Case study then Direct-Form FIR Poly-phase Linear Interpolator Filter Design with Symmetric Structure

Mandeep Singh Saini¹ and Dr. Kuldeep Kaur²

²Ph. D (Asst. Prof.), Faculty Member of B.A.M. Khalsa College Garhshankar. (Punjab)

ABSTRACT — FIR poly-phase decomposition filters reduce of computation complexity. However, FIR poly-phase filters most of advantage they require a very small number of multipliers to implement but The Direct-Form Symmetric FIR shows 50% reduction in multipliers as compared to Direct-Form FIR Poly-phase Linear Interpolator, Direct-Form FIR Transposed and Direct-Form I.

In this paper, Cost effective techniques for design of Pulse Shaping Filter have been presented to improve the computational and implementation complexity.

Keywords:- FIR, Interpolators, Decimators, Transposed, Symmetric.

[A] INTRODUCTION

1. Need of Pulse Shaping Design

Multirate digital signal processing is required in digital system where more than one sampling rate is required. In digital audio, the different sampling rates used are 32 KHz for broadcasting, 44.1 KHz for compact disc and 48 KHz for audio tape. In digital video, the sampling rates for composite video signal are 14.3181818 MHz and 17.734475 MHz for NTSC and PAL respectively. But the sampling rates for digital component of video signals are 13.5 MHz and 6.75 MHz for luminance and colour difference signal. Different sampling rates can be obtained using as up sampler and down sampler.

The digital signal processing is used for communication system, speech and audio processing systems, antenna systems and radar system. They have also advantages of multirate signal processing are computational requirements is less, storage for filter coefficient is less, finite arithmetic efficient are less, finite order required in multirate application is low and sensitivity to filter coefficient length is less.

While design multirate systems, effect of aliasing for decimation and pseudo image for interpolation should be avoided.

1. SAMPLING

Let signal be $x = (10 \, 9 \, 8 \, 7 \, 6 \, 5 \, 4 \, 3 \, 2 \, 1)$ and $t$ (sec) is the signal time, then $x(t)$ be a continuous time varying signal is sampled at regular intervals of time with sampling period $T$. Then sampling signal $x(t)$ is given by

$$x(nT) = x(t) \bigg|_{t=nT} = x(nT) \quad - \infty < n < \infty \quad (1.1)$$
The continuous time signal $x(t)$ is multiplied by the sampling function $s(t)$ which is series of impulse, the resultant signal is discrete time signal $x(n)$.

$$x(n) = x(t) s(t) \mid t = nT \quad - \infty < n < \infty \quad (1.2)$$

The given signal output is shown in Fig. 1.2

$x(0) = 10; x(1) = 9; x(2) = 8; x(3) = 7; x(4) = 6; x(5) = 5; x(6) = 4; x(7) = 3; x(8) = 2; x(9) = 1$

### 2. Sampling rate conversion

Sampling rate conversion is the process of converting the sequence $x(n)$ which is got from sampling the continuous time signal $x(t)$ with a period $T$, to another sequence $y(k)$ obtained from sampling $x(t)$ with a period $T'$. The new sequence $y(k)$ can be obtained by first reconstruction the original signal $x(t)$ from the sequence $x(n)$ and then sampling the reconstruction signal with a period $T'$.

![Sampling of a discrete time signal](image)

**The new sampling rate $F'$ become**

$$F' = \frac{1}{T'} = \frac{1}{MT} = \frac{F}{M} \quad (1.3)$$

Let the signal $x(n)$ be a full band signal, with non-zero values in the frequency range $-\frac{F}{2} \leq f \leq \frac{F}{2}$, where $w = 2\pi f T$

$$|X(e^{jw})| \neq 0, |w| = |2\pi f T| \leq 2\pi f \frac{T}{2} = \pi \quad (1.4)$$

To avoid aliasing in the decimation process, the signal $x(n)$ is filtered with a digital low pass filter which has the following spectral response

$$|H(e^{jw})| = \begin{cases} 
1 & \text{if } |w| = 2\pi F \frac{T}{2} = \frac{\pi}{M} \\
0 & \text{otherwise} 
\end{cases} \quad (1.5)$$

The sequence $y(k)$ is obtained by selecting only the $M^{th}$ sample of the filtered output which results in sampling rate reduction. Let the impulse response of the filter be $h(n)$, then the filtered output $w(n)$ is given by

$$w(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k) \quad (1.6)$$

$W(e^{jw})$ is the spectrum of $w(n)$.

The decimation signal $y(m)$ is

$$y(m) = w(Mm) \quad (1.7)$$

![Conversion of a sequence x(n) to another sequence y(k)](image)

### 3. Decimation

The process of reducing the sampling rate of a signal is called decimation (sampling rate compression). Let $M$ be the integer sampling rate reduction factor for the signal $x(n)$.

$$\frac{T'}{T} = M$$
The process of interpolation the sampling rate of a signal is interpolation. Let L be an interpolation factor, of the signal x(n), then

\[ \frac{T'}{T} = \frac{1}{L} \]

The sampling rate is given by

\[ F' = \frac{1}{T'} = \frac{L}{T} = LF \]  \hspace{1cm} (1.9)

Interpolation of a signal x(n) be a factor L refers to the process of interpolation L-1 samples between each pair of samples of x(n)

\[ w(m) = \begin{cases} x\left(\frac{m}{L}\right), & m = 0, \pm L, \pm 2L, \ldots \ldots \text{otherwise} \\ 0 & \end{cases} \]  \hspace{1cm} (1.10)

The Z-transform of the signal w(m) is given by

\[ W(z) = \sum_{m=-\infty}^{\infty} w(m)z^{-m} = \sum_{m=-\infty}^{\infty} x(m)z^{-mL} = X(z^L) \]  \hspace{1cm} (1.11)

When considered over the unit circle, \( z = e^{jw'} \),

\[ w\left(e^{jw'}\right) = X\left(e^{-jw'\cdot L}\right) \]

Where \( w' = \frac{2\pi}{T'} \)  \hspace{1cm} (1.12)

The spectrum of the signal w(m) contain the image of baseband at the harmonics of the sampling frequency ± \( \frac{2\pi}{L}, \pm \frac{4\pi}{L} \). To remove the image, an anti-imaging (low-pass) filter is used. The ideal characteristics of the low pass filter is given by

\[ H\left(e^{jw'}\right) = \begin{cases} G, & |w'| \leq \frac{2\pi}{2}\frac{T'}{L} = \frac{\pi}{L} \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} (1.13)

Where G is the gain of the filter and should be Lin the pass band. The frequency response of the output signal is given by

\[ Y\left(e^{jw'}\right) = H\left(e^{jw'}\right)X\left(e^{-jw'\cdot L}\right) \]

\[ Y\left(e^{jw'}\right) = \begin{cases} Gx\left(e^{-jw'\cdot L}\right), & |w'| = \frac{\pi}{L} \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} (1.14)

The output signal y(m) is given by

\[ y(m) = \sum_{k=-\infty}^{\infty} h(m-k)w(k) \]
Interpolation with L=2
Then output signal is shown in Fig.1.8

\[ y(0) = 10; \ y(1) = 0; \ y(2) = 9; \ y(3) = 0; \ y(4) = 8; \]
\[ y(5) = 0; \ y(6) = 7; \ y(7) = 0; \ y(8) = 6; \ y(9) = 0; \]
\[ y(10) = 5; \ y(11) = 0; \ y(12) = 4; \ y(13) = 0; \]
\[ y(14) = 3; \ y(15) = 0; \ y(16) = 2; \]
\[ y(17) = 0; \ y(18) = 1; \]

[B] PROPOSED FILTER DESIGN

2.1 FIR Direct-Form I Structure

The general input-output relationship of an FIR filter is given by

\[ y(n) = \sum_{k=0}^{N-1} h(k)x(n - k) \quad 2.1 \]

The filter is represented by N-point impulse response h(k) takes value only in the range \( k = 0, \ldots, N-1 \), outside this interval h(k) is zero fig. 2.1 shown the graph representation of an FIR filter.

The z-transform direct-from I filter is

\[ H(z) = \sum_{n=0}^{\infty} h(n)z^{-1} \quad 2.2 \]

Where

\[ H(z) = h(0) + h(1)z^{-1} + \ldots \]

From Fig. 2.2, we get

\[ h(0) = 0.125 \quad h(1) = 0.25 \quad h(2) = 0.375 \quad h(3) = 0.5 \]
\[ h(4) = 0.625 \quad h(5) = 0.75 \quad h(6) = 0.875 \quad h(7) = 1 \]
\[ h(8) = 0.875 \quad h(9) = 0.75 \quad h(10) = 0.625 \quad h(11) = 0.5 \]
\[ h(12) = 0.375 \quad h(13) = 0.25 \quad h(14) = 0.125 \quad h(15) = 0 \]

\[ y(m) = \sum_{k=-\infty}^{\infty} h(m-k)x(K/L), \text{ where } k/L \text{ an integer} \quad 1.15 \]

Input signal is shown in Fig. 1.2.

\[ x(0) = 10; \ x(1) = 9; \ x(2) = 8; \ x(3) = 7; \ x(4) = 6; \]
\[ x(5) = 5; \ x(6) = 4; \ x(7) = 3; \ x(8) = 2; \ x(9) = 1 \]
To implement a filter with a fractional multi-rate factor of \(L/M\), the data is first upsampled by \(L\) and then a filter is applied to both interpolate between the data points as well as provide anti-alias protection, and then finally downsampled by \(M\). These three designs are diagrammed below.

Using \(M\) decimation factor then Equ. 2.2 is given as

\[
H(z) = \sum_{n=0}^{\infty} h(n)z^{-M}
\]

\[
H(z) = h(0) + h(z)z^{-M} + h(2M)z^{-2M} + \cdots \quad (2.3)
\]

Rearranging the above equation, we get
\[ H(z) = h(0) + z^{-2}h(2) + z^{-4}h(4) + z^{-6}h(6) + z^{-8}h(8) + z^{-10}h(10) + z^{-12}h(12) + z^{-14}h(14) + z^{-1}[h(1) + z^{-1}h(2) + z^{-3}h(4) + z^{-5}h(6) + z^{-7}h(8) + z^{-9}h(10) + z^{-11}h(12) + z^{-13}h(14)] \]

\[ H(z) = E_0(z^2) + z^{-1}E_1(z^2) \quad (2.4) \]

Where \( E_0(z^2) \) and \( E_1(z^2) \) are poly phase components for a factor of two

### 2.3 Direct-Form FIR Transposed Form

If two digital filter structures have the same transfer function, then they are called equivalent structure. A simple way to generate an equivalent structure from a given realization structure is via the transpose operation. The transposed from is obtained by (1) reversing the paths (2) replacing pick-off nodes by adder and viva-versa,(3) interchange the input and output nodes. For a signal input output system the transposed structure has the same transfer function as an original realization structure.

<table>
<thead>
<tr>
<th>cost</th>
<th>MULT</th>
<th>ADD</th>
<th>STA</th>
<th>MPIS</th>
<th>APIS</th>
</tr>
</thead>
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<td>14</td>
<td>07</td>
<td>1</td>
<td>14</td>
<td>7</td>
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</table>
\[ h(0) = 0.125 \quad h(14) = 0.125 \quad h(1) = 0.25 \quad h(13) = 0.25 \]
\[ h(2) = 0.375 \quad h(12) = 0.375 \quad h(3) = 0.5h(11) = 0.5 \]
\[ h(4) = 0.625 \quad h(10) = 0.625 \quad h(5) = 0.75 \quad h(9) = 0.75 \]
\[ h(6) = 0.875 \quad h(8) = 0.875 \quad h(7) = 1 \quad h(15) = 0 \]

### Table 2.3: Cost Analysis Direct-Form FIR Transposed Structure

<table>
<thead>
<tr>
<th>cost</th>
<th>MULT</th>
<th>ADD</th>
<th>STA</th>
<th>MPIS</th>
<th>APIS</th>
</tr>
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<tr>
<td>Direct-Form FIR Transpose Structure</td>
<td>14</td>
<td>14</td>
<td>15</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

### 2.4 Direct-Form FIR Symmetrical Form

In Symmetric structures filter coefficients are symmetric and linear phase. Linear phase digital filters allow all the frequency components of an input signal to pass through the filter with the same delay, which means that the group delay through the filter is a constant value independent of the frequency. Linear phase filters are useful in filtering applications in which you want to minimize signal distortion and spreading over time. It has the advantage to reduce number of multipliers (MULT) and multi per input sample (MPIS).
[B] PROPOSED FILTER DESIGN

A symmetrical structure is used to shape and oversample a symbol stream before modulation/transmission as well as after modulation and demodulation. It is used to reduce the bandwidth of the oversampled symbol stream without introducing inter symbol interference.

In this proposed work Direct-From I has been designed using filter order 14 with no of tap 14 and numerator length is 16 and denominator length is 1 shown in Fig. 2.1 Direct-From FIR transposed structure using filter order 14 with no of tap 14 and filter length is 16 shown in Fig. 2.6. Direct-From FIR Polyphase linear interpolation filter contain order 14 with interpolation factor 8 shown in Fig. 2.4 as compared to Direct-From I and Direct-From FIR Transposed. Results shown Hardware requirement is less in Direct-From FIR Symmetrical structure shown in Fig. 2.8.

So the proposed design is based on Park-McClellan algorithm technique which provides same stop band attenuation and transition width.

The comparison of all filter design is shown in Fig. 3.1.

Table 2.4: cost analysis Direct-Form FIR Symmetrical Structure

<table>
<thead>
<tr>
<th>cost</th>
<th>MULT</th>
<th>ADD</th>
<th>STA</th>
<th>MPIS</th>
<th>APIS</th>
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<tr>
<td>Direct-Form FIR Symmetrical Structure</td>
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<td>15</td>
<td>15</td>
<td>7</td>
<td>15</td>
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</tbody>
</table>

This method achieves computational costs Direct-Form Symmetrical Structure lower than only 7 MPIS on average compared to 14 MPIS that of Cascade Direct-From I, Direct-Form FIR Polyphase Structure and Direct-Form FIR Transpose Structure design.

[D] CONCLUSION

In this paper, four optimized alternatives to cosine filter are presented. The performance of the four designs (Direct-From I, Direct-Form FIR Transpose structure, FIR Polyphase linear interpolation structure, Direct-Form FIR Symmetrical structure) is almost identical in terms of FIR Symmetrical filters have traditionally been considered much more efficient than their FIR design.

The results showed FIR Symmetrical 50% saving in terms of hardware requirement. So proposed alternative designs Multirate Direct-Form FIR Symmetrical filter may be used to provide cost effective solution for the different sampling rates.

Acknowledgment

RESULT & DISCUSSION

The performance and cost of all the all designs have been analyzed.

TABLE 4.1: performance comparisons of all the four (4) designs

<table>
<thead>
<tr>
<th>Technique</th>
<th>Number of Multipliers</th>
<th>Number of Addressed ADD</th>
<th>Number of States</th>
<th>Multi Per Input Sample (MPIS)</th>
<th>Add Per Input Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct-Form I FIR Structure</td>
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<td>14</td>
<td>15</td>
<td>14</td>
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<tr>
<td>Direct-Form I FIR Transpose</td>
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<td>1</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>Direct-Form I FIR Symmetrical Structure</td>
<td>7</td>
<td>15</td>
<td>15</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

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REFERENCES


MANDEEP SINGH SAINI was born in Paschim, India on 28th January 1975. He completed his B. Tech in Electronics Engineering from Nagpur University (Maharashtra State) in 1999. He has also completed Masters of Technology (M Tech) from Guru Nanak Dev Engg College Ludhiana (Punjab) in 2012. His current research and teaching interest are in Digital Signal Processing and Very Large Scale Integration Design. He has also worked as M. Tech Research guide with more than 15 Journal and Conferences.