Approximate Analysis of SIR for OFDM Transmissions in the Presence of Time-Varying Channel Impairments

Vien Nguyen-Duy-Nhat

Abstract— **In this paper, we concerned with the detrimental effect of phase noise (PHN) and carrier frequency offset (CFO) on the performance of orthogonal frequency division multiplexing (OFDM) transmissions over time-selective channels. The approximate formulation of signal-to-interference (SIR) at an OFDM receiver in the presence of PHN, CFO and normalized Doppler frequency (NDF) is derived. Simulation and analytical results under various OFDM system settings are in good agreement and reveal the contributions of these channel impairments in the SIR degradation.**

Index Terms— **phase noise, carrier frequency offset, time-variant channel, SIR, Taylor series expansion.**

I. INTRODUCTION

Recently, orthogonal frequency division multiplexing (OFDM) has been recognized as a promising solution to facilitate the explosive growth in broadband data traffic of wireless multimedia services [1]. However, the superior advantages of OFDM only exist under the condition of perfect synchronization and quasi-static fading channel [2]. Unfortunately, such system conditions are rarely attainable in practice, whereas imperfect synchronization and time-selective fading most likely happen in OFDM transmissions. In particular, synchronization impairments (e.g., CFO and PHN) give rise to inter-carrier interference (ICI) that would significantly degrade the performance of OFDM transmissions [3], [4]. In addition, the presence of high-speed moving subscribers (in 4G mobile networks) causes time-selective channel response that also leads to ICI in OFDM systems [5].

For inter-carrier interference management, accurate estimates of CFO, phase noise and channel responses are needed to compensate those channel impairments at OFDM receivers. In a noisy channel, existing estimation techniques always produce imperfect estimates of those channel impairments [6], [7]. Reducing the discrepancy between the estimated and actual values needs an increase in transmission overhead (i.e., spectral efficiency loss in pilot-aided estimation [8]) or in computational complexity (in blind estimation [9]). After compensating those channel impairments by using the imperfect estimates, residual channel impairments still cause inter-carrier interference that would reduce the SIR of the system [10]. An analytical SIR expression that indicates their effects on the residual SIR would be useful for system performance analysis and system design specifications. In another words, a SIR expression can help one to determine just enough amount of system resources for the estimation of channel impairments (e.g., CFO, phase noise and doubly selective fading gains).

In the literature, most of existing studies consider one or two of these channel impairments in system analysis. In particular, the CFO effect on OFDM systems has been extensively studied in [3] while the investigation of phase noise has been addressed in [4]. Besides imperfect synchronization conditions, the effect of time-selective channels has been considered in [5], [11]. Combined time-selective fading and phase noise effects on OFDM systems have been analyzed in [12], [14]. In addition, the effect of CFO and time-selective channels in SIR analysis has been well documented in [13], [15] while the impacts of CFO and phase noise have been investigated in [16].

Different from [3] - [5], [11]-[16], this paper considers the effect of CFO, phase noise and time-selective channel responses in deriving an exact expression of SIR. The SIR analysis helps to reveal the impacts of these channel impairments on the OFDM system performance. However, the exact, but complex. This paper considers the joint effect of both time-varying fading, phase noise and carrier frequency offset in deriving an approximate closed-form expression of SIR by using Taylor series expansion. Under different OFDM system settings, this paper provides several numerical results to illustrate the tightness between empirical and analytical values of the approximate SIR expression.

The rest of this paper is organized as follows. Section II describes the considered OFDM system and the modeling of the aforementioned channel impairments. An exact expression of SIR is developed in Section III. Numerical results and related discussions are located in Section IV. Finally, Section V provides some concluding remarks.

Notations: $E[\cdot]$ stands for expectation operator, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

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II. System Model

A. Transmitted Signal Model

Consider an OFDM system using N-point fast Fourier transform (FFT) for multicarrier transmission. After IFFT and cyclic prefix (CP) insertion, the transmitted baseband samples in an OFDM symbol can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(j\frac{2\pi kn}{N}\right), \quad (1)$$

where $\in \{0, \ldots, N-1\}$, X_k is the k th data-modulated subcarrier in the considered OFDM symbol. It is assumed that the power of transmitted subcarriers has been normalized to one, i.e., $E[|X_k|^2] = 1$.

B. Doubly Selective Channel Model

In the considered OFDM transmission, the transmitted baseband signal x_n undergoes a time- and frequency-selective (doubly selective) channel. For the multipath fading channel between the transmit antenna and the receive antenna, the lth (time-selective) channel tap gain that includes the effect of transmit-receive filters and doubly selective propagation is denoted by $h_{l,n}$ where n stands for the index of the time-domain sample. The correlation of the channel coefficients for path l can be described as [17]

$$R_h(\tau) = J_0(2\pi f_d T_s \tau) \quad (2)$$

where f_d is the Doppler shift ($f_d = \frac{v f_c}{c_0}$, v is the mobile speed, f_c denotes the carrier frequency, c_0 is the speed of light), T_s is the OFDM symbol duration.

In the presence of doubly selective fading, carrier frequency offset and phase noise, the complex baseband received signal in an OFDM symbol can be written by [12], [13], [16]

$$y_n = e^{\frac{j2\pi\varepsilon n}{N}} e^{j\phi_n} \sum_{l=0}^{L-1} x_{n-l} h_{l,n} + z_n, \quad (3)$$

where ε denotes the fractional carrier frequency offset, ϕ_n denotes phase noise, and z_n is the additive white Gaussian noise (AWGN) sample with variance of N_0. As a result, the signal-to-noise ratio (SNR) can be determined by $SNR = \frac{1}{N_0}$.

C. Effect of Phase Noise

Phase noise $\theta(t)$ denotes rapid, short-term, random fluctuations in the phase of the transmitter and receiver oscillators which are caused by time domain instabilities [12], [4]. Phase noise can be described as a continuous Brownian motion process. In this paper, we consider discrete Brownian motion for phase noise modeling, i.e., $\theta_n = \theta(nT_s)$. As a result, we have $\theta_{n+1} = \theta_n + \varsigma_n$, where ς_n denotes mutually independent Gaussian random variables having zero mean and variance $\sigma_\varsigma^2 = 2\pi\beta T_s/N$, β stands for the two-sided 3 dB line-width of the Lorentzian power density spectrum of the free-running carrier generator [4]. This model is valid when the β is smallas compared to the subcarrier spacing $1/T_s$ [18].

As shown in [4], the autocorrelation function of θ_n can be computed by

$$R_\theta(\tau) = e^{-j2\pi\beta T_s|\tau|} \quad (4)$$

D. Effect of Carieer Frequency Offset

Let f_c and f_0 denote the carrier frequencies in the transmitter and receiver, respectively Let we define $\varepsilon f_c - f_0$ is the carrier frequency. The autocorrelation function of ϵ can be computed by

$$R_\varepsilon(\tau) = e^{j2\pi\varepsilon T_s \tau}. \quad (5)$$

E. Received signal model

In the presence of both time-selective channel and phase noise, the n^{th} received sample in an OFDM symbol (after CP removal) can be represented by

$$y_n = e^{\frac{j2\pi\varepsilon n}{N}} e^{j\theta(n)} \sum_{l=0}^{L-1} h_{l,n} x_{n-l} + z_n, \quad (6)$$

where $n = 0, \ldots, N-1$ and z_n is the additive white Gaussian noise (AWGN) with variance N_o. In this paper, the powers of both the transmitted signals and channel impulse response (CIR) are normalized to one and the resulting signal-to-noise ratio (SNR) can be determined by $SNR = \frac{1}{N_o}$.

As observed in (6), the presence of phase noise and carrier frequency offset introduces a time-domain phase rotation that will translate into ICI in the frequency domain as presented by the next formulations. In addition, the time-variation of the multipath channels also induces ICI in the frequency domain [16]. Consequently, the presence of both phase noise and time-selective channels would incur significant ICI power at an OFDM receiver, giving rise to an irreducible error floor in the receiver performance.

After performing FFT at receiver, the received frequency-domain signals can be determined by

$$Y_k = C_0 H_k X_k + I_k + Z_k, \quad (7)$$

where $I_k = \sum_{p=1}^{N-1} C_p H_{k-p} X_{k-p}$ denotes inter-carrier-interference (ICI), $C_p = \frac{1}{N}\sum_{n=0}^{N} e^{-\frac{j2\pi np}{N}} e^{\frac{j2\pi\varepsilon n}{N}} e^{i\theta(n)}$, and $H_k = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h_{l,n} e^{-\frac{j2\pi kl}{N}}$.

Based on (7), the formulation of an approximate SIR expression is derived in the next section as follows.

III. Interference Analysis

A. SIR formulation

In (7), the power of the kth subcarrier at receiver can be expressed by

$$P_r = P_{des} + P_{ICI} + N_0, \quad (8)$$

where $P_r = E[|Y_k|^2]$, $P_s = E[|C_0 H_k X_k|^2]$, $P_{ICI} = E[|I_k|^2]$, and $N_0 = E[|Z_k|^2]$.

Hence, the value of signal-to-interference-plus-noise ratio SIR can be determined by

$$\text{SIR} = \frac{P_{des}}{P_{ICI}}. \quad (9)$$

In (9), the ICI power can be computed by

$$P_{ICI} = \int_{-1}^{1} (1-|x|)(1 - R(T_s x))\,dx \quad (10)$$

where $R(T_s x)$ denotes the autocorrelation function of effective channel response [9].

In (9), the power of the desired signal can be calculated by

$$P_{des} = E[|C_0|^2|H_k|^2] = (1 - P_{ICI_c})(1 - P_{ICI_h}) \quad (11)$$

where $P_{ICI_c} = E\left[\sum_{p=1}^{N-1} |C_p|^2\right]$, and $P_{ICI_h} = E\left[\sum_{p=1}^{N-1} |H_{k-p}|^2\right]$.

By using (2), (5), (6) and (10), one can obtain

$$P_{ICI_c} = \int_{-1}^{1} (1-|x|)(1 - R_\varepsilon(T_s x)R_\theta(T_s x))dx$$

$$= \frac{1}{4\pi^2(\beta-\varepsilon)^2 T_s^2}(e^{-2j\pi(\beta-\varepsilon)T_s} - 2 + e^{2j\pi(\beta-\varepsilon)T_s} + 4\pi^2\beta^2T_s^2$$

and

$$P_{ICI_h} = \int_{-1}^{1} (1-|x|)(1 - R_h(T_s x))dx$$

$$\approx \frac{{}_0F_1[2, -(\pi f_d T_s)^2]}{\Gamma(2)} - 2\ {}_pF_q\left[t\left\{\frac{1}{2}\right\}, \left\{1, \frac{3}{2}\right\}, -(f_d \pi T_s)^2\right] \quad (13)$$

where ${}_0F_1(a,z)$, $\Gamma(a)$, and ${}_pF_q(a;b;z)$ are the regularized confluent hypergeometric function, gamma function, and generalized hypergeometric function, respectively.

From (10), (12), and (13), we obtain

$$P_{des} = \left(1 - \frac{2e^{-\pi\beta T_s}(1 - e^{\pi\beta T_s} + e^{\pi\beta T_s}\pi\beta T_s)}{\pi^2\beta^2T_s^2}\right) \times \left(1 - \frac{{}_0F_1[2, -(\pi f_d T_s)^2]}{\Gamma(2)} - 2\ {}_pF_q\left[\left\{\frac{1}{2}\right\}, \left\{1, \frac{3}{2}\right\}, -(f_d \pi T_s)^2\right]\right). \quad (14)$$

By assuming fading channel response and phase noise are statistically independent, one can deduce

$$R(T_s x) = R_\varepsilon(T_s x)R_\theta(T_s x)R_h(T_s x). \quad (15)$$

Based on (2), (5), (6), (10) and (15), we can obtain P_{ICI} Substituting P_{des}, P_{ICI} into (9), we can obtain a close-form expression of SIR that considers the joint effect of time-varying channel and phase noise. However, the closed-form SIR expression is relatively complicated. In this paper, we explore Taylor series expansion to obtain an approximate expression of SIR as presented in the next subsection.

B. *Approximate SIR formulation*

In order to separate the signal and noise terms, let us suppose that the phase noise and carrier frequency offset are small, so that:

$$e^{v(t)} \approx 1 + v(t) + \frac{v(t)^2}{2}. \quad (17)$$

From (12), we have

$$P_{ICI_c} \approx \frac{1}{3}(\pi^2\beta^2T_s^2 - 2\pi^2\beta\varepsilon T_s^2 + \pi^2\varepsilon^2T_s^2) \quad (18)$$

Applying Taylor series expansion for (13) to first-order terms, we get

$$P_{ICI_h} \approx \frac{(\pi f_d T_s)^2}{6}. \quad (19)$$

Similarly, one can rewrite (14) by

$$P_{des} \approx \left(1 - \frac{(\pi f_d T_s)^2}{6}\right)\left(1 - \frac{1}{3}(\pi^2\beta^2T_s^2 - 2\pi^2\beta\varepsilon T_s^2 + \pi^2\varepsilon^2T_s^2)\right)$$

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Hence, the approximate ICI power can be calculated as

$$P_{ICI} \approx \frac{1}{840}(35 f_d^2 T_s^2 + 280\pi^2\beta^2T_s^2 - 28 f_d^2 T_s^2\pi^2\beta^2T_s^2 - 560\pi^2\beta\varepsilon T_s^2 + 56 f_d^2 T_s^2\pi^2\beta\varepsilon T_s^2 + 280\pi^2\varepsilon^2T_s^2 - 28 f_d^2 T_s^2\pi^2\varepsilon^2T_s^2 - 224\pi^4\beta^2\varepsilon^2T_s^2 + 30 f_d^2 T_s^2\pi^4\beta^2T_s^2\varepsilon^2T_s^2)$$

Using (9), (21), (19), and (21) and first-order terms in Taylor series, we can obtain a SIR expression as a function of the NDF $f_d T_s$, and the product of the 3dB two-side phase noise line-width βT_s and the normalize carrier frequency offset εT_s. In particular, the approximate expression of SIR can be expressed by

$$\mathrm{SINR}_{approx} \approx \frac{(1 - \frac{\Phi}{3})(1 - \frac{\vartheta^2}{6})}{\frac{\Phi}{3} + \frac{\vartheta^2}{3}(\frac{1}{2} - \frac{3\Phi}{10}) + N_0} \quad (22)$$

where $\Phi = \pi\beta T_s - \pi\varepsilon T_s$, and $\vartheta = \pi f_d T_s$.

IV. SIMULATION RESULTS

In this section, computer simulation was conducted to evaluate the tightness of the derived SIR expression in the presence of time-selective channels, phase noise and carrier frequency offset. Unless otherwise indicated, the considered system settings are described in Table 1.

Table 1. System parameters

	Notation	Value
Number of resolvable paths Number of sampled points FFT	L N_{FFT}	5 512
CP length	N_{CP} (samples)	40
Sampling frequency	f_s (MHz)	5.6
Carrier frequency	f_c(GHz)	3.5
Channel realizations	N_{trial}	10000

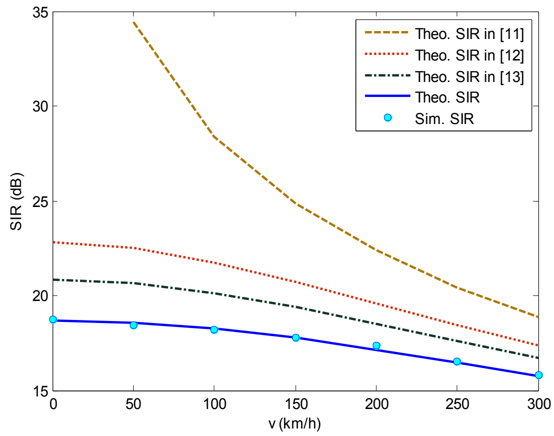

Fig. 1. SIR versus the NDF when $\varepsilon = 0.05$ nd $\beta T_s = 0.005$.

To show the difference between the SIR expression in (22) and other existing ones in the literature [11] - [13], Fig. 1 shows numerical results of the SIR expression and other ones in the literature. In the considered system settings, one can find that ignoring only phase noise incurs the smallest gap between the theoretical and simulated SIR values.

Fig. 2 shows the ICI effect of decreasing SIR as CFO increases. In the considered system settings, CFO is the primary performance-limiting factor when $\varepsilon \geq 0.3$. In the presence of large CFO values, the effect of phase noise and time-selective channels becomes quite insignificant.

To verify the validity of SIR analysis, numerical results of SIR versus PHN level βT_s are shown in Fig. 3. SIR curves are provided under different CFO values. It is observed that the SIR decreases as synchronization impairments increases. In addition, Fig. 3 shows a good agreement between simulated and theoretical results.

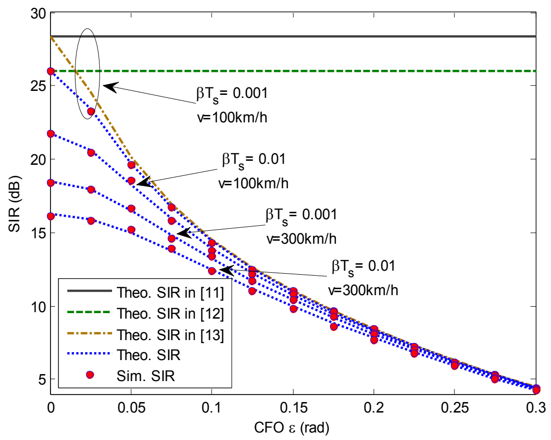

Fig. 2. SIR versus CFO when βT_s = 0.001 and 0.01 under $v = 100\ km/h$.

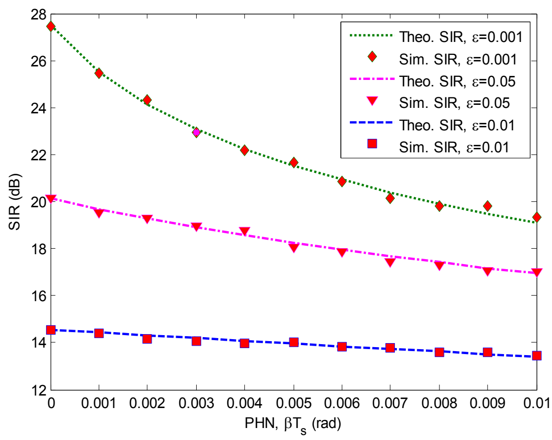

Fig.3. SIR versus PHN level βT_s under $f_d T_s = 0.03\ (v = 100\ km/h)$ and $\varepsilon = 0.05, 0.01$.

V. Conclusion

This paper developed an approximate SIR expression for OFDM transmissions for OFDM transmissions in the presence of phase noise, carrier frequency offset, and time-selective channels. The analytical results are in a good agreement with the simulation results over wide ranges of mobile speeds, CFO, and PHN.

VI. References

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