FIXED POINT THEOREM IN MENGER SPACE

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Abstract— In this paper, fixed point theorem for two pairs of weakly compatible mappings in Menger space without appeal to continuity. Our main results extend the result of Pant , B.D. et al. [7].

Index Terms— Fixed point, compatibility, Menger space and weakly compatible.

I INTRODUCTION

The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the reverse is not true. Recently in this line, Singh and jain [12] introduced the notion of weakly compatible maps in Menger space to establish a common fixed point theorem. There has been a number of generalizations of metric space . One such generalization is Menger space initiated by Menger [4] It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$ .Schweizer and Sklar [8] studied this concept and gave some fundamental results on this space. It is observed by many authors that contraction condition in metric space may be exactly translated into PM space endowed with the min norm.

Sehgal and Bharucha-Reid [9] obtained a generalization of Banach contraction principle in a complete Menger space. Sessa [10] initiated the tradition of improving commutativity conditions

In fixed point theorems by introducing the notion of weakly commuting maps in metric spaces.

Jungck [2] soon enlarged this concept to compatible maps. Recently, Jungck and Rohades [3] termed a pair of self – maps to be coincidentally commuting or equivalently weak-compatible if

They commute at their coincidence points. Menger , K [4] introduced the notion of a probabilistic metric space in 1942 and since the theory of probabilistic metric space has developed in many directions, especially in nonlinear analysis and applications. The idea of Menger was to use distribution functions instead of nonnegative real numbers as values of the metric space. Schweizer and Sklar [8] studied this concept and gave some fundamental results on this space.

The important development of fixed points theory in Menger spaces was due to Sehgal and Bharucha-Reid [9] . In this paper , we establish a common fixed point theorem for two pairs of weakly compatible mappings in Menger space without appeal to continuity.

II PRELIMINARIES

Definition 2.1 A triangular norm T (shortly t-norm) is a binary operation on the unit interval [0,1] such that for all a, b, c, d $\in$ [0,1] the following conditions are satisfied:

(i) $T(a, 1) = a$;
(ii) $T(a, b) = T(b, a)$;
(iii) $T(a, b) \leq T(c, d)$ whenever $a \leq c$ and $b \leq d$;
(iv) $T(a,T(b,c)) = T(T(a,b),c)$;

Two typical examples of continuous t-norm are T (a, b) = ab and T(a, b) = min{a, b}.

Now t-norms are recursively defined by $T^1 = T$ and

$T^n(x_1, \ldots, x_{n+1}) = T(T^{n-1}(x_1,\ldots,x_n), x_{n+1})$, for n ≥ 2 and $x_i \in [0,1]$, for all

$i \in \{1, 2, \ldots, n+1\}$.

Definition 2.2 A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is said to be distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$. We will denote by $\mathfrak{I}$ the set of all distribution functions defined on [- ∞, ∞] while H(t) will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, if\ t \le 0; \\ 1, if\ t > 0. \end{cases}$$

If X is a non-empty set $\mathcal{F} : XxX \to \mathfrak{I}$ is called a probabilistic distance on X and the Value of $\mathcal{F}$ at $(x, y) \in XxX$ is represented by $F_{x,y}$.

Definition 2.3 The ordered pair (X , $\mathcal{F}$) is called a probabilistic metric space (shortly PM-space) , If X is a non-empty set and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions for all x, y, z ∈ X and t, s > 0 ;

(i) $F_{x,y}(t) = H(t)$ for all t > 0 if and only if x = y ;

(ii) $F_{x,y}(t) = 0$;

(iii) $F_{x,y}(t) = F_{y,\ x}(t)$;

(iv) $F_{x,y}(t) = 1, F_{y,z}(s) = 1 \Rightarrow F_{x,z}(t + s) = 1$.

The ordered triple (X, $\mathcal{F}$, T) is called a Menger space if (X , $\mathcal{F}$) is a PM-space, T is a t-norm and the following inequality hold :

(v) $F_{x,z}(t + s) \ge T\big(F_{x,y}(t), F_{y,z}(s)\big)$.

Every metric space (X, d) can be realized as a PM-space by taking $\mathcal{F} : XxX \to \mathfrak{I}$ defined by $F_{x,y}(t) = H\big(t - d(x,y)\big)$ for all x, y ∈ X .

Definition 2.4 Let (X , $\mathcal{F}$, T) be a Menger space and T be a continuous t-norm.

(i) A sequence $\{x_n\}$ in X is said to be converge to a point x in X iff for every $\varepsilon > 0$ and $\lambda \in (0, 1)$ there exists an integer N such that $F_{x_n,y}(\varepsilon) > 1 - \lambda$ for all n ≥ N.

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$ there exists an integer N such that $F_{x_n,y_m}(\varepsilon) > 1 - \lambda$ for all n, m ≥ N.

(iii) A Menger space in which every Cauchy sequence is convergent is said to be Complete.

Definition 2.5 [5] Self maps A and B of a Menger space (X , $\mathcal{F}$, T) are said to be compatible if

$F_{ABx_n,BAx_n}(t) \to H(t)$ for all t > 0,

whenever $\{x_n\}$ is a sequence in X such that

$Ax_n, Bx_n \to x$ for some x in X as $n \to \infty$.

Definition 2.6 [12] Self maps A and B of a Menger space (X , $\mathcal{F}$, T) are said to be weakly compatible (or coincidently commuting) if they commute at their coincidence points , that is if Ax = Bx for some x ∈ X then ABx = BAx.

Remark 2.1 [12] If self maps A and B of a Menger space (X , $\mathcal{F}$, T) are compatible then they are weakly compatible.

Lemma 2.1 [6,11] Let (X , $\mathcal{F}$, T) be a Menger space and define $E_{\lambda,F} : X^2 \to R^+ \cup \{0\}$ by

$E_{\lambda,F}(x, y) = \inf\{t > 0 : F_{x,y}(t) > 1 - \lambda\}$, for each $\lambda \in (0, 1)$ and x , y ∈ X. Then we have

(i) For any $\mu \in (0,1)$ there exists $\lambda \in (0, 1)$ such that $E_{\mu,F}(x, z) \le E_{\lambda,F}(x, y) + E_{\lambda,F}(y, z)$. For any x, y, z ∈ X ;

(ii) The sequence $\{x_n\}_{n \in N}$ is convergent with respect to Menger Probabilistic metric $\mathcal{F}$ if and only if $E_{\lambda,F}(x_n, x) \to 0$. Also the sequence $\{x_n\}_{n \in N}$ is

Cauchy sequence with respect to Menger probabilistic metric $\mathcal{F}$ if and only if it is a Cauchy sequence with $E_{\lambda,F}$.

Lemma 2.2 [5] Let (X , $\mathcal{F}$, T) be a Menger space. If there exists a constant $k \in (0,1)$ such that

$F_{x,y}(kt) \geq F_{x,y}(t)$, for all x , y ∈ X and t > 0 then x = y.

III. MAIN RESULTS

Theorem 3.1 Let A, L, M and S be self maps on a complete Menger space (X , $\mathcal{F}$, T) and suppose the following are satisfied :

(3.1.1) L(X) ⊆ S(X) , M(X) ⊆ A(X) ;

(3.1.2) One of S(X) and A(X) is a closed subset of X;

(3.1.3) There exists a constant $k \in (0,1)$ such that

$$F_{Lx,My}(kt) \geq \min \begin{Bmatrix} F_{Ax,Lx}(t), F_{Sy,My}(t), \frac{F_{Ax,Lx}(t)+F_{Sy,My}(t)}{2}, \\ F_{Sy,Lx}(\alpha t), F_{Ax,My}((2-\alpha)t), F_{Ax,Sy}(t) \end{Bmatrix},$$

for all x , y ∈ X , $\alpha \in (0, 2)$ and t > 0;

(3.1.4) The pairs (L , A) and (M ,S) are weakly compatible.In addition assume that

$E_{\lambda,F}$ (x ,y) = inf {t > 0: $F_{x,y}$ (t) > 1 - λ}, for each $\lambda \in (0,1)$ and x , y ∈ X.Then A, L, M and S have a unique common fixed points in X.

Proof : Let x_0 be an arbitrary point in X . Since L(X) ⊆ S(X) , one can find a point x_1 in X

With Lx_0 = Sx_1 = y_0 . Again , as M(X) ⊆ A(X) , one can also choose a point x_2 ∈ X

With Mx_1 = Ax_2 = y_2 . Inductively, we construct a sequences $\{x_n\}$ and $\{y_n\}$ in X

Such that Lx_{2n} = Sx_{2n+1} = y_{2n} and Mx_{2n+1} = Ax_{2n+1} = y_{2n+1} for n = 0 ,1 , 2, … .

Using (3.1.3) putting x = x_{2n} and y = x_{2n+1} for t > 0 and α = 1- q with q ∈ (0 ,1) , then We get

$$F_{Lx_{2n}, Mx_{2n+1}}(kt) \geq$$

$$\min \begin{Bmatrix} F_{Ax_{2n},Lx_{2n}}(t), F_{Sx_{2n+1},Mx_{2n+1}}(t), \frac{F_{Ax_{2n},Lx_{2n}}(t)+F_{Sx_{2n+1},Mx_{2n+1}}(}{2} \\ F_{Sx_{2n+1},Lx_{2n}}((1-q)t), F_{Ax_{2n},Mx_{2n+1}}((1+q)t), F_{Ax_{2n},Sx_{2n+1}} \end{Bmatrix}$$

,

$$F_{y_{2n},y_{2n+1}}(kt) \geq$$

$$\min \begin{Bmatrix} F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t), \frac{F_{y_{2n-1},y_{2n}}(t)+F_{y_{2n},y_{2n+1}}(t)}{2}, \\ F_{y_{2n-1},y_{2n+1}}((1+q)t), F_{y_{2n-1},y_{2n}}(t) \end{Bmatrix}$$

,

$$F_{y_{2n},y_{2n+1}}(kt) \geq$$

$$\min \begin{Bmatrix} F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t), \frac{F_{y_{2n-1},y_{2n}}(t)+F_{y_{2n},y_{2n+1}}(t)}{2}, \\ F_{y_{2n-1},y_{2n}}(t), F_{y_{2n+1},y_{2n}}(qt) \end{Bmatrix}$$

,

$$= \min\{ F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t), 1, F_{y_{2n+1},y_{2n}}(qt)\},$$

Letting q → 1, we get

$$F_{y_{2n},y_{2n+1}}(kt) \geq \min\{ F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t), 1, F_{y_{2n+1},y_{2n}}(t)\},$$

$$= \min\{ F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t), 1, F_{y_{2n+1},y_{2n}}(t)\},$$

$$= \min\{ F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t)\},$$

Hence ,

$$F_{y_{2n},y_{2n+1}}(kt) \geq \min\{ F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n+1}}(t)\}.$$

Similarly ,

$$F_{y_{2n+1},y_{2n+2}}(kt) \geq \min\{ F_{y_{2n},y_{2n+1}}(t), F_{y_{2n+1},y_{2n+2}}(t)\}.$$

Therefore , for all n we have ,

$F_{y_n,y_{n+1}}(kt) \geq \min\{F_{y_{n-1},y_n}(t), F_{y_n,y_{n+1}}(t)\}$.

Consequently,

$F_{y_n,y_{n+1}}(kt) \geq \min\{F_{y_{n-1},y_n}(k^{-1}t), F_{y_n,y_{n+1}}(k^{-1}t)\}$.

By repeated application of above inequality, we get for each j ∈ {1,2,3…}

$$F_{y_n,y_{n+1}}(kt) \geq \min\{F_{y_{n-1},y_n}(k^{-1}t), F_{y_{n-1},y_n}(k^{-2}t), F_{y_n,y_{n+1}}(k^{-2}t)\},$$

$$= \min\{F_{y_{n-1},y_n}(k^{-1}t), F_{y_n,y_{n+1}}(k^{-2}t)\},$$

$$\geq \ldots \geq \min\{F_{y_{n-1},y_n}(k^{-1}t), F_{y_n,y_{n+1}}(k^{-j}t)\},$$

And so for each $\lambda \in (0, 1)$ we have

$$E_{\lambda,F}(y_n, y_{n+1}) = \inf\{t > 0: F_{y_n,y_{n+1}}(t) > 1 - \lambda\},$$

$$\leq \inf\{t > 0: \min\{F_{y_{n-1},y_n}(k^{-1}t), F_{y_n,y_{n+1}}(k^{-j}t)\} > 1 - \lambda\},$$

$$\leq \max\begin{Bmatrix}\inf\{t > 0: F_{y_{n-1},y_n}(k^{-1}t) > 1 - \lambda\}, \\ \inf\{t > 0: F_{y_n,y_{n+1}}(k^{-j}t) > 1 - \lambda\}\end{Bmatrix}$$

$$\leq \max\{k\, E_{\lambda,F}(y_{n-1}, y_n), k^j\, E_{\lambda,F}(y_n, y_{n+1})\}.$$

Since , $k^j E_{\lambda,F}(y_n, y_{n+1}) \to 0$ as $j \to \infty$, it follows that :

$$E_{\lambda,F}(y_n, y_{n+1}) \leq k\, E_{\lambda,F}(y_{n-1}, y_n) \leq k^n E_{\lambda,F}(y_0, y_1)$$ for every $\lambda \in (0, 1)$.

Now , we show that $\{y_n\}$ is a Cauchy sequence. For every $\mu \in (0, 1)$, there exists $\gamma \in (0, 1)$ such that , for $m \geq n$,

$$E_{\mu,F}(y_n, y_m) \leq E_{\mu,F}(y_{m-1}, y_m) + E_{\mu,F}(y_{m-2}, y_{m-1}) + \ldots + E_{\mu,F}(y_n, y_{n+1})$$

$$\leq E_{\mu,F}(y_0, y_1) \sum_{i=n}^{m-1} k^i \to 0,$$

As m, n → ∞ . Hence by lemma 2.1, $\{y_n\}$ is a Cauchy sequence in X. since X is complete then $\{y_n\}$ converges to z ∈ X . i.e.

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Lx_{2n} = \lim_{n\to\infty} Mx_{2n+1} = \lim_{n\to\infty} Sx_{2n+1} = \lim_{n\to\infty} Ax_{2n} = z.$$

Suppose that S(X) is closed then for some $v \in X$ we have $S(v) = z$.

Using (3.1.3) putting $x = x_{2n}$, $y = v$ and $\alpha = 1$, then we get

$$F_{Lx_{2n},Mv}(kt) \geq \min\begin{Bmatrix}F_{Ax_{2n},Lx_{2n}}(t), F_{Sv,Mv}(t), \frac{F_{Ax_{2n},Lx_{2n}}(t) + F_{Sv,Mv}(t)}{2}, \\ F_{Sv,Lx_{2n}}(t), F_{Ax_{2n},Mv}(t), F_{Ax_{2n},Sv}(t)\end{Bmatrix},$$

As n →∞ , we have

$$F_{z,Mv}(kt) \geq \min\left\{F_{z,z}(t), F_{z,Mv}(t), \frac{F_{z,z}(t) + F_{z,Mv}(t)}{2}, F_{z,z}(t), F_{z,Mv}(t), F_{z,z}(t)\right\},$$

$$F_{z,Mv}(kt) \geq \min\left\{1, F_{z,Mv}(t), \frac{F_{z,z}(t) + F_{z,Mv}(t)}{2}, 1, F_{z,Mv}(t), 1\right\},$$

Then on simplification

$$F_{z,Mv}(kt) \geq F_{z,Mv}(t).$$

Thus , by lemma 2.2, $z = Mv$. Therefore $M(v) = S(v) = z$ .From weak compatibility of (M, S) , we have $M(S(v)) = S(M(v))$, hence Mz = Sz.

Using (3.1.3) putting $x = x_{2n}$, y = z and $\alpha = 1$, then we get

$$F_{Lx_{2n},Mz}(kt) \geq \min\begin{Bmatrix}F_{Ax_{2n},Lx_{2n}}(t), F_{Sz,Mz}(t), \frac{F_{Ax_{2n},Lx_{2n}}(t) + F_{Sz,Mz}(t)}{2}, \\ F_{Sz,Lx_{2n}}(t), F_{Ax_{2n},Mz}(t), F_{Ax_{2n},Sz}(t)\end{Bmatrix},$$

As n → ∞ , we have

$$F_{z,Sz}(kt) \geq \min\begin{Bmatrix}F_{z,z}(t), F_{Sz,Sz}(t), \frac{F_{z,z}(t) + F_{Sz,Sz}(t)}{2}, \\ F_{Sz,z}(t), F_{z,Sz}(t), F_{z,Sz}(t)\end{Bmatrix},$$

$$F_{z,Sz}(\mathrm{kt}) \geq \min\{1,1,1,F_{z,Sz}(\mathrm{t})\},$$

Then on simplification

$$F_{z,Sz}(\mathrm{kt}) \geq F_{z,Sz}(\mathrm{t}).$$

Thus by lemma 2.2, z = Sz. Therefore z = Mz = Sz. Since M(X) ⊆ A(X) , then there exists w ∈ X such that A(w) = Mz = Sz = z.

Using (3.1.3) putting x = w, y = z and α = 1, then we get

$$F_{Lw,Mz}(\mathrm{kt}) \geq \min \left\{ \begin{matrix} F_{Aw,Lw}(\mathrm{t}), F_{Sz,Mz}(\mathrm{t}), \frac{F_{Aw,Lw}(\mathrm{t})+F_{Sz,Mz}(\mathrm{t})}{2}, \\ F_{Sz,Lw}(\mathrm{t}), F_{Aw,Mz}(\mathrm{t}), F_{Aw,Sz}(\mathrm{t}) \end{matrix} \right\},$$

and so we have

$$F_{Lw,z}(\mathrm{kt}) \geq \min \left\{ \begin{matrix} F_{z,Lw}(\mathrm{t}), F_{z,z}(\mathrm{t}), \frac{F_{z,Lw}(\mathrm{t})+F_{z,z}(\mathrm{t})}{2}, \\ F_{z,Lw}(\mathrm{t}), F_{z,z}(\mathrm{t}), F_{z,z}(\mathrm{t}) \end{matrix} \right\},$$

$$F_{Lw,z}(\mathrm{kt}) \geq \min \left\{ F_{z,Lw}(\mathrm{t}), 1, \frac{F_{z,Lw}(\mathrm{t})+F_{z,z}(\mathrm{t})}{2}, F_{z,Lw}(\mathrm{t}), 1,1 \right\}$$

$$F_{Lw,z}(\mathrm{kt}) \geq F_{Lw,z}(\mathrm{t}).$$

Thus , by lemma 2.2, z = Lw . Therefore Lw = Aw = z . Also it is given that the pair (L, A) is weakly compatible then $L(A(w)) = A(L(w)),$ i.e. $Lz = Az$.

Using (3.1.3) putting x = z , y = x_{2n+1} and α = 1, then we get

$$F_{Lz,Mx_{2n+1}}(\mathrm{kt}) \geq \min \left\{ \begin{matrix} F_{Az,Lz}(\mathrm{t}), F_{Sx_{2n+1},Mx_{2n+1}}(\mathrm{t}), \frac{F_{Az,Lz}(\mathrm{t})+F_{Sx_{2n+1},Mx_{2n+1}}(\mathrm{t})}{2}, \\ F_{Sx_{2n+1},Lz}(\mathrm{t}), F_{Az,Mx_{2n+1}}(\mathrm{t}), F_{Az,Sx_{2n+1}}(\mathrm{t}) \end{matrix} \right\},$$

As n → ∞ we have

$$F_{Lz,z}(\mathrm{kt} \geq \min \left\{ F_{z,Lz}(\mathrm{t}), F_{z,z}(\mathrm{t}), \frac{F_{z,Lz}(\mathrm{t})+F_{z,z}(\mathrm{t})}{2}, F_{z,Lz}(\mathrm{t}), F_{z,z}(\mathrm{t}), F_{z,z}(\mathrm{t}) \right\},$$

$$F_{Lz,z}(\mathrm{kt} \geq \min \left\{ F_{z,Lz}(\mathrm{t}), 1, \frac{F_{z,Lz}(\mathrm{t})+F_{z,z}(\mathrm{t})}{2}, F_{z,Lz}(\mathrm{t}), 1,1 \right\},$$

$$F_{Lz,z}(\mathrm{kt} \geq F_{Lz,z}(\mathrm{t}).$$

Thus , by lemma 2.2, z = Lz . Therefore Lz = Az = z.

Now, combine all the results it is clear that z = Az = $Lz = Mz = Sz$.

i.e. the common fixed point . The proof is similar when A(X) is assumed to be a closed subset of X.

Uniqueness: Let $u(u \neq z)$ be another common fixed point of A, L, M and S.

Using (3.1.3) putting x = z , y = u and α = 1, then we get

$$F_{Lz,Mu}(\mathrm{kt}) \geq \min \left\{ \begin{matrix} F_{Az,Lz}(\mathrm{t}), F_{Su,Mu}(\mathrm{t}), \frac{F_{Az,Lz}(\mathrm{t})+F_{Su,Mu}(\mathrm{t})}{2}, \\ F_{Su,Lz}(\mathrm{t}), F_{Az,Mu}(\mathrm{t}), F_{Az,Su}(\mathrm{t}) \end{matrix} \right\},$$

$$F_{z,u}(\mathrm{kt}) \geq \min \left\{ \begin{matrix} F_{z,z}(\mathrm{t}), F_{u,u}(\mathrm{t}), \frac{F_{z,z}(\mathrm{t})+F_{u,u}(\mathrm{t})}{2}, \\ F_{u,z}(\mathrm{t}), F_{z,u}(\mathrm{t}), F_{z,u}(\mathrm{t}) \end{matrix} \right\},$$

$$F_{z,u}(\mathrm{kt} \geq \min\{1,1,1,F_{u,z}(\mathrm{t})\}$$

$$F_{z,u}(\mathrm{kt} \geq F_{z,u}(\mathrm{t}).$$

Thus , by lemma 2.2, z = u and so the uniqueness of the common fixed point .

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