Case study then Direct-Form FIR Poly-phase Linear Interpolator Filter Design with Symmetric Structure

Mandeep Singh Saini[1] and Dr. Kuldeep Kaur[2]

[1]M.Tech. (E.C.E), Guru Nanak Dev Engineering College, Ludhiana P.T.U Regional Centre, (Punjab) India.

[2]Ph. D (Asst. Prof.), Faculty Member of B.A.M. Khalsa College Garhshankar. (Punjab)

***ABSTRACT* — FIR poly-phase decomposition filters reduce of computation complexity. However, FIR poly-phase filters most of advantage they require a very small number of multipliers to implement but The Direct-Form Symmetric FIR shows 50% reduction in multipliers as compared to Direct-Form FIR Poly-phase Linear Interpolator , Direct-Form FIR Transposed and Direct-Form I.**

In this paper, Cost effective techniques for design of Pulse Shaping Filter have been presented to improve the computational and implementation complexity.

Keywords:- FIR, Interpolators, Decimators, Transposed, Symmetric.

[A] INTRODUCTION

1. Need of Pulse Shaping Design

Multirate digital signal processing is required in digital system where more than one sampling rate is required. In digital audio, the different sampling rates used are 32 KHz for broadcasting, 44.1 KHz for compact disc and 48 KHz for audio tape. In digital video, the sampling rates for composite video signal are 14.3181818 MHz and 17.734475 MHz for NTSC and PAL respectively. But the sampling rates for digital component of video signals are 13.5 MHz and 6.75 MHz for luminance and colour difference signal. Different sampling rates can be obtained using as up sampler and down sampler.
The digital signal processing is used for communication system, speech and audio processing systems, antenna systems and radar system.
They have also advantages of multirate signal processing are computational requirements is less, storage for filter coefficient is less, finite arithmetic efficient are less, finite order required in multirate application is low and sensitivity to filter coefficient length is less.

While design multirate systems, effect of aliasing for decimation and pseudo image for interpolation should be avoided.

1. SAMPLING

Let signal be $x = (10\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1)$ and t (sec) is the signal time, then x (t) be a continuous time varying signal is sampled at regular intervals of time with sampling period T. Then sampling signal x (t) is given by

$$x(nT) = x(t)\big|_{t=nT} = x(nT) \qquad -\infty < n < \infty \quad (1.1)$$

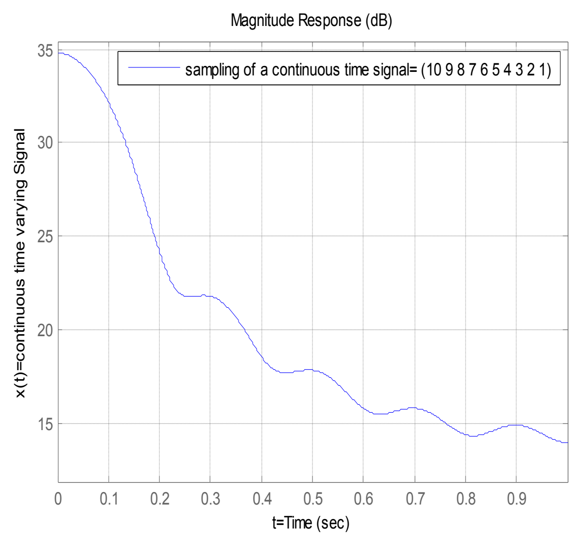

Fig.1 .1 sampling of a continuous time signal

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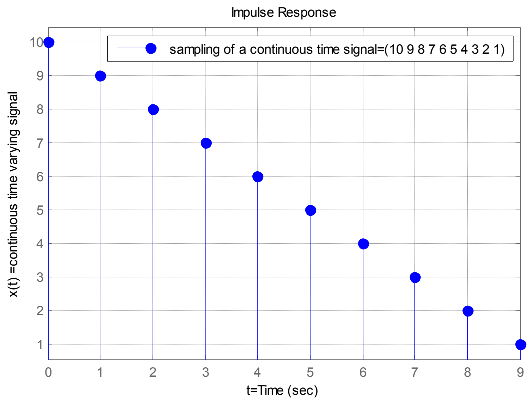

Fig.1 .2 sampling of a discrete time signal

The continuous time signal x(t) is multiplied by the sampling function s(t) which is series of impulse, the resultant signal is discrete time signal x(n).

$$x(n) = x(t)s(t)\ |_{t=nT} \qquad -\infty < n < \infty \qquad (1.2)$$

The given signal output is shown in Fig. 1.2

$x(0) = 10; x(1) = 9; x(2) = 8; x(3) = 7; x(4) = 6;$
$x(5) = 5; x(6) = 4; x(7) = 3; x(8) = 2; x(9) = 1$

2. Sampling rate conversion

Sampling rate conversion is the process of converting the sequence x(n) which is got from sampling the continuous time signal x(t) with a period T, to another sequence y(k) obtained from sampling x(t) with a period T'.
The new sequence y(k) can be obtained by first reconstruction the original signal x(t) from the sequence x(n) and then sampling the reconstruction signal with a period T'.

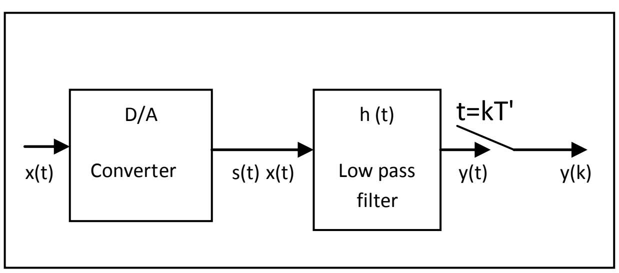

Fig. 1.3 conversion of a sequence x(n) to another sequence y(k)

3. Decimation

The process of reducing the sampling rate of a signal is called decimation (sampling rate compression). Let M be the integer sampling rate reduction factor for the signal x(n).

$$\frac{T'}{T} = M$$

The new sampling rate F' become

$$F' = \frac{1}{T'} = \frac{1}{MT} = \frac{F}{M} \qquad (1.3)$$

Let the signal x(n) be a full band signal, with non-zero values in the frequency range $-F/_2 \le f \le F/_2$, $where\ w = 2\pi fT$

$$\left|X(e^{jw})\right| \neq 0, |w| = |2\pi fT|, \le 2\pi f\frac{T}{2} = \pi \qquad (1.4)$$

To avoid aliasing in the decimation process, the signal x(n) is filtered with a digital low pass filter which has the following spectral response

$$\left|H(e^{jw})\right| = \begin{cases} 1 & |w| = 2\pi F'\frac{T}{2} = \frac{\pi}{M} \\ 0 & otherwise \end{cases} \qquad (1.5)$$

The sequence y (k) is obtained by selecting only the M^th sample of the filtered output which results in sampling rate reduction. Let the impulse response of the filter be h(n), then the filtered output w(n) is given by

$$w(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \qquad (1.6)$$

$W(e^{jw})\ is\ the\ spectrum\ of\ w(n).$

The decimation signal y(m) is

$$y(m) = w(Mm) \qquad (1.7)$$

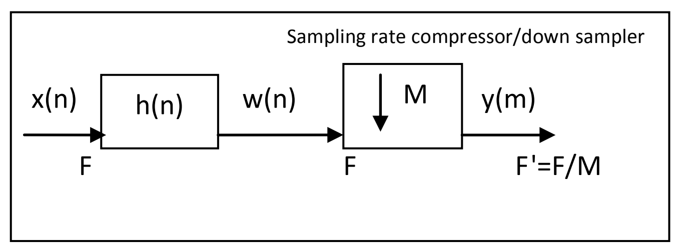

Fig. 1.4 conversion of a sequence x(n) with down sample M

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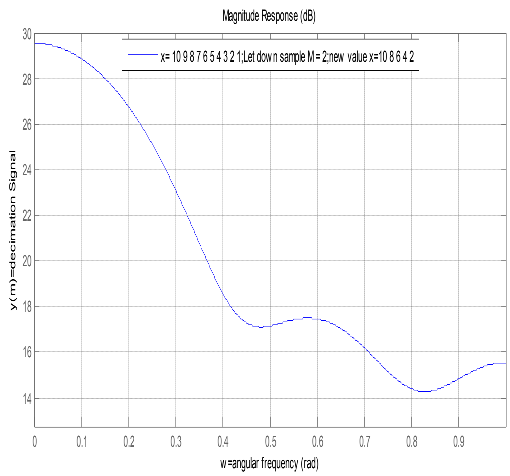

Fig.1 .5 sampling of a continuous time signal M=2

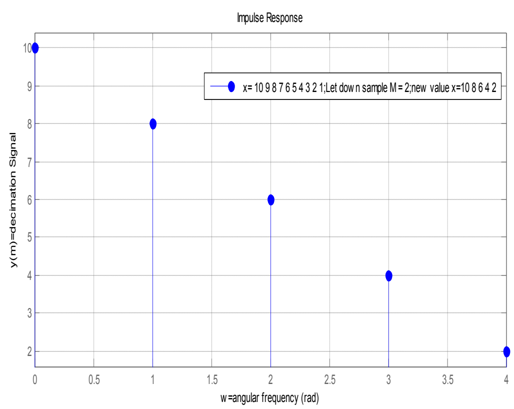

Fig.1 .6 sampling of a discrete time signal M=2

Compare the equation 1.6 and 1.7, y(m) becomes

$$\boldsymbol{y(m)} = \sum_{k=-\infty}^{\infty} h(k)x(Mm-k)$$

or

$$\boldsymbol{y(m)} = \sum_{k=-\infty}^{\infty} h(Mm-n)x(n) \qquad (1.8)$$

Input signal is shown in Fig. 1.2.

$x(0)=10; x(1)=9; x(2)=8; x(3)=7; x(4)=6;$
$x(5)=5; x(6)=4; x(7)=3; x(8)=2; x(9)=1$

Decimation with M=2
Then output signal is shown in fig.1.6

$$y(0)=10; y(1)=8; y(2)=6; y(3)=4; y(4)=2;$$

4. Interpolation

The process of interpolation the sampling rate of a signal is interpolation. Let L be an interpolation factor, of the signal x(n), then

$$\frac{T'}{T} = {}^{1}/_{L}$$

The sampling rate is given by

$$F' = \frac{1}{T'} = \frac{L}{T} = LF \qquad (1.9)$$

Interpolation of a signal x(n) be a factor L refers to the process of interpolation L-1 samples between each pair of samples of x(n)

$$w(m) = \begin{cases} x\left({}^{m}/_{L}\right), & m = 0, \pm L, \pm 2L, \dots\dots\dots \\ 0 & otherwise \end{cases} \qquad (1.10)$$

The Z-transform of the signal w(m) is given by

$$\mathrm{W(z)} = \sum_{m=-\infty}^{\infty} w(m)z^{-m} = \sum_{m=-\infty}^{\infty} x(m)z^{-mL} = X(z^{L}) \qquad (1.11)$$

When considered over the unit circle, $z = e^{jw'}$,

$$w\left(e^{jw'}\right) = X(e^{-jw'L}) \text{ Where } w' = {}^{2\pi}/_{T'} \qquad (1.12)$$

The spectrum of the signal w(m) contain the image of baseband at the harmonics of the sampling frequency $\pm {}^{2\pi}/_{L}, \pm {}^{4\pi}/_{L}$. To remove the image, an anti-imaging (low-pass) filter is used. The ideal characteristics of the low pass filter is given by

$$H\left(e^{jw'}\right) = \begin{cases} G & |w'| \le 2\pi f \frac{T'}{2} = \frac{\pi}{L} \\ 0 & otherwise \end{cases} \qquad (1.13)$$

Where G is the gain of the filter and should be Lin the pass band. The frequency response of the output signal is given by

$$Y\left(e^{jw'}\right) = H\left(e^{jw'}\right)X(e^{-jw'L})$$

$$Y\left(e^{jw'}\right) = \begin{cases} GX(e^{-jw'L}) & |w'| = \frac{\pi}{L} \\ 0 & otherwise \end{cases} \qquad (1.14)$$

The output signal y(m) is given by

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k)w(k)$$

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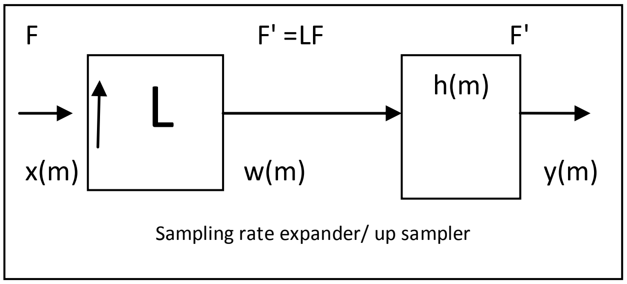
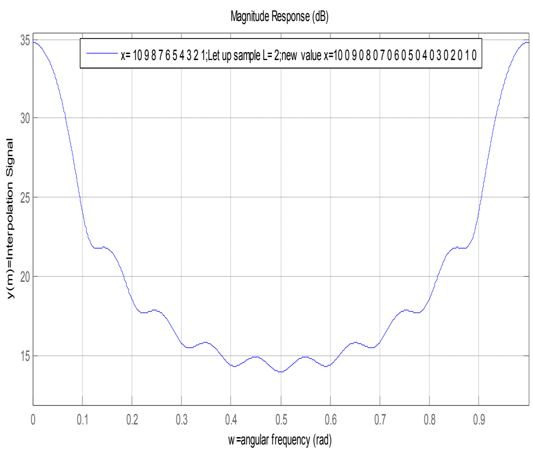


Fig. 1.7 conversion of a sequence x(n) with up sample L



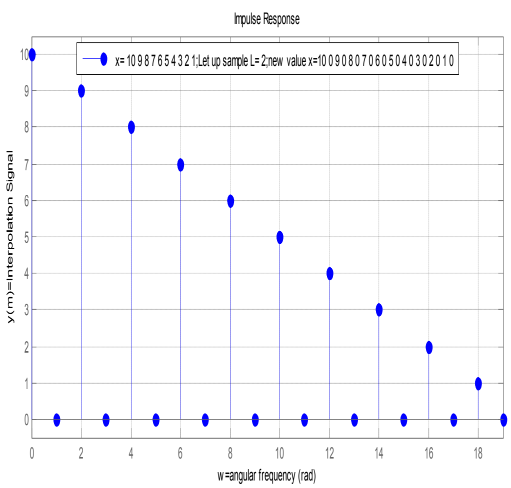


Fig.1 .8 sampling of a discrete time signal L=2

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k)x(^K/_L), ^k/_L \; an \; integer \quad (1.15)$$

Input signal is shown in Fig. 1.2.

$x(0) = 10; x(1) = 9; x(2) = 8; x(3) = 7; x(4) = 6;$
$x(5) = 5; x(6) = 4; x(7) = 3; x(8) = 2; x(9) = 1$

Interpolation with L=2
Then output signal is shown in Fig.1.8

$y(0) = 10; y(1) = 0; y(2) = 9; y(3) = 0; y(4) = 8;$
$y(5) = 0; y(6) = 7; y(7) = 0; y(8) = 6; y(9) = 0;$
$y(10) = 5; y(11) = 0; y(12) = 4; y(13) = 0;$
$y(14) = 3; y(15) = 0; y(16) = 2;$
$y(17) = 0; y(18) = 1;$

[B] PROPOSED FILTER DESIGN

2.1 FIR Direct-Form I Structure

The general input-output relationship of an FIR filter is given by

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \qquad 2.1$$

The filter is represented by N-point impulse response h(k) takes value only in the range k = 0,…,N-1,outside this interval h(k) is zero fig. 2.1 shown the graph representation of an FIR filter.
The z-transform direct-from I filter is

$$H(z) = \sum_{n=0}^{\infty} \mathrm{h(n)}Z^{-1} \qquad 2.2$$

Where
$H(z) = h(0) + h(1)z^{-1} + \cdots$

From Fig. 2.2, we get

$h(0) = 0.125\ h(1) = 0.25\ h(2) = 0.375\ h(3) = 0.5$
$h(4) = 0.625\ h(5) = 0.75\ h(6) = 0.875\ h(7) = 1$
$h(8) = 0.875\ h(9) = 0.75\ h(10) = 0.625\ h(11) = 0.5$
$h(12) = 0.375\ h(13) = 0.25\ h(14) = 0.125\ h(15) = 0$

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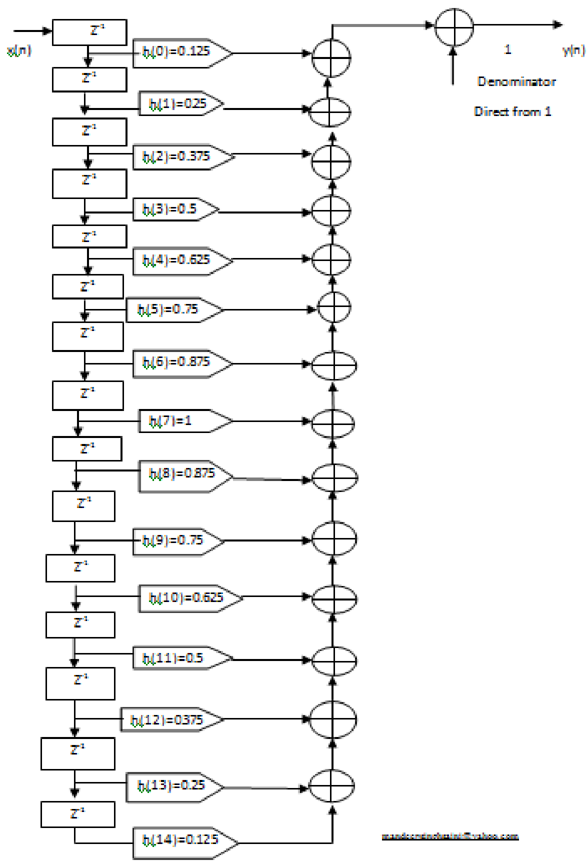


Fig 2.1 FIR Direct-Form I Structure

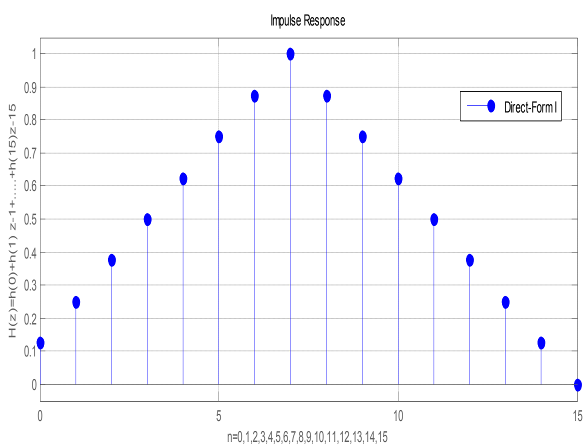
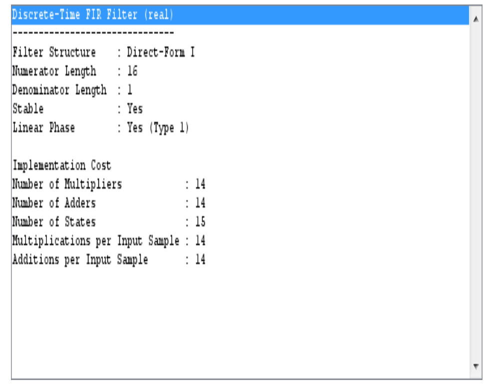


Fig 2.2 FIR Direct-Form I Structure (Impulse response)


Discrete-Time FIR Filter (real)

Filter Structure : Direct-Form I
Numerator Length : 16
Denominator Length : 1
Stable : Yes
Linear Phase : Yes (Type 1)

Implementation Cost
Number of Multipliers : 14
Number of Adders : 14
Number of States : 15
Multiplications per Input Sample : 14
Additions per Input Sample : 14

Table 2.1: cost analysis Direct-Form I FIR Structure

cost	MULT	ADD	STA	MPIS	APIS
Direct-Form I FIR Structure	14	14	15	14	14

2.2 Direct-Form FIR Poly-phase Linear Interpolator

To implement a filter with a fractional multi-rate factor of L/M, the data is first up sampled by L and then a filter is applied to both interpolate between the data points as well as provide anti-alias protection, and then finally down sampled by M. These three designs are diagrammed below.

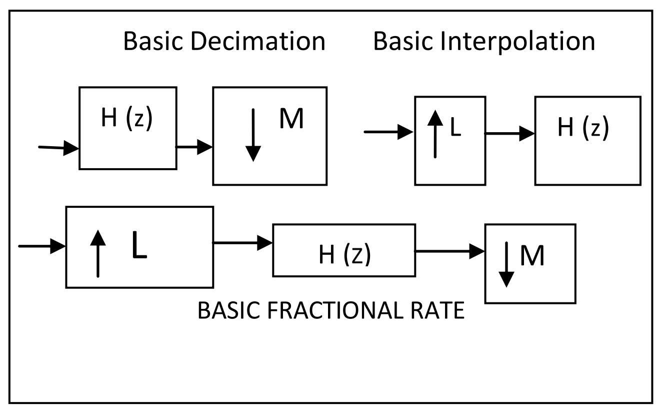


Fig.2.3 Polyphase Filter Structure

Using M decimation factor then Equ. 2.2 is given as

$$H(z) = \sum_{n=0}^{\infty} h(z) z^{-M}$$

$$H(z) = h(0) + h(z)z^{-M} + h(2M)z^{-2M} + \cdots \qquad (2.3)$$

Rearranging the above equation, we get

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$$H(z) = h(0) + z^{-2}h(2) + z^{-4}h(4) + z^{-6}h(6) + z^{-8}h(8) + z^{-10}h(10) + z^{-12}h(12) + z^{-14}h(14) +$$

$$z^{-1}[h(1) + z^{-2}h(2) + z^{-4}h(4) + z^{-6}h(6) + z^{-8}h(8) + z^{-10}h(10) + z^{-12}h(12) + z^{-14}h(14)]$$

$$H(z) = E_0(z^2) + z^{-1}E_1(z^2) \qquad (2.4)$$

Where $E_0(z^2)$ and $E_1(z^2)$ are poly phase components for a factor of two

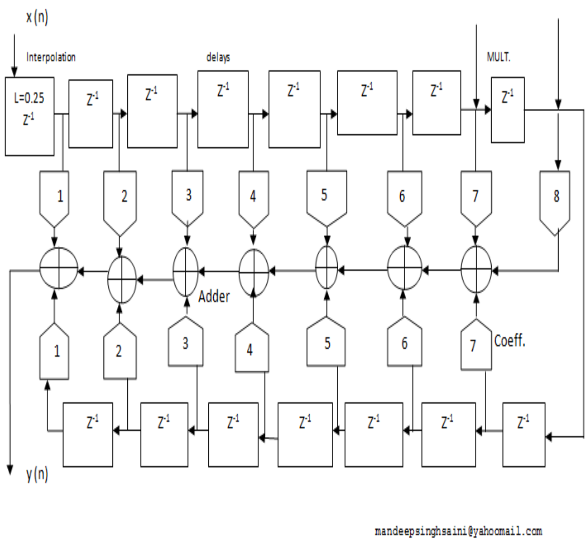


Fig 2.4 FIR Direct-Form Polyphase linear Structure

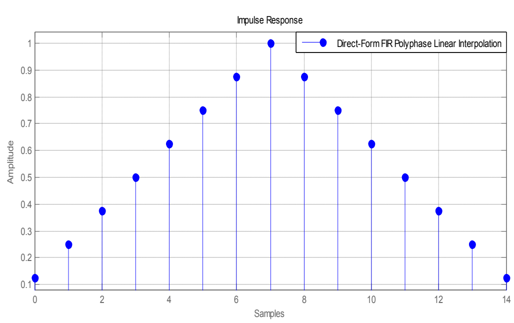
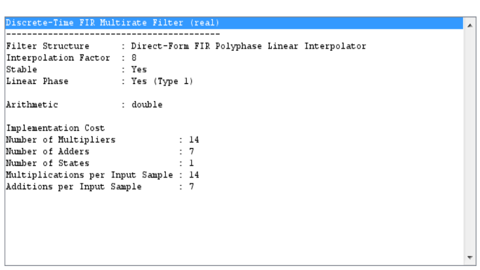


Fig 2.5 FIR Direct-Form polyphase linear Structure (Impulse response)



```
Discrete-Time FIR Multirate Filter (real)
-----------------------------------------
Filter Structure       : Direct-Form FIR Polyphase Linear Interpolator
Interpolation Factor   : 8
Stable                 : Yes
Linear Phase           : Yes (Type 1)

Arithmetic             : double

Implementation Cost
Number of Multipliers            : 14
Number of Adders                 : 7
Number of States                 : 1
Multiplications per Input Sample : 14
Additions per Input Sample       : 7
```

Table 2.2: cost analysis Direct-Form FIR Polyphase Linear Structure

cost	MULT	ADD	STA	MPIS	APIS
Direct-Form FIR Polyphase Linear Structure	14	07	1	14	7

2.3 Direct-Form FIR Transposed Form

If two digital filter structures have the same transfer function, then they are called equivalent structure. A simple way to generate an equivalent structure from a given realization structure is via the transpose operation. The transposed from is obtained by (1) reversing the paths (2) replacing pick-off nodes by adder and viva-versa.(3) interchange the input and output nodes. For a signal input output system the transposed structure has the same transfer function as an original realization structure.

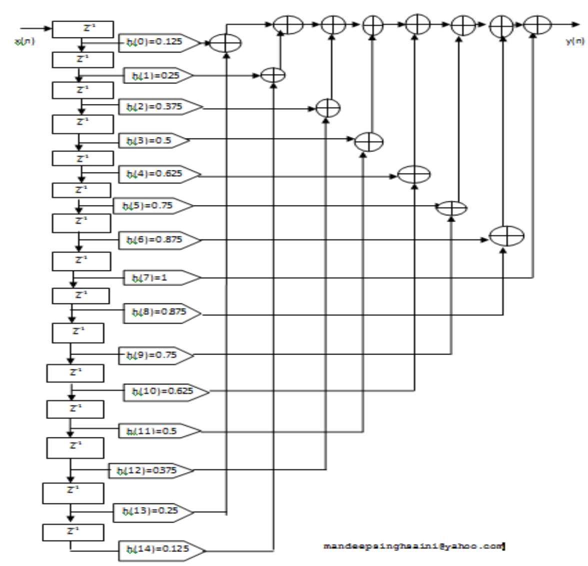


Fig 2.6 FIR Direct-Form FIR Transposed Structure

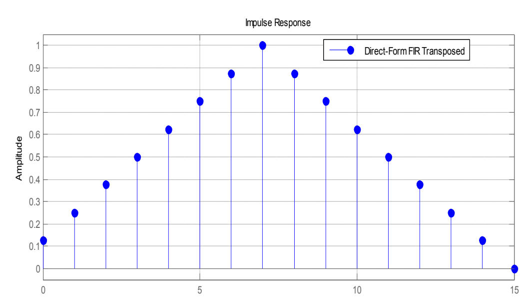


Fig 2.7 FIR Direct-Form Transpose Structure (Impulse response)

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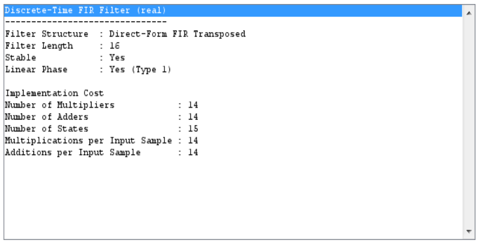
$h(0) = 0.125 \;\; h(14) = 0.125 \;\; ; h(1) = 0.25 \;\;\; h(13) = 0.25$
$h(2) = 0.375\, h(12) = 0.375\, ; h(3) = 0.5 h(11) = 0.5$
$h(4) = 0.625\, h(10) = 0.625;\; h(5) = 0.75\, h(9) = 0.75$
$h(6) = 0.875\, h(8) = 0.875;\; h(7) = 1; h(15) = 0$



```
Discrete-Time FIR Filter (real)
-------------------------------
Filter Structure  : Direct-Form FIR Transposed
Filter Length     : 16
Stable            : Yes
Linear Phase      : Yes (Type 1)

Implementation Cost
Number of Multipliers            : 14
Number of Adders                 : 14
Number of States                 : 15
Multiplications per Input Sample : 14
Additions per Input Sample       : 14
```

Table 2.3: cost analysis Direct-Form FIR Transposed Structure

cost	MULT	ADD	STA	MPIS	APIS
Direct-Form FIR Transpose Structure	14	14	15	14	14

2.4 Direct-Form FIR Symmetrical Form

In Symmetric structures filter coefficients are symmetric and linear phase. Linear phase digital filters allow all the frequency components of an input signal to pass through the filter with the same delay, which means that the group delay through the filter is a constant value independent of the frequency. Linear phase filters are useful in filtering applications in which you want to minimize signal distortion and spreading over time.
It has the advantage to reduce number of multipliers (MULT) and multi per input sample (MPIS).

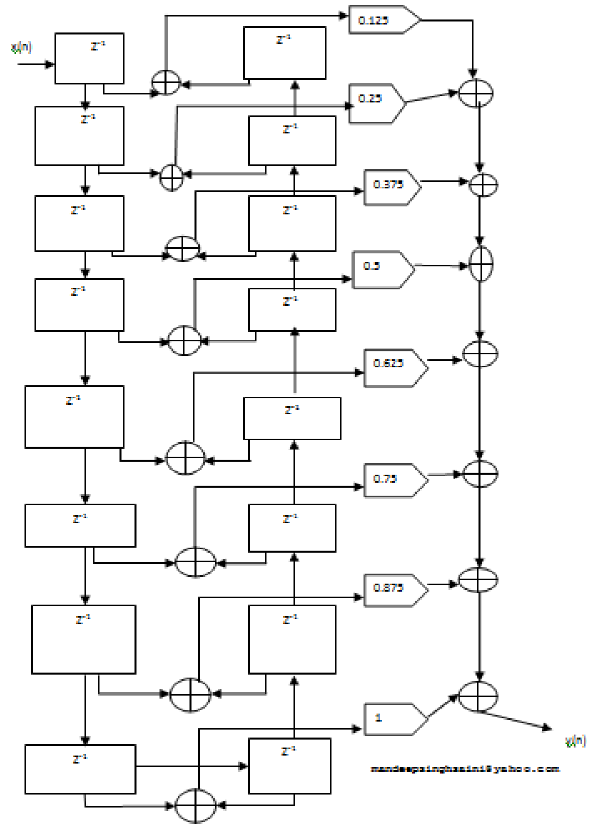


Fig 2.8 FIR Direct-Form FIR symmetrical Structure

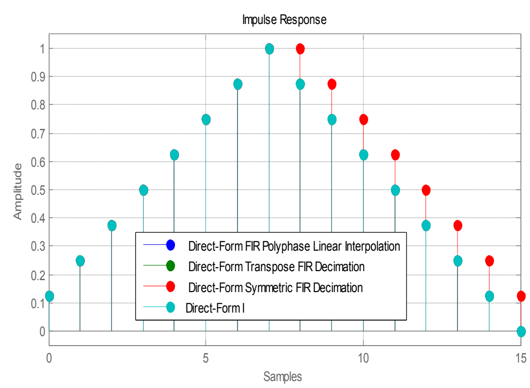


Fig 2.9 FIR Direct-Form Symmetrical Structure (Impulse response)

$h(0) = 0.125 \;\; h(15) = 0.125 \;\; ; h(1) = 0.25 \;\;\; h(14) = 0.25$
$h(2) = 0.375\, h(13) = 0.375\, ; h(3) = 0.5 h(12) = 0.5$
$h(4) = 0.625\, h(11) = 0.625;\; h(5) = 0.75\, h(10) = 0.75$
$h(6) = 0.875\, h(9) = 0.875;\; h(7) = 1; h(8) = 1$

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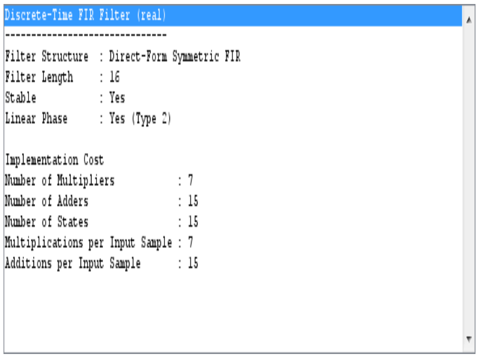

```
Discrete-Time FIR Filter (real)
-------------------------------
Filter Structure  : Direct-Form Symmetric FIR
Filter Length     : 16
Stable            : Yes
Linear Phase      : Yes (Type 2)

Implementation Cost
Number of Multipliers          : 7
Number of Adders               : 15
Number of States               : 15
Multiplications per Input Sample : 7
Additions per Input Sample     : 15
```

Table 2.4: cost analysis Direct-Form FIR Symmetrical Structure

cost	MULT	ADD	STA	MPIS	APIS
Direct-Form FIR Symmetrical Structure	7	15	15	7	15

[B] PROPOSED FILTER DESIGN

A symmetrical structure is used to shape and oversample a symbol stream before modulation/transmission as well as after modulation and demodulation. It is used to reduce the bandwidth of the oversampled symbol stream without introducing inter symbol interference.

In this proposed work Direct-From I has been designed using filter order 14 with no of tap 14 and numerator length is 16 and denominator length is 1 shown in Fig. 2.1

Direct- From FIR transposed structure using filter order 14 with no of tap 14 and filter length is 16 shown in Fig. 2.6.

Direct-From FIR Polyphase linear interpolation filter contain order 14 with interpolation factor 8 shown in Fig. 2.4 as compared to Direct-From I and Direct-From FIR Transposed. Results shown Hardware requirement is less in Direct-Form FIR Symmetrical structure shown in Fig. 2.8.

So the proposed design is based on Park-McClellan algorithm technique which provides same stop band attenuation and transition width.

The comparison of all filter design is shown in Fig. 3.1.

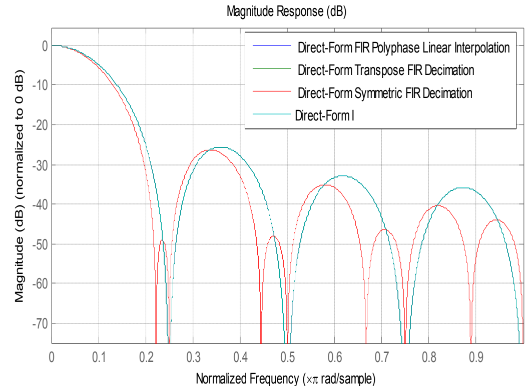

Fig. 3.1 all filter design performance

[C] RSEULTS & DISCUSSION

The performance and cost of all the all designs have been analyzed.

TABLE 4.1: performance comparisons of all the four (4) designs

Technique	Number of Multi pliers MULTI	Numb er of Adder s ADD	Numb er of States	Mult Per Input Sample (MPIS)	Add Per Input Sample
Direct-Form I FIR Structure	14	14	15	14	14
Direct-Form FIR Transpose Structure	14	14	15	14	14
Direct-Form FIR Polyphase Linear Structure	14	7	1	14	7
Direct-Form FIR Symmetrical Structure	**7**	**15**	**15**	**7**	**15**

This method achieves computational costs Direct-Form Symmetrical Structure lower than only 7 MPIS on average compared to 14 MPIS that of Cascade Direct-Form I, Direct-Form FIR Polyphase Structure and Direct-Form FIR Transpose Structure design.

[D] CONCLUSION

In this paper, four optimized alternatives to cosine filter are presented. The performance of the four designs (Direct-Form I, Direct-Form FIR Transpose structure, FIR Polyphase linear interpolation structure, Direct-Form FIR Symmetrical structure) is almost identical in terms of FIR Symmetrical filters have traditionally been considered much more efficient than their FIR design.

The results showed FIR Symmetrical 50% saving in terms of hardware requirement. So proposed alternative designs Multirate Direct-Form FIR Symmetrical filter may be used to provide cost effective solution for the different sampling rates.

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MANDEEP SINGH SAINI was born in Punjab, India on 26th January 1975. He completed his B. Tech in Electronics Engineering from Nagpur University (Maharashtra State) in 2001. He has also completed Master of Technology (M. Tech) from Guru Nanak Dev Eng. College Ludhiana (Punjab) in 2012. His current research and teaching interests are in Digital Signal Processing and Very Large Scale Integration Design. He has also worked as M. Tech Research guide with more than 22 Journal and Conferences.

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