Introduction of Fourier Series to First Year Undergraduate Engineering Students

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Abstract: In this paper we have introduced the general form of Fourier series to simplify the better understanding of the concept of Fourier expansion of functions.

Keywords: Cosine series, Engineering student, Fourier expansion, Sine series.

Introduction: For first year undergraduate engineering students it is quite difficult to recall the results of Fourier series for given example. Because almost in all the books of engineering mathematics number of results are defined interval wise. For example $(c, c+2\pi)$ $(0,2\pi)$ $(-\pi,\pi)$, $(0,L)$, $(0,\pi)$, $(c,c+2L)$, $(0,2L)$ $(-L,L)$ etc. Usually many students get confuse about the selection of the results.

The form which we have introduced here can be used to find the Fourier expansion of function for any given interval. Our attempt is to introduce the Fourier series to first year undergraduate engineering students in such a way that they can easily obtain the Fourier expansion for given function in any interval without any confusion and memorize the minimum number of formulae.

General Formula for Fourier Series:

It is not necessary to recall all set of formulae, it is sufficient to remember only three formulae given below.

1) Fourier series for a function in $(c, c+2L)$ which is stated below

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

Where

$$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\, dx \;,\quad a_n = \frac{1}{L}\int_c^{c+2L} f(x)\cos\frac{n\pi x}{L}\, dx \text{ and } b_n = \frac{1}{L}\int_c^{c+2L} f(x)\sin\frac{n\pi x}{L}\, dx$$

2) Half range cosine series in (0,L):

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

Where

$$a_0 = \frac{2}{L}\int_0^{L} f(x)\, dx \qquad a_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}\, dx$$

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and

3) Half range sine series in (0,L):

$$f(x) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}$$

Where

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

Above results can be converted to any suitable given interval and for that students are advised to understand the following things for given problem.

1) To decide whether result 1, 2 or 3 is suitable that depends upon question.

2) Understand LL (Lower limit)

3) Understand UL (Upper limit)

4) The value of L which can be calculated by simple result.

Total length of general interval = Total length of given interval.

And make the changes from 2, 3, 4 in 1 that becomes suitable result for given problem.

For example:

Que 1) obtain the Fourier series for $f(x) = (\frac{\pi - x}{2})^2 \; in \; (0,2\pi)$

Fourier series is expected so we shall focus on result 1 given below

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L})$$

where

$$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\,dx \qquad a_n = \frac{1}{L}\int_c^{c+2L} f(x)\cos\frac{n\pi x}{L}\,dx$$

and

$$b_n = \frac{1}{L}\int_c^{c+2L} f(x)\sin\frac{n\pi x}{L}\,dx$$

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Here

1) LL=0

2) UL=2π

3) Length of general interval =2L

Length of given interval =2π

Hence 2L=2π

Therefore L= π

We shall make above three changes at a time in above selected results.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx \ dx$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \ dx$$

Here after calculation for above constants to be made as usual.

Que.2) Find half range cosine series for $f(x) = \sqrt{1 - \cos x}$ in $(0, \pi)$

Fourier series is expected so we shall focus on result 2 given below

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} \quad \text{where} \quad a_0 = \frac{2}{L} \int_0^{L} f(x)\, dx$$

$$a_n = \frac{2}{L} \int_0^{L} f(x) \cos \frac{n\pi x}{L} \ dx$$

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Here

1) LL=0

2) UL= π

3) Length of general interval =L

Length of given interval = π

Hence L=π

We shall make above three changes at a time in above selected results in 2.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

where

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx \, dx$$

Here after, calculation for above constants to be made as usual.
Similarly half range sine series can also be obtained.

Conclusion:
For engineering students finding an efficient way of learning mathematics is very important. To make their learning better mathematics should be introduced to them in a general way. As they do not have sufficient intention and time to study mathematics rigorously. In this paper we have presented introduction of Fourier series in a simple and general way. Engineering students will find this very easy and better to understand the concepts Fourier series.

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We all the four authors have read this article completely and we all have contributed equally towards the completion of this article.

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