

Analysis and control of non-linear system using three-mode controller

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Abstract— This paper explores the recent developments in nonlinear system control and controller tuning methods. Most systems are inherently nonlinear in nature. As the non-linear systems are difficult to solve, they are estimated using linear equations. It is well known that the ability of Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers to compensate several practical industrial processes has led to their wide acceptance in various industrial applications. The requirement to choose either two or three controller parameters is most easily done using tuning rules.

Robotics is one of the most promising emerging field in engineering. Various tuning methods are applied to the PI and PID controllers that are used in non-linear cruise control system of the robotic vehicle. Performance indices such as Integral Square Error (ISE), Integral Absolute Error (IAE) and Integral Time Absolute Error (ITAE) are compared and analyzed. Implementation has been carried out using MATLAB/Simulink programming environment.

Index Terms—cruise control, non-linear control, PID controller, robot, tuning methods

I. INTRODUCTION

There is perhaps no better symbol of the 21st century than the intelligent robots. In the field of robotics, opportunity is endless and discovery is on-going. It is one of the most promising emerging fields in engineering. In the past, robots are used mainly for industrial purposes but intelligent mobile robot's usability and necessity is greatly increased in recent years.

In this work, the first order non-linear adaptive cruise control system is taken into consideration. The Adaptive Cruise Control is based on two controllers; Proportional-Integral-Derivative (PID) control for velocity tracking, and PI control for distance tracking. The distance tracking controller calculates the velocity set point difference of the current inter-distance between the leading and following robots from the desired distance. The velocity tracking controller computes the velocity set point, to maintain the constant desired cruising speed. The model of adaptive cruise control system of the differential driven wheeled mobile robot is expressed by the state space representation. The model is controlled by PI and PID

controller with different tuning algorithms in simulation environment. This analysis is simulated in MATLAB/Simulink environment.

II. ANALYSIS OF NON-LINEAR CONTROL

Nonlinear systems are usually approximated by linear equation because nonlinear equations are difficult to solve. This works well up to certain accuracy for the input values. The behavior of a non-linear system appears to be chaotic, unpredictable or counterintuitive. In [1], with the help of feedback and coordinates, a system is transformed into a linear one. In [2], further reports on local model network are presented, and states that this approach is one of the standard techniques to combine linear models and artificial neural network to characterize the non-linearity. In [3], results of input to state stabilizability are discussed to hold even for systems which are not linear in controls, provided that a feedback is allowed.

In [4], stabilization problem for the feed-forward systems are solved by using Lyapunov function and feedback. In [5], PID-P controller is designed for non-linear processes using Cuckoo optimization technique, where proportional controller is used in the feedback path to increase the stability of the non-linear process. PID controller is used in the feed forward path to obtain the desired response. In [6], proportional-integral-derivative, Fuzzy and linear quadratic regulator controllers were designed to select speed controllers for a non-linear autonomous ground vehicle. Tuning of the controllers are carried out and comparative analysis of their performance is conducted.

III. PID CONTROLLER ANALYSIS

PID controllers are the most widely used controller in several industries because of their simplicity, robustness and successful practical applications. A PID controller is a control loop feedback mechanism. It operates on the error in the feedback and does the following operations. It calculates a term proportional to the error called the P term. And calculates a term proportional to the integral of the error, which is called the I term and calculates a term

proportional to the derivative of the error called the D term. The three terms the P, I and D, are added together to generate a control signal which is applied to the system being controlled. PID controllers are also called as three-term controllers or three-mode controllers. The “three-mode” functionalities of the controller are highlighted by the following. a) The proportional(P) term provides an overall control action proportional to the error signal through the all-pass gain factor. b) Steady-state errors are reduced by the integral(I) term through low-frequency compensation by an integrator. c) The transient response is improved by the derivative(D) term through high-frequency compensation by a differentiator. Here's a block diagram representation of the PID.

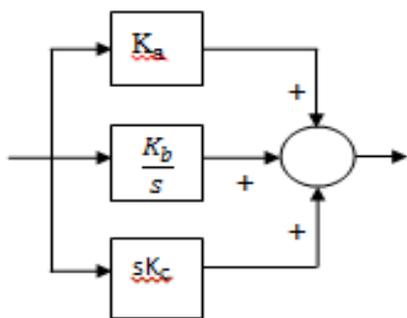


Fig.1 PID controller

The PID controller transfer function can be obtained by adding the three terms and is generally written in the parallel form given by

$$PID(s) = K_a + K_b(1/s) + sK_c(1) \tag{1}$$

Where K_a is the proportional gain, K_b is the integral gain and K_c is the derivative gain.

The transfer function of PID controller is combined into a pole-zero form.

$$PID(s) = [sK_a + K_b + s^2K_c] / s \tag{2}$$

As there is a quadratic in the numerator, there are two zeroes in the transfer function as well as a pole at the origin, $s = 0$. The effect of using a PID controller is that the transfer function of PID controller adds a pole at the origin, and the two zeroes that can be anywhere in the s-plane, depending upon the designer's choice of the three gains.

IV. MATHEMATICAL PRELIMINARIES

This paper focuses on cruise control of differential driven robot which has a nonlinear model and certain considerations are to be made for better control. A cruise control system needs to attain the desired speed in a short time. Also, it needs to maintain the speed with little deviation, when the robot travels up or down. An important consideration is made to incorporate air resistance and include as a system disturbance.

The Newton's law of motion states that given mass m , air resistance B and force $cu(t)$, where c is proportionality constant, $F(t) = mv(t) = cu(t)$, $t \geq 0$.

Air resistance, proportional to the velocity squared times constant B , produces drag on the robot. Additionally when it encounters an angle ϕ , gravity creates a second counter force termed as $mg \sin(\phi)$, where g is the gravitational constant of value 9.8 m/s^2 . A small angle is assumed in this case, so the following consideration is valid, $\sin\phi \approx \phi$.

The equation of motion is

$$mdv/dt = Cu(t) - Bv^2(t) - mg\phi \tag{3}$$

$$dv/dt = C/m[u(t)] - B/m[v^2(t)] - g\phi \tag{4}$$

A. Observation of non-linear differential equation

Robot's velocity is a nonlinear differential equation in terms of $v(t)$ because of the air resistance term $v^2(t)$. Although the nonlinear differential equation is not in a form suitable for design of an adaptive speed control, some observations have to be made.

Case 1: When the robot is on the horizontal floor ($\phi = 0$), the reduction of nonlinear differential equation is

$$dv/dt = C/m[u(t)] - B/m[V^2(t)] \tag{5}$$

The highest velocity will be reached when 't' becomes large. At highest velocity the acceleration must be zero, so $dv/dt \rightarrow 0$ when $t \rightarrow \infty$

Equation(5) further simplifies to

$$v_{\max}^2 = C/B \text{ (or) } v_{\max} = \sqrt{C/B} \tag{6}$$

Case 2: When the robot moves up and down at full speed, it stalls for some critical angle ϕ_c . The stall represents that the vehicle velocity is zero and the acceleration also becomes zero.

From the nonlinear differential equation,

$$C/mg = \sin(\phi_c) \tag{7}$$

$$\phi_c = \sin^{-1}[C/(mg)] \tag{8}$$

Case 3: The solution to the nonlinear differential equation at highest speed & $\phi = 0$. Defining the constant $\beta = \sqrt{BC/m}$, Consider $x = v/v_{\max}$ and $v_{\max} = \sqrt{C/B}$

$$dv/dt = C/m[u(t)] - B/m[V^2(t)] \tag{9}$$

where $x = \sqrt{B/C} * v$

Differentiating with respect to t, we get

$$dv/dt = \sqrt{C/B} dx/dt \tag{10}$$

Substitute in (4)

$$\sqrt{C/B} dx/dt = (C/m)u(t) - (B/m)x^2(t) \tag{11}$$

$$dx/dt = \sqrt{BC}/mu(t) - \sqrt{BC}/mx^2(t) \tag{12}$$

Hence the state space robot model equation is

$$dx(t)/dt = \beta u(t) - \beta x^2(t) \tag{13}$$

The absolute solution to the above simplified form is $x(t) = v_{\max} \tanh(t)$. This final information will provide the velocity profile versus time with the accelerator held to the floor.

B. Open loop Ziegler-Nichols tuning method

Many tuning methods have been proposed for PID controllers. They are open loop methods and closed loop methods. Here Ziegler-Nichols open loop method is used. This method is also known as reaction curve method. In this technique, the process dynamics is modelled by a first order plus dead time model. The open loop testing philosophy is to begin with a steady-state process and to make a step change to the final control element and it also records the results of the process output. Ziegler-Nichols tuning rules are widely used to tune PID controllers in process control where the plant dynamics are not precisely known.

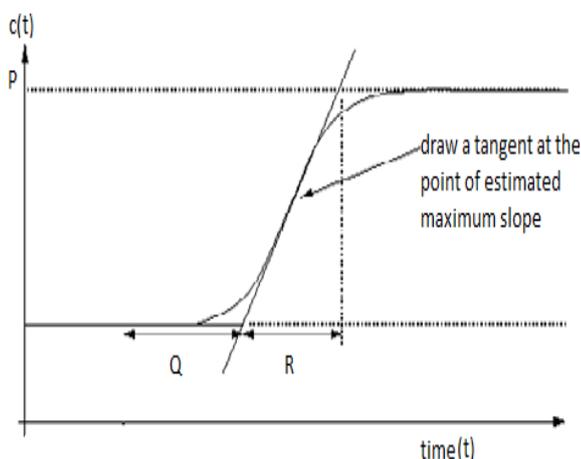


Fig.2 Open-loop step response

Over many years such tuning rules are proved to be useful. Zeigler-Nichols' transient response method will work on any system that has an open-loop step response that is an essentially critically damped or over damped. Information produced by the open-loop test is the open-loop gain P, the loop apparent dead time Q, and the loop time constant, R. Zeigler-Nichols tuning methods, tend to produce systems whose transient response is rather oscillatory and so will need to be tuned further prior to put the system into closed-loop operation.

V. SIMULATIONRESULTS

The Simulation results provided in fig.3 represents the system model $((dx(t)/dt=\beta u(t)-\beta x^2(t))$. The fig.4(A) and fig.4(B) is based on Murrill proposed tuning algorithm and it represents the servo response and regulatory response of Murrill tuning algorithm respectively. Fig.5(A) and fig.5(B) represents the servo response and regulatory response of Ziegler Nichols tuning method respectively. Table (I) provides the integral error analysis of PI and PID controller using four among several tuning algorithm.

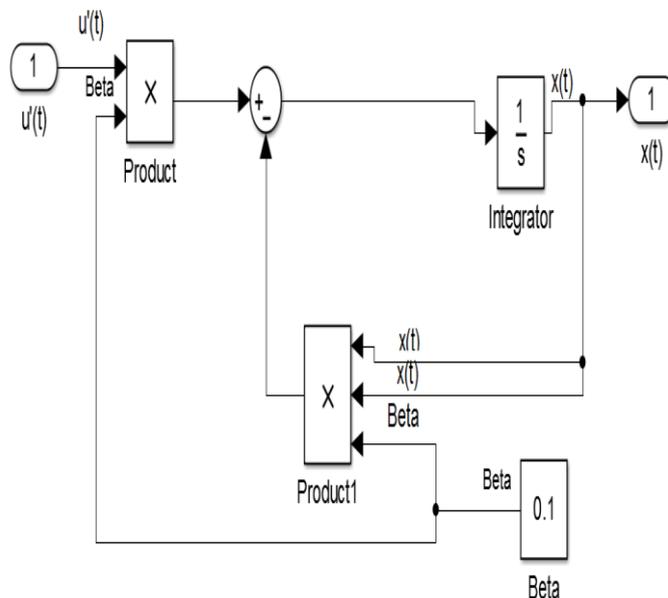


Fig.3 System model

The above system model is fed up with step input and the output of the system will be a S-type curve. Using open loop Ziegler-Nichols transient response method, by drawing tangent to the curve, the open-loop gain P, the loop apparent dead time Q, and the loop time constant, R can be determined.

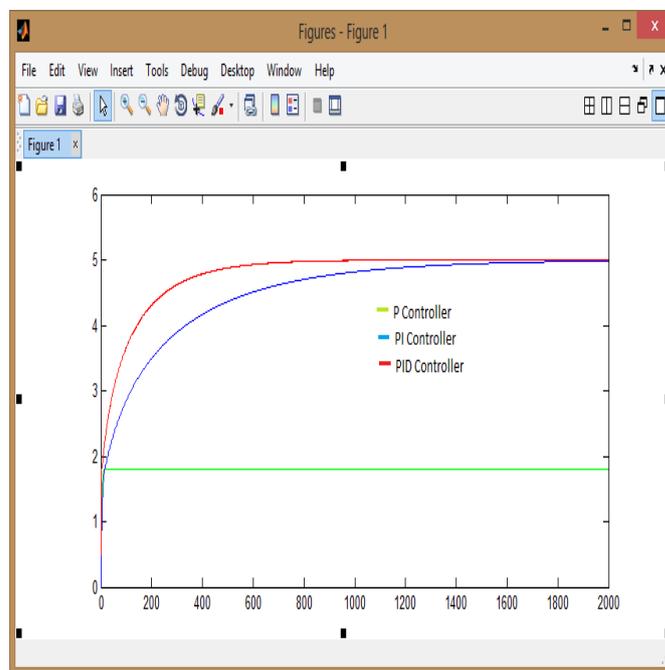


Fig.4(A) Servo response

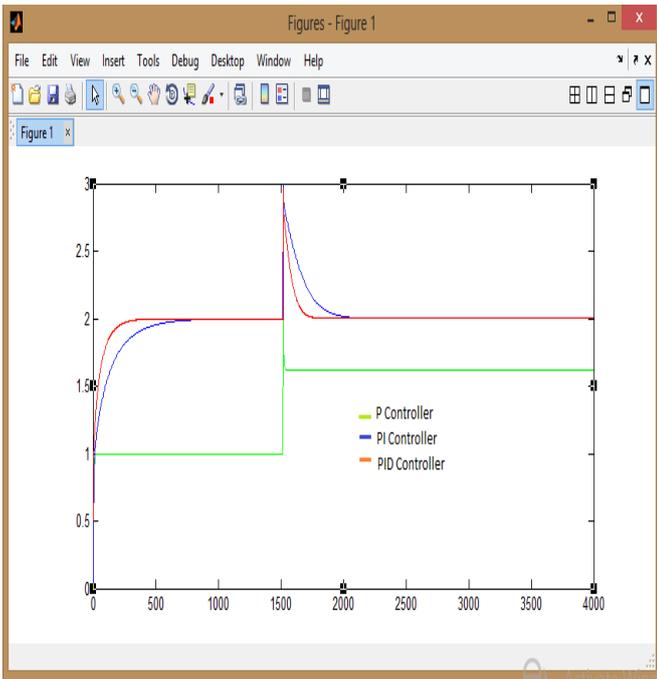


Fig.4(B) Regulatory response

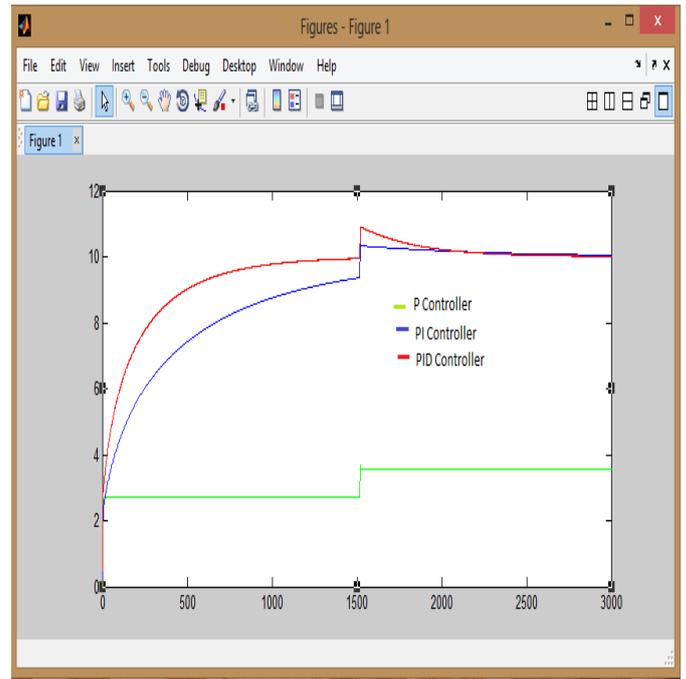


Fig.5(B) Regulatory response

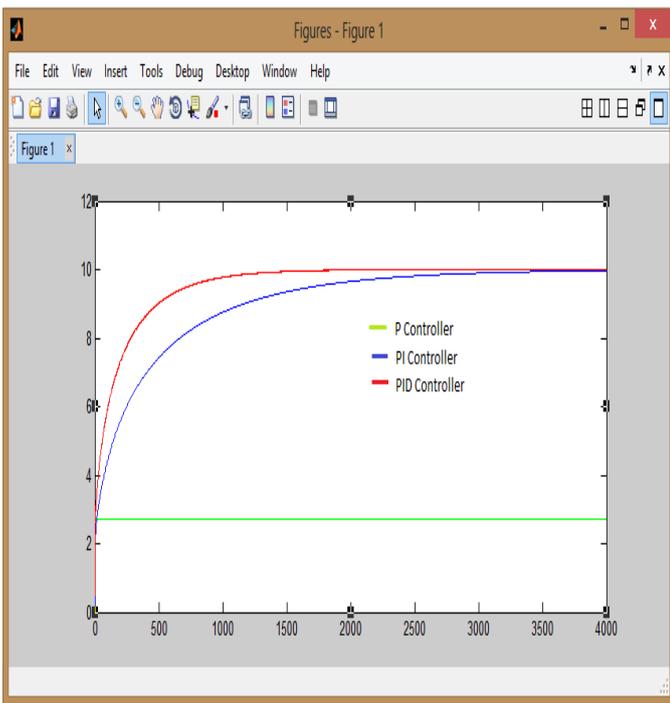


Fig.5(A) Servo response

I. Integral error analysis

Rule name	Controller type	ISE	IAE	ITAE
Ziegler and Nichols	PI	51.01	103.84	1.165e+004
	PID	35.26	60.43	4296
Astrom and Hagglund	PI	54.35	108.97	1.001e+004
	PID	38.34	61.07	3975
Chien et al	PI	33.59	37.26	1040
	PID	28.52	38	725.6
Murrill	PI	22.21	25.63	767.43
	PID	23.27	26.89	801.8

VI. CONCLUSION

The nonlinear cruise control system of the car-like robotic vehicle is modelled using state space representation and it is approximated using open loop Ziegler-Nichols transient response method. ISE, IAE, and ITAE are used for comparison as performance index. This analysis is performed in MATLAB/Simulink simulation environment.

The simulation results suggest that the Murrill tuning proposed PID algorithm gives better and reasonable results, since it provides minimum ISE, IAE, and ITAE values. This suggests that the Murrill tuning algorithm can be used confidently for majority of systems, which confirms again wide applicability of this method.

VII. FUTURE WORK

Further extensions can be directed towards building a real-time model of a car-like robotic vehicle and to analyse the performance of the nonlinear system and to employ adaptive cruise control algorithm for the robotic vehicle using micro-controller.

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