}^{t_2} Q_{w,2}(t)dt\right) \end{array} \right\}$$

$$= P_c \left(W(1 - e^{\alpha t_1}) + \left(\frac{D}{\alpha}(e^{\alpha(t_2 - t_1)} - \alpha(t_2 - t_1) - 1)\right) + \left(\frac{D}{\beta}(e^{\beta t_1} - \beta t_1 - 1)\right)\right)$$

5. Present worth of backlogging cost is

$$b_c \left(\int_{t_2}^{T} S(t)dt\right) = \frac{b_c\ D}{\delta^2}\left(\delta(T - t_2)e^{-\delta(T - t_2)} - (1 - e^{-\delta(T - t_2)})\right)$$

6. Present worth of lost sales cost is

$$l_c \left(\int_{t_2}^{T} (1 - B(t)D(t))dt\right) = l_c \left(\frac{l_c\ D}{\delta}\left(\delta(T - t_2) - (1 - e^{-\delta(T - t_2)})\right)\right)$$

7. The interest payable and earned by the retailer during permissible delay period and non-permissible delay period based on the parameter values t_1 , t_2 , M and T , the following cases are arise and discussed separately

Case-1:$0 < M \le t_1$
Case-2:$t_1 < M \le t_2$
Case-3:$t_2 < M \le T$
Case-4:$T < M$
Now we discussed each case separately in section 4.0 as follows:

4.0 CASE DISCUSSION

4.1 Case-1: $0 < M \le t_1$

In this case the length of positive stock period is larger than permissible delay period therefore retailer has to be financed stock at the interest rate I_p after M and hence interest paid is given by

$$IP_1 = P_c I_p \left(\int_M^{t_1} Q_r(t)dt + \int_0^{t_1} Q_{w,1}(t)dt + \int_{t_1}^{t_2} Q_{w,2}(t)dt\right);$$

Total interest paid at the time of paying purchased cost to the supplier is

$$TP_1 = IP_1 + I_p \int_{t_2}^{T} P_c \left(\left(\int_M^{t_1} Q_r(t)dt + \int_0^{t_1} Q_{w,1}(t)dt + \int_{t_1}^{t_2} Q_{w,2}(t)dt\right)dt\right.$$

$$= P_c I_p \left(\left(\frac{D}{\beta^2}(e^{\beta(t_1 - M)} - \beta(t_1 - M) - 1)\right) + \frac{W}{\alpha}(e^{-\alpha M} - e^{-\alpha t_1}) + \left(\frac{D}{\alpha^2}(e^{\alpha(t_2 - t_1)} - \alpha(t_2 - t_1) - 1)\right)\right)(1 + \quad (T - t_2)); \qquad (1.1)$$

On the other hand retailer starts selling the product; as a result he accumulates revenue and earned interest at the rate I_e on the sales revenue. Therefore interest earned is given by

$$IE_1 = S_P I_e D \left(\int_0^{M} tD(t)dt + I_e \int_M^{T} \left(S_P I_e \int_0^{M} tD(t)\ dt\right)dt + \int_0^{T} S_P I_e IB(t = T)dt\right)$$

$$= \frac{S_P I_e}{2}\left(M^2 + I_e(T - M)M^2 + \frac{2T}{\delta}\ (e^{-\delta(T - t_2)} - 1)\right)\ ; \qquad (1.2)$$

4.2: Case-2: $t_1 < M \le t_2$

In this case also the length of positive stock period is larger than permissible delay period therefore retailer has to befinanced stock at the interestrateI_p after M and hence interest paid is given by

$$IP_2 = P_c I_p \left(\int_M^{t_2} Q_{w,2}(t)dt\right);$$

$$= \frac{P_c I_p D}{\alpha^2}(e^{\alpha(t_2 - M)} - \alpha(t_2 - M) - 1)(T - M); \qquad (2.1)$$

Total interest paid at the time of paying purchased cost to the supplier is

$$TP_2 = IP_2 + I_p \int_{t_2}^{T} P_c \left(\int_M^{t_2} Q_{w,2}(t)dt\right)dt$$

$$= \frac{P_c I_p D}{\alpha^2}(e^{\alpha(t_2 - M)} - \alpha(t_2 - M) - 1)(T - M)(1 + (T - t_2)); \qquad (2.2)$$

On the other hand retailer starts selling the product; as a result he accumulates revenue and earned interest at the rate I_e on the sales revenue. Therefore interest earned is given by

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$$IE_2 = S_P I_e \left(\int_0^M tD(t)dt + I_e \int_M^T \left(S_P I_e \int_0^M tD(t)\ dt \right) dt + I_e \int_0^T S_P IB(t = T)dt \right)$$

$$= \frac{S_P I_e D}{2} \left(M^2 + I_e(T - M)M^2 + \frac{2T}{\delta}\ (e^{-\delta(T - t_2)} - 1) \right); \quad (2.3)$$

4.3: Case-3: $t_2 < M \leq T$

In this case the length of positive stock period is less than permissible delay period therefore retailer has not to pay any interest for the positive stock held till M but he is paying at the end of cycle length therefore he has to pay interest for the period (T-M) on the total purchase cost.Therefore

$$IP_3 = I_p \int_M^T P_c \left(\int_0^{t_1} Q_r(t)dt + \int_0^{t_1} Q_{w,1}(t)dt + \int_{t_1}^{t_2} Q_{w,2}(t)dt \right) dt; \quad = P_c I_p \left(\left(\frac{D}{\beta^2} (e^{\beta(t_1 - M)} - \beta(t_1 - M) - 1) \right) + \frac{W}{\alpha} (e^{-\alpha M} - e^{-\alpha t_1}) + \left(\frac{D}{\alpha^2} \left(e^{\alpha(t_2 - t_1)} - \alpha(t_2 - t_1) - 1 \right) \right) \right) (T - t_o); \quad (3.1)$$

On the other hand retailer starts selling the product; as a result he accumulates revenue and earned interest at the rate I_e on the sales revenue till he pays to the supplier. Interest earned is given by

$$IE_1 = S_P I_e \left(\int_0^{t_2} tD(t)dt + I_e \int_M^T \left(S_P I_e \int_0^{t_2} tD(t)\ dt \right) dt + I_e \int_0^T S_P IB(t = T)dt \right)$$

$$= \frac{S_P I_e D}{2} \left({t_2}^2 + I_e(T - M){t_2}^2 + \frac{2T}{\delta}\ (e^{-\delta(T - t_2)} - 1) \right); \quad (3.2)$$

4.4: Case-4.0: $T < M$

In this case also retailer has not to pay any interest charged and accumulate interest on revenue collected from the sales, therefore

$$IP_4 = 0; \quad (4.1)$$

$$IE_4 = S_P I_e \left(\int_0^{t_2} tD(t)dt + I_e \int_{t_2}^M \left(S_P I_e \int_0^{t_2} tD(t)\ dt \right) dt + I_e \int_0^M S_P IB(t = T)dt \right)$$

$$= \frac{S_P I_{eD}}{2} \left({t_2}^2 + I_e(M - t_2){t_2}^2 + \frac{2M}{\delta}\ (e^{-\delta(T - t_2)} - 1) \right); \quad (4.2)$$

Hence the present worth of the profit per unit of time during the replenishment period is

$$Ʈ^m(t_1, \text{T}) = \begin{cases} Ʈ^1(t_1, \text{T}) & if \quad 0 < M \leq t_1 \\ Ʈ^2(t_1, \text{T}) & if \ t_\mu < M \leq t_2 \\ Ʈ^3(t_1, \text{T}) & if \ t_2 < M \leq T \\ Ʈ^4(t_1, \text{T}) & if T < M \end{cases} \quad (4.3)$$

Where
$Ʈ^i(t_1, \text{T}) =$
OC+HC in RW & OW DC in RW & OW + $Shortages\ cost + Lost\ sale\ cost + IP -$ IE];
$for\ i = 1,2,3,4$

Note: From eq. (10) it is observed that t_2 is the function of t_1 and if the value of t_1 is known, t_2 can be determined hence t_1 and T are the only decision variables for the developed model.

5.0 OPTIMALITY CONDITION

The necessary condition to minimize the total relevant inventory cost given in the eq. (4.3) is

$$\frac{\partial Ʈ^i(t_1, \text{T})}{\partial t_1} = 0; \ \frac{\partial Ʈ^i(t_1, \text{T})}{\partial T} = 0 \ ; \quad (4.4)$$

Provided

$$\left(\frac{\partial^2 Ʈ^i(t_1, \text{T})}{\partial t_1^2} \right) \left(\frac{\partial^2 Ʈ^i(t_1, \text{T})}{\partial T^2} \right) - \left(\frac{\partial^2 Ʈ^i(1, \text{T})}{\partial t_1 \partial \text{T}} \right)^2 > 0; \quad (4.5)$$

holds for the optimal value of decision variables obtained from eq. (4.4) i.e. Hessian matrix must be of positive definite.

Solving eq. (4.4) ,we can obtained the values of decision variables under restriction given in each case and substituting back in the eq. (4.3) we get the optimal solution as we required in each case and among these ,the case having minimum value may selected depending upon the value of permissible delay period M.

6.0 SOLUTION PROCEDURE

Summarizing the above arguments, we establish the following solution procedure to find the optimal solution.

Step 0: Input all the initial value of parameters.

Step 1: Find the optimal solution for case-1.

1.1 :Solve eq. (4.4) to get values of decision variables.

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1.2 : Using eq. (10) find the optimal value t_{2*} corresponding to t_{1*}.

1.3 : Using optimal values of decision variables in eq.(4.5) check the optimality condition.

1.4 : Using optimal values of decision variables in eq. (4.3) find total relevant inventory cost and store the result as $\Upsilon^{1*}(t_{1*}, T_*)$ and go to Step 2.

Step 2: Find the optimal solution for case-2.

2.1:Solve eq. (4.4) to get values of decision variables.

2.2: Using eq. (10) find the optimal value t_{2*} corresponding to t_{1*}.

2.3: Using optimal values of decision variables in eq. (4.5) check the optimality condition.

2.4: Using optimal values of decision variables in eq. (4.3) find total relevant inventory cost and store the result as $\Upsilon^{2*}(t_{w*}, T_*)$ and go to Step 3.

Step 3: Find the optimal solution for case-3.

3.1:Solve eq. (4.4) to get values of decision variables.

3.2: Using eq. (10) find the optimal value t_{2*} corresponding to t_{w*}.

3.3: Using optimal values of decision variables in eq. (4.5) check the optimality condition.

3.4: Using optimal values of decision variables in eq. (4.3) find total relevant inventory cost and store the result as $\Upsilon^{3*}(t_{1*}, T_*)$ and go to Step 4.

Step 4: Find the optimal solution for case-4.

4.1:Solve eq. (4.4) to get values of decision variables.

4.2: Using eq. (10) find the optimal value t_{2*} corresponding to t_{1*}.

4.3: Using optimal values of decision variables in eq.(4.5) check the optimality condition.

4.4: Using optimal values of decision variables ineq. (4.3) find total relevant inventory cost and store the result as $\Upsilon^{4*}(t_{w*}, T_*)$ and go to Step 5.

Step 5: Find

$min \{\Upsilon^{1*}(t_{1*}, T_*), \Upsilon^{2*}(t_{1*}, T_*), \Upsilon^{3*}(t_{1*}, T_*), \Upsilon^{4*}(t_{1*}, T_*)\}$.

Set $\Pi^m(t_{\mu 1*}, T_{\mu*}) = min. \{\Upsilon^{1*}(t_{1*}, T_*), \Upsilon^{2*}(t_{1*}, T_*), \Upsilon^{3*}(t_{1*}, T_*), \Upsilon^{4*}(t_{1*}, T_*)\}$, then $\Pi^m(t_{\mu*}, T_*)$ *is* optimal solution and $t_{\mu 1*}$ and $T_{\mu*}$ are optimal decision variables.

7.0 NUMERICAL ANALYSIS

In order to illustrate the above model with the help of above solution procedure, we consider the following examples where the parameter values are not taken from any case study and selected randomly.

Example-1.Let $A_o = 100, D = 400, W = 100, h_r = 0.6, h_w = 0.2, b_c = 3.0, l_c = 15, S_p = 15, P_c = 10, I_p = 0.015, I_e = 0.012, \delta = 0.6, \alpha = 0.05, \beta = 0.03$ in appropriate unit and $M = 0.25, 0.30, 0.50$ year .The computational result is shown in Table-1.

Example-2.Let $A_o = 1500, D = 2000, W = 100, h_r = 16, h_w = 15, b_c = 3.0, l_c = 15, S_p = 15, P_c = 10, I_p = 0.015, I_e = 0.012, \delta = 0.6, \alpha = 0.05, \beta = 0.03$ in appropriate unit and $M = 0.60$ year..

Table-1

Case	M=0.25				
	t_{1*}	t_{2*}	T_*	Q_*	$\Pi^i(t_{1*}, T_*)$
1	0.3099	0.8872	1.1055	311.86	189.12
2	0.3328	0.9032	1.1196	320.32	184.64
3	0.3497	1.0302	1.2245	318.35	164.40
4	0.5306	1.4609	1.6784	400.93	140.39

Case	M=0.30				
	t_{1*}	t_{2*}	T_*	Q_*	$\Pi^i(t_{1*}, T_*)$
1	0.3139	0.8860	1.1035	313.19	187.27
2	0.3288	0.9042	1.1154	318.15	170.52
3	0.3517	1.0352	1.2288	318.85	144.53
4	0.5306	1.4609	1.6784	400.93	140.39

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Case	M=0.50				
	t_{1*}	t_{2*}	T_*	Q_*	$\Pi^i(t_{1*}, T_*)$
1	0.3299	0.8803	1.0932	324.20	179.86
2	0.3080	0.8806	1.0883	315.04	176.49
3	0.3596	1.0559	1.2466	320.92	159.79
4	0.5306	1.4609	1.6784	400.93	140.39

Table-2

Case	M=0.60				
	t_{1*}	t_{2*}	T_*	Q_*	$\Pi^i(t_{1*}, T_*)$
1	0.0758	0.1958	0.8508	1561.77	3589.37
2	0.0663	0.1855	0.8337	1529.13	3518.98
3	0.0703	0.1940	0.8542	1561.15	3588.79
4	0.0707	0.1948	0.8555	1562.95	3587.52

7.1Nnmerical observations

- From Table-1, it is observed that in case-4, when permissible delay period is beyond the replenishment cycle length, the total relevant inventory cost is minimum and as the permissible delay period increases and ordering cycle length decreases as well as the total relevant inventory cost also decreases .
- If permissible delay period is less than replenishment cycle length then total relevant inventory cost is minimum in case-3 for all values of M and is decreases as the permissible delay period increases.
- From Table-2,it is observed that for example-2,when demand as well as holding costs in both warehouses are very high as compared to example-1then ordered quantity as well as the total relevant inventory cost is minimum for case-2.
- The graphs for total average inventory cost in each case have been drawn fixing the value of cycle length and varying t_{1*}, t_{2*}.
- The convexity of model in each case is depicted in Figure-2 with the help of 3-D graphical representation.

8.0 SENSITIVITY ANALYSIS

Considering example-1 (when M=0.50) mentioned in section 7.0, we have performed the sensitivity analysis to study the effect of changes for selected parameters on the optimal policy and the results are given in Table-3.

8.1Observations

From Table-3, the following observations can be made:

1) When warehouse capacity W of the retailer's is increasing, the optimal replenishment cycle length is decreasing but the total relevant inventory cost per unit of time is increasing. Forexample, when$A_o = 200$ and D=800,W increases from 100 unit to 500 units,T_* decreases form 1.1322 to 0.8528 but $\Pi^1(t_{1*}t_{2*}, T_*)$ increases from 338.50 to 383.19.This imply that retailer can order quantity less frequent to reduce cost when retailer owns larger storage space.
2) As the order quantity increases, the optimal replenishment cycle length and total relevant inventory cost increase simultaneously. For example, when=100,D=500 and A_o increase from 200 to 300 then T_*increases from 1.4152 to 1.7322and $\Pi^1(t_{1*}t_{2*}, T_*)$ increases from 279.17 to 342.68.This imply that retailer may order more quantity to reduce total average inventory cost.
3) It is observed that when demand is increasing the replenishment cycle length is decreasing. For example, when W=100, A_o=200 and demand D increase from 500 to 12000, T_*decreases from 1.4152 to 0.9292 year. It is clear that 0.9292 is more close to M=0.50 year than 1.4152 year, which implies that the retailer may order less quantity to take the benefits of the permissible delay period more frequently.
4) It is observed that when demand and capacity of retailer's owns warehouse are increasing ,the number of quantity ordered and total relevant inventory cost per unit of time is increasingsimultaneously. For example, When W= 500, $A_o = 200$ and D=1200, the ordered quantity is 860.99 and replenishment cycle length is 0.7681 year, which is very close to M=0.50 year. This may also impliesthat the retailer may order less quantity to take the benefits of thepermissible delay period more frequently.

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Table-3:Sensitivity in parameters changed for example-1when M=0.50

W	A_o	D	t_{1*}	t_{2*}	T_*	Q_*	$\Pi^1(t_{1*}, T_*)$
100	200	500	0.4772	1.1542	1.4152	479.29	279.17
		800	0.4062	0.9264	1.1322	591.58	338.50
		1200	0.3499	0.7628	0.9292	721.76	399.02
	300	500	0.5990	1.4203	1.7322	588.14	342.68
		800	0.5035	1.1319	1.3879	704.88	417.83
		1200	0.4302	0.9386	1.1416	863.17	495.56
	400	500	0.7002	1.6413	1.9938	630.03	396.34
		800	0.5844	1.3159	1.5991	798.18	484.78
		1200	0.4969	1.0846	1.3173	574.69	576.89
300	200	500	0.2108	0.9599	1.2369	544.23	310.94
		800	0.2462	0.8187	1.0361	671.61	374.72
		1200	0.2455	0.6958	0.8706	8058.45	437.35
	300	500	0.3453	1.2540	1.5861	639.59	381.75
		800	0.3501	1.0458	1.3084	791.63	459.98
		1200	0.3298	0.8802	1.0931	953.20	539.16
	400	500	0.4536	1.4904	1.8647	715.49	439.69
		800	0.4346	1.2306	1.5286	886.35	530.46
		1200	0.3987	1.0311	1.2742	1073.02	623.63
500	200	700	0.0123	0.6373	0.8681	670.17	359.73
		800	0.0515	0.6359	0.8528	714.75	383.19
		1200	0.1239	0.5914	0.7681	860.99	455.30
	300	500	0.0346	0.9650	1.3029	686.26	391.55
		800	0.1717	0.8988	1.1676	852.75	482.10
		1200	0.2165	0.7942	1.0126	1022.72	567.56
	400	500	0.1615	1.2122	1.6292	774.45	459.73
		800	0.2644	1.1015	1.4089	958.28	559.70
		1200	0.2899	0.9547	1.2510	1149.87	657.73

Note: Because D=500 when W=500 &$A_o = 200$ is infeasible, therefore we performed sensitivity for D=700 units.

Case-1

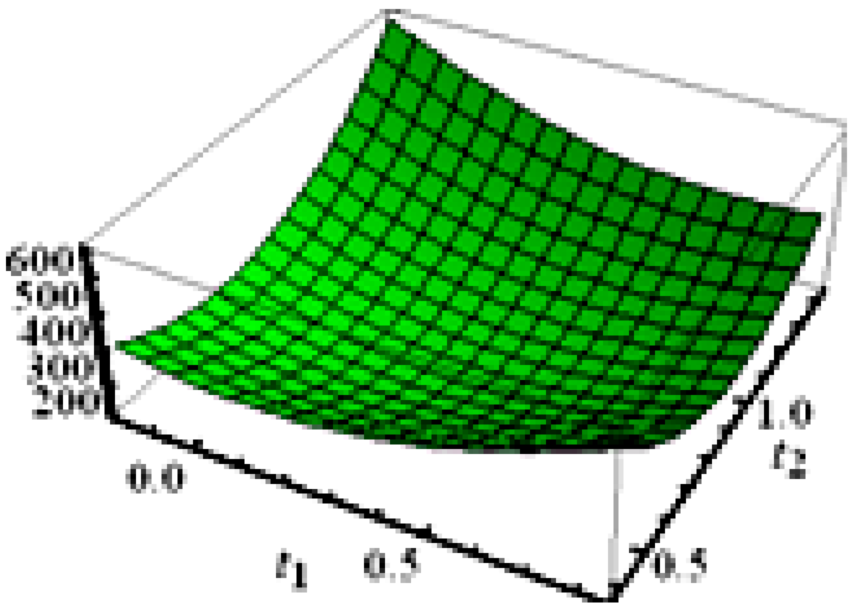

Figure-2: Graphical representation showing convexity of Inventory model

(t_{1*} ,t_{2*} verses average inventory cost)

Case-2

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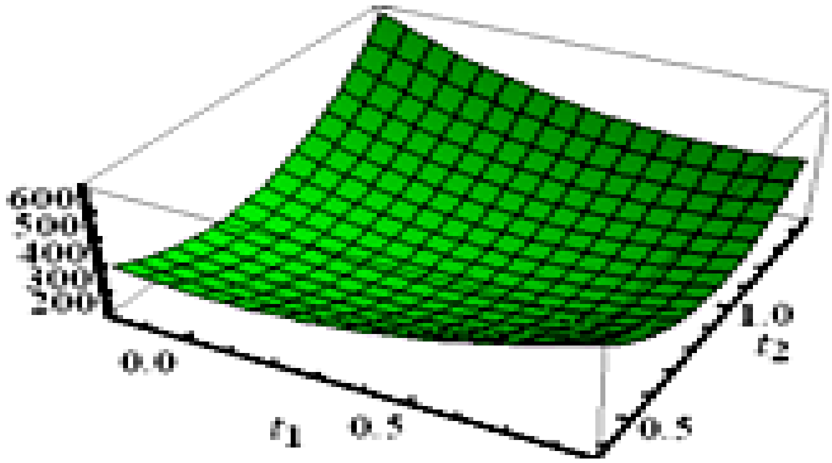
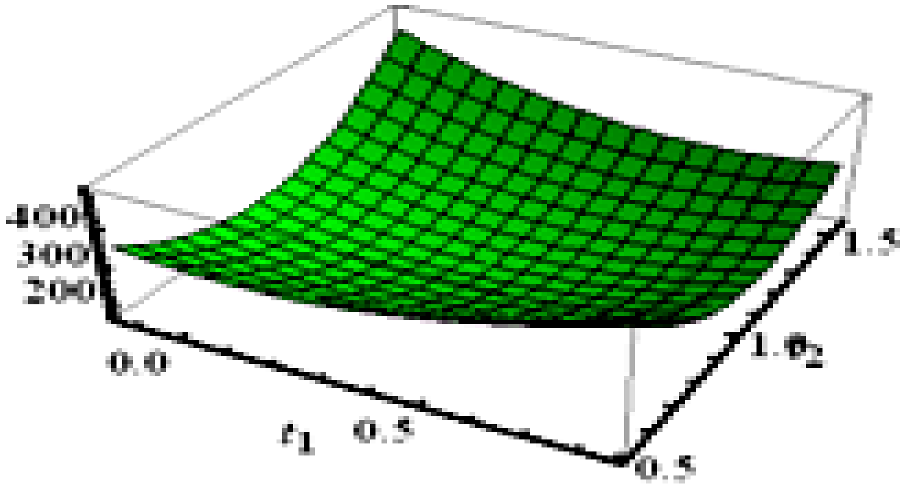


Case-3



Case-4

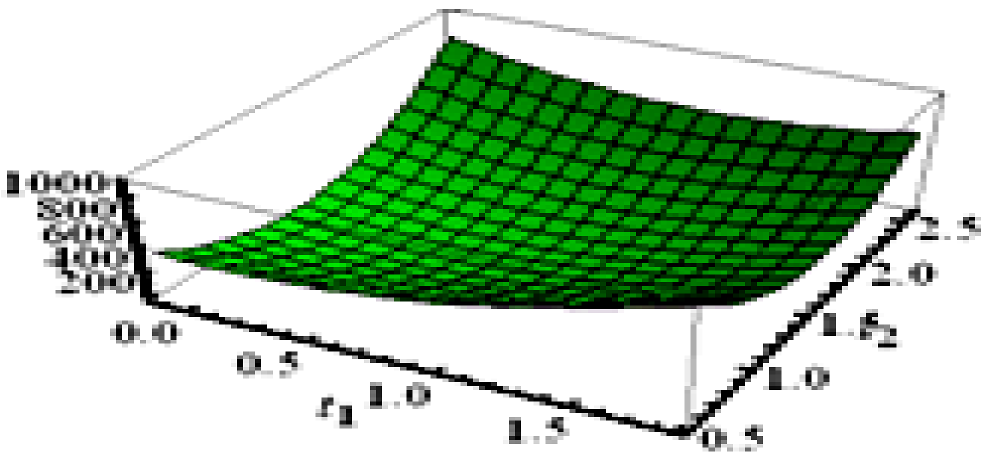


Figure-2: Graphical representationshowing convexity of Inventory model

(t_{1*} ,t_{2*} verses average inventory cost)

9.0 CONCLUSION

In this paper, we proposed a deterministic two-warehouse inventory model for deteriorating items with constant demand and permissible delay period in

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payment under conditionwith the objective of minimizing the total relevant inventory cost per unit of time. Shortages are allowed and backlogged at exponential rate of waiting time. An algorithm is proposed to find the optimal solution of the model. Finally, numerical examples are given to illustrate and validate the model. Sensitivity analysis is also performed on the model for some selected parameters and results are analysed. The optimal replenishment policy is sensitive to demand, ordering cost and own ware-house capacity.Furthermore the proposed model can be extended in several ways such as by incorporatingtime/price dependent demand, stock dependent demandor probabilisticdemand pattern and variable holding cost, deterioration rate with other realistic combinations.

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