Robust Quaternion LMS Algorithm for Adaptive Filtering of Hyper complex real world processes

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***Abstract-*. Experimental results shows that absolute error is decreases when number of iterations increases for all models among that WL-QLMS has less absolute error compares to other methods. The quaternion least mean square (QLMS) calculation is presented for adaptive shifting of three- and four-dimensional methods, for example, those can be observed in environmental displaying (wind, vector fields). These techniques display complex nonlinear elements furthermore coupling between the measurements, which make their segment knowledge handling by different univariate LMS, bivariate complex LMS (CLMS), or multichannel LMS (MLMS) calculations insufficient. The QLMS represents these issues characteristically, as it is determined straightforwardly in the quaternion space. The dissection demonstrates that QLMS works inherently focused around which is based on "increased" detail, that is, both the covariance and pseudo covariance of the tap data vector are considered.**

***Index Terms*—ℚ Properness, quaternion adaptive filtering, quaternion LMS (QLMS), AQLMS (argumented quaternion least mean square, widely linear model, widely linear quaternion least mean square model.**

I. INTRODUCTION

Mean square error (MSE) is the major problem in communication system. To reduce MSE we go for filtering in this we are describing adaptive filter due to simplicity and less cost. Least mean square(LMS) algorithm is first proposed and has been at the core of adaptive filtering applications [1], [2], and its online adaptive mode of operation makes it suited for the processing of non stationary real world signals. Due attractive property of adaptive filters , it have led to its applications in noise reduction, radar/sonar signal processing, channel equalization for cellular mobile phones, echo cancelation, and low delay speech coding [3].

The LMS update can be expressed as

$$w(n+1) = w(n) + \mu e(n)x(n) \qquad (1)$$

Where $w(n)$, $e(n)$, μ and x(n) denote, respectively, the adaptive weight vector, instantaneous output error, step size, and theinput data vector of length L.

To process *bi-variate* signals, the LMS algorithm was extended to the complex domain [9]. Recently,Mandic*et al.* have exploited the bivariate model of wind [10] in this context; this was achieved by using the so-called augmented statistics [11]. As the quaternion domain represents an extension of the complex field, it is natural to ask whether we can extend the class of LMS algorithms to cater for adaptive filtering of three- and four- dimensional (hyper-complex) signals.

Quaternion's can be regarded as a non commutative extension of complex numbers, and comprise at most four variables [12], [13]. A quaternion variable $q \in$ Hwhich has a real/scalar part $\Re\{q\}$ (denoted with subscript a) and a vector part $\Im\{q\}$ comprising of three imaginary parts (denoted with subscripts b ,c , and d), can be expressed as

$$\begin{aligned} q &= [\Re\{q\}, \Im\{q\}] = [q_a, q] \in \mathrm{H} \\ &= [q_a, (q_b, q_c, q_d)] \\ &= q_a + q_b t + q_c j + q_d\, k \{q_a, q_b, q_c, q_d \in R\} \end{aligned} \qquad (2)$$

Quaternions have been used for more than 150 years (conceived by Hamilton in 1843) and have found applications in computer graphics, for the modeling of three-dimensional (3-D) rotations [14], in robotics [15], and molecular modeling [16].Although the standard least squares problem has also been addressed in the quaternion domain [16], [25], [26], adaptive filtering algorithms for the processing of quaternion valued signals are lacking. The recent progress in technology, environmental sciences, robotics, and biomedicine, has highlighted the need for adaptive filtering of several important classes of multidimensional signals, for instance, 3-D wind field measured by three axis anemometers. By processing those data directly in the multidimensional domain where they reside, we can exploit the correlation and coupling between each dimension and therefore provide enhanced modeling. That is the quaternion least mean square (QLMS) algorithm.

Although these results provide an initial insight into the processing of general quaternionic signals, they are not straightforward to apply in the context of adaptive filtering applications. To this end, we first propose the quaternion widely linear model, specifically designed for the unified modeling of the generality of quaternion signals, both -proper and -improper. The benefits of such an approach are shown to be analogous to the benefits that the augmented statistics provides for complex valued data [7]. Next, the QWL model is incorporated into the quaternion LMS [3] to yield the widely linear QLMS (WL-QLMS), and its theoretical and practical advantages are demonstrated through analysis and simulations.

This paper is organized as follows. Section II presents the quaternion algebra and statistics After that, Section III presents the QLMS and its varients AQLMS Then, Section IV and V introduces the quaternion widely linear model and widely linear quaternion least mean square algorithm Simulation results as highlighted in Section VI and finally, conclusion and future work are discussed in section VII.

***Manuscript received Nov 15, 2014*.**

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ISSN: 2278 – 1323

II QUATERNION ALGEBRA AND STATISTICS

A). Quaternion Algebra

The properties of the orthogonal unit vectors i, j, k, describing the three vector dimensions of a quaternion are

$$ij = k$$
$$jk = i$$
$$ki = j$$
$$ijk = i^2 = j^2 = k^2 = -1 \quad (3)$$

Due to the non commutativity of the quaternion, for example, $ji \neq ij$, instead $ji = -k$. Other elements of quaternion algebra that are used in this work include the multiplication given by

$$q_1 q_2 = [q_a, 1, q_1][q_a, 1, q_2]$$
$$=[q_{a,1} q_{a,2} - q_1 . q_2, q_{a,1} q_2 + q_{a,2} q_1 + q_1 \times q_2] \quad (4)$$

Where$q = q_a + q_b i + q_c i + q_d k = [q_a, q]$, Symbols "**.**" And "x" denote respectively the dot-product and the cross-product, the conjugate of a quaternion $q^* = [q_a, q]^* = [q_a, -q]$, andthe norm $\|q\|_2^2 = qq^*$. Note, that quaternion conjugation is antiinvolution,that is, $(q_1 q_2)^* = q_2{}^* q_1{}^*$. A quaternion is said to bepure, if its real part vanishes.

The quaternion vector space $\mathbb{H}$forms a non commutative or med division algebra, that is

$$q_1 q_2 \neq q_2 q_1$$

B). Quaternion Statistics

The complete second order statistical description in H is obtained when the real valued qua drivariate covariance's are expressed in terms of their quaternion valued counterparts However, unlike the complex domain $\mathbb{C}$, where for this purpose it is sufficient to combine the complex vector z and its conjugate $\boldsymbol{z}^*$ into theaugmented complex vector $\boldsymbol{z}^a = [\boldsymbol{z}^T \boldsymbol{z}^H]^T$, in the quaternion domain.We can therefore build an augmented quaternion vector, comprising of any four of the five quantities$\{\boldsymbol{q}, \boldsymbol{q}^*, \boldsymbol{q}^i, \boldsymbol{q}^j, \boldsymbol{q}^k\}$ or their conjugates. One convenient augmentedbasis is$\boldsymbol{q}^a = \{\boldsymbol{q}, \boldsymbol{q}^*, \boldsymbol{q}^{i*}, \boldsymbol{q}^{j*}\}$ and will be used in this work. Then, theaugmented vector $\boldsymbol{q}^a = [\boldsymbol{q}^T \boldsymbol{q}^H \boldsymbol{q}^{iH} \boldsymbol{q}^{jH}]^T$ contains all the necessarysecond order statistical information and its augmented covariancematrix is given by

$$\mathcal{C}_{\mathbf{q}}^a = E\{\mathbf{q}^a \mathbf{q}^{aH}\} = \begin{bmatrix} \mathcal{C}_{\mathbf{q}} & \mathcal{P}_{\mathbf{q}} & \mathcal{P}_{\mathbf{q}}^i & \mathcal{P}_{\mathbf{q}}^j \\ \mathcal{P}_{\mathbf{q}}^H & \tilde{\mathcal{C}}_{\mathbf{q}} & \tilde{\mathcal{C}}_{\mathbf{q}\mathbf{q}^i} & \tilde{\mathcal{C}}_{\mathbf{q}\mathbf{q}^j} \\ \mathcal{P}_{\mathbf{q}}^{iH} & \tilde{\mathcal{C}}_{\mathbf{q}\mathbf{q}^i}^H & \tilde{\mathcal{C}}_{\mathbf{q}^i} & \tilde{\mathcal{C}}_{\mathbf{q}^i\mathbf{q}^j} \\ \mathcal{P}_{\mathbf{q}}^{jH} & \tilde{\mathcal{C}}_{\mathbf{q}\mathbf{q}^j}^H & \tilde{\mathcal{C}}_{\mathbf{q}^i\mathbf{q}^j}^H & \tilde{\mathcal{C}}_{\mathbf{q}^j} \end{bmatrix}. \quad (5)$$

The sub matrices in (5) are calculated according to

$$\mathcal{C}_\alpha = E\{\alpha\alpha^H\} \hat{\mathcal{C}}_\alpha = E\{\alpha^* \alpha^T\} \hat{\mathcal{C}}_{\alpha\beta} = E\{\alpha^* \beta^T\}$$
$$\mathcal{P}_\alpha = E\{\alpha\alpha^T\} \quad \alpha, \beta \in \{\boldsymbol{q}, \boldsymbol{q}^i, \boldsymbol{q}^j\}$$

We refer to $\hat{\mathcal{C}}_\alpha$ as the quasi-covariance and $\hat{\mathcal{C}}_{\alpha\beta}$ as the cross-quasicovariancematrices.

B.1:Circularity in$\mathbb{H}$ and $\mathbb{Q}$ Properness

For a quaternion valued variable to be second order circular, its probability distribution should be rotation-invariant with respect to the six pairs of axes {1,i}, {1,j}, {1,k}, {i,j}, {k,j} and {k,i}, where '1' represents the real axis and i,j,k denote the corresponding imaginary axes. In other words, a $\mathbb{Q}$ – proper quaternionrandom variable should satisfy the following properties [8]:

$$P1: \; E\{q_\delta^2\} = E\{q_\epsilon^2\} = \sigma^2 \quad \forall\, \delta, \varepsilon = a, b, c, d$$
$$P2: \; E\{q_\delta q_\epsilon\} = 0 \quad \forall\, \delta, \varepsilon = a, b, c, d \text{ and } \delta \neq \epsilon$$
$$P3: \; E\{qq\} = -2E\{q_\delta^2\} = -2\sigma^2 \quad \forall\, \delta = a, b, c, d$$
$$P4: \; E\{|q^2|\} = 4E\{q_\delta^2\} = 4\sigma^2 \quad \forall\, \delta = a, b, c, d \quad (6)$$

he first property, P1, states that all the four-signal components of a quaternion valued variable have equal variances. The property P2 implies that the components of q are not correlated. Property P3 suggests that the pseudo covariance matrix of $\mathbb{Q}$ -proper signals does not vanish, in contrast to the complex case. Finally, the fourth property states that the power of a quaternion variable is a sum of the powers of the signal components. Observe that properties P1 and P2 imply properties P3and P4.

B.2: Augmented Statistics of $\mathbb{Q}$ -Proper Variables

Notice that $\mathbb{Q}$ -properness also implies that the quaternion vector q is uncorrelated with the vector involutions $\boldsymbol{q}^i, \boldsymbol{q}^j, \boldsymbol{q}^k$ that is,1

$$E\{\mathrm{qq}^{iH}\} = 0 \quad E\{\mathrm{qq}^{jH}\} = 0 \quad E\{\mathrm{qq}^{kH}\} = 0 \quad (7)$$

This simplifies the structure of the augmented covariance matrix $\mathcal{C}_q{}^a$ of a $\mathbb{Q}$ -proper random vector, as

$$\mathcal{C}_{\mathbf{q}}^a = \begin{bmatrix} \mathcal{C}_{\mathbf{q}} & \mathcal{P}_{\mathbf{q}} & \mathcal{P}_{\mathbf{q}}^i & \mathcal{P}_{\mathbf{q}}^j \\ \mathcal{P}_{\mathbf{q}} & \mathcal{C}_{\mathbf{q}} & 0 & 0 \\ \mathcal{P}_{\mathbf{q}}^i & 0 & \mathcal{C}_{\mathbf{q}_\mathrm{I}} & 0 \\ \mathcal{P}_{\mathbf{q}}^j & 0 & 0 & \mathcal{C}_{\mathbf{q}} \end{bmatrix} = 2\sigma^2 \begin{bmatrix} 2\mathbf{I} & -\mathbf{I} & \mathbf{I} & \mathbf{I} \\ -\mathbf{I} & 2\mathbf{I} & 0 & 0 \\ \mathbf{I} & 0 & 2\mathbf{I} & 0 \\ \mathbf{I} & 0 & 0 & 2\mathbf{I} \end{bmatrix} \quad (8)$$

that is, the cross-quasi covariance matrices $\hat{\mathcal{C}}_{\alpha\beta}$ all vanish.

III DERIVATION OF THE QLMS AND ITS VARIANT AQLMS

Based on the quaternion algebra, and standard stochastic gradient approximation, we shall now derive the quaternion LMS (QLMS) algorithm for quaternion-valued linear adaptive finite impulse response (FIR) filters.

A). The Quaternion LMS

The same real-valued quadratic cost function (the quaternion norm) as in LMS and CLMS is used, that is

$$\mathcal{J}(n) = e(n)e^*(n)$$
$$= e_a^2(n) + e_b^2(n) + e_c^2(n) + e_d^2(n) \quad (9)$$

Where the error $e(n) = d(n) - w^T(n)x(n)$ with d(n), $w(n)$and x(n) denoting respectively the desired signal, the adaptiveweight vector, and the filter input. Symbols $(\cdot)^T, (\cdot)^H and (\cdot)^*$ denote respectively the transpose, Hermitian, and quaternionconjugate operator. Based on the cost function (9), within the steepest descent optimization, the following gradients need to be calculated

ISSN: 2278 – 1323

$$\nabla_w\big(e(n)e^*(n)\big) = \nabla_{wa}\big(e(n)e^*(n)\big) + \nabla_{wb}\big(e(n)e^*(n)\big)i + \nabla_{wc}\big(e(n)e^*(n)\big)j + \nabla_{wd}\big(e(n)e^*(n)\big)k \quad (10)$$

Where $w = w_a + w_b i + w_c j + w_d k$ and

$$\nabla_{wa}\big(e(n)e^*(n)\big) = e(n)\nabla_{wa}\big(e^*(n)\big) + \nabla_{wa}\big(e(n)\big)e^*(n)$$
$$\nabla_{wb}\big(e(n)e^*(n)\big) = e(n)\nabla_{wb}\big(e^*(n)\big) + \nabla_{wb}\big(e(n)\big)e^*(n)$$
$$\nabla_{wc}\big(e(n)e^*(n)\big) = e(n)\nabla_{wc}\big(e^*(n)\big) + \nabla_{wc}\big(e(n)\big)e^*(n)$$
$$\nabla_{wd}\big(e(n)e^*(n)\big) = e(n)\nabla_{wd}\big(e^*(n)\big) + \nabla_{wd}\big(e(n)\big)e^*(n) \quad (11)$$

Subsequently, the update of the adaptive weight vector of QLMS can be expressed as

$$w(n+1) = w(n) + \mu\big(2e(n)x^*(n) - x^*(n)e^*(n)\big) \quad (12)$$

Due to the non commutativity of the quaternion product, there are two terms within the gradient of the cost function, that is, $2e(n)x^*(n) and x^*(n)e^*(n)$. The QLMS update includes the term $\mu e(n)x^*(n)$which is similar to that within the complexLMS [9], together with an additional term$x^*(n)e^*(n)$ which is specific to the quaternion domain. To answer whether QLMS simplify exactly into CLMS when quaternion-valued signals are limited to two dimensions, let the imaginary parts j and k of quaternion's x(n) and e(n) vanish. The QLMS for such a case simplifies into

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu([\boldsymbol{e_a}(n)\boldsymbol{x_a}(n) + 3\boldsymbol{e_b}(n)\boldsymbol{x_b}(n)] + i[-\boldsymbol{e_a}(n)\boldsymbol{x_b}(n) + 3\boldsymbol{e_b}(n)\boldsymbol{x_a}(n)]) \quad (13)$$

A comparison with the CLMS update

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu([\boldsymbol{e_a}(n)\boldsymbol{x_a}(n) + \boldsymbol{e_b}(n)\boldsymbol{x_b}(n)] + i[-\boldsymbol{e_a}(n)\boldsymbol{x_b}(n) + \boldsymbol{e_b}(n)\boldsymbol{x_a}(n)]) \quad (14)$$

shows that QLMS does not simplify exactly into CLMS, highlighting the direct multidimensional mode of operation.

The non commutativity of quaternion product make their algebraic manipulation demanding. One way to circumvent this problem is to treat the real/scalar $\Re\{.\}$and the vector part $\Im\{.\}$of a quaternion separately, similarly to [42]. The analysis willbe based on the following two observations:

1).Property 1:

$$y = -y^* \;\; iff \quad \Re\{y\} = 0 \quad (15)$$

2).Property 2:

$$y = y^* \;\; iff \quad \Im\{y\} = 0 \quad (16)$$

To analyze the QLMS algorithms, we shall now make the standard assumption in adaptive filtering that $d(n) = w_{opt}^T x(n)$. Following the standard analysis of the convergence in the mean [43], the weight error vector is defined as

$$v(n) = w(n) - w_{opt} \quad (17)$$

Where w_{opt} is the optimal weight vector, while the error e(n) between the desired signal d(n) and its estimate y(n) is given by

$$e(n) = d(n) - y(n) = w_{opt}^T\; x(n) - w^T(n)x(n) = \big(w_{opt}^T - w^T(n)\big)x(n) = -v^T(n)x(n) \quad (18)$$

A.1: Analysis of QLMS

From the QLMS update (9), the real part can be computed as

$$\Re\{w(n+1)\} = \Re\{w(n)\} + \Re\{\mu\big(2e(n)x^*(n) - x^*(n)e^*(n)\big)\} \quad (19)$$

it can be shown that (19) is equivalent to

$$\Re\{w(n+1)\} = \Re\{w(n)\} + 2\Re\{\mu\big(e(n)x^*(n)\big)\} - \Re\{\,\mu(e(n)x(n)\} \quad (20)$$

Substitute (18) into (20) to yield

$$\Re\{\boldsymbol{w}(n+1)\} = \Re\{\boldsymbol{w}(n)\} - 2\Re\{\mu\, \boldsymbol{v^T}(n)\boldsymbol{x}(n)\boldsymbol{x}^*(n)\} + \Re\{\,\mu v^T(n)x(n)x(n)\} = \Re\{\boldsymbol{w}(n)\} - 2\Re\{\mu\, (\boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^H(n))^T\} + \Re\left\{\mu\left(\boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^T(n)\right)^T\right\} \quad (21)$$

Subtract w_{opt} from both sides of (21) to give

$$\Re\{\boldsymbol{v}(n+1)\} = \Re\{\boldsymbol{v}(n)\} - 2\Re\{\mu\, (\boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^H(n))^T\} + \Re\left\{\mu\left(\boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^T(n)\right)^T\right\}$$

$$\Re\{\boldsymbol{v^T}(n+1)\} = \Re\{\boldsymbol{v^T}(n)(I + \mu[\boldsymbol{x}(n)\boldsymbol{x}^T(n) - 2\boldsymbol{x}(n)\boldsymbol{x}^H(n)]\} \quad (22)$$

From (22), we can see that in terms of statistics, QLMS includes both the pseudo covariance $P_{xx} = E\{XX^T\}$andthe covariance $C_{xx} = E\{XX^H\}$. This is a major differenceas compared with CLMS, and therefore, it is expectedthat the QLMS and augmented QLMS will have similarperformance.

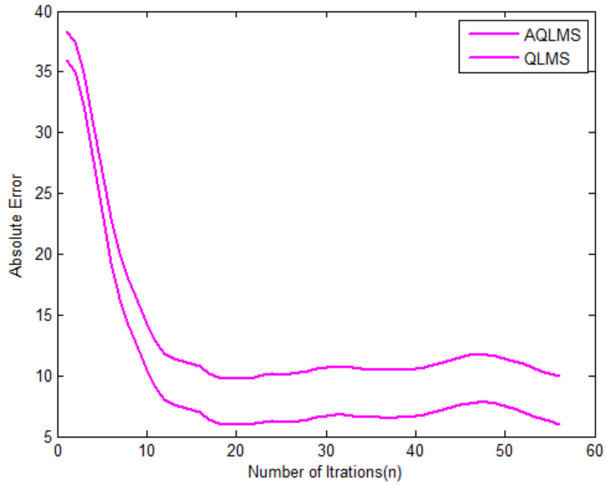

Fig 1 : Comparison Between 3D WINDQLMS and AQLMS

The vector part $\Im\{.\}$of the QLMS update (9) can be analyzedusing Property 1 and (20), that is

$$\Im\{w(n+1)\} = \Im\{w(n)\} + \Im\{\mu\big(2e(n)x^*(n) - x^*(n)e^*(n)\big)\} \quad (23)$$

$$= \Im\{w(n)\} + \Im\{2\mu\big(e(n)x^*(n)\big)\} + \Im\{\,\mu(e(n)x(n)\} = \Im\{\boldsymbol{w}(n)\} - \Im\left\{\mu\left(\boldsymbol{2v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^H(n)\right)^T\right\} - \Im\left\{\mu\left(\boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^T(n)\right)^T\right\} \quad (24)$$

Yielding

$$\Im\{\boldsymbol{w^T}(n+1)\} = \Im\{\boldsymbol{w^T}(n)\} - \Im\{\mu\, \boldsymbol{2}\, \boldsymbol{v^T}(n)\boldsymbol{x}(n)\boldsymbol{x}^H(n)\} - \Im\{\,\mu\, \boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^T(n)\} \quad (25)$$

Subtract w_{opt} from both sides of (25) to give

$$\Im\{\boldsymbol{v^T}(n+1)\} = \Im\{\boldsymbol{v^T}(n)\} - 2\Im\{\mu\, \boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^H(n)\} - \Im\{\,\mu\, \boldsymbol{v}^T(n)\boldsymbol{x}(n)\boldsymbol{x}^T(n)\}$$

$$\Im\{\boldsymbol{v^T}(n+1)\} = \Im\{\boldsymbol{v^T}(n)(\boldsymbol{I} + \mu[-\boldsymbol{x}(n)\boldsymbol{x}^T(n) - 2\boldsymbol{x}(n)\boldsymbol{x}^H(n)]\} \quad (26)$$

Again, both the pseudo covariance and the covariance estimates are involved in the weight update of QLMS. Fig 1 : shows the Comparison Between 3D WIND QLMS and AQLMS This indicates that the "augmented" statistics is inherent to the QLMS, which is a unique property of this class of algorithms.

ISSN: 2278 – 1323

B. The Augmented QLMS (AQLMS)

The augmented QLMS algorithm, which is capable of dealing with the generality of quaternion data. It is usually assumed that the statistics in are a simple extension of the statistics in , obtained by replacing the operator by the operator in the corresponding second order statistical moments. For example, the covariance in the real domain is replaced by in the complex domain. The use of augmented statistics is crucially important when processing noncircular complex signals; non circularity or improperness is a second order statistical property, which can be defined as

$$C_{xx} = E\{XX^H\} \neq 0 \qquad P_{xx} = E\{XX^H\} \neq 0 \qquad (27)$$

that is, the pseudo covariance does not vanish for a noncircular complex signal.3 In the context of quaternion statistics, properness (known as -properness) is defined as the invariance of the probability density function (pdf) under some special angle rotations. More specifically, the pseudo-covariance does not vanish even for a -proper signal.4 Motivated by the augmented CLMS [28], augmented CRTRL [40], and augmented statistics for wind profile [8], we now investigate the benefits of including the pseudo covariance into the QLMS algorithm .In order for QLMS to cater for *general* quaternion processes, we employ a quaternion-valued widely linear model [37], given by

$$y(n) = w^T(n)\,x(n) + g^T(n)x^*(n) \qquad (28)$$

This model incorporates both the information contained in the covariance and pseudo covariance (for more detail, see [27]). For the quaternion scenario, the update for vector in (28) can be found similarly to that for QLMS, and is given by

$$g(n+1) = g(n) + \mu\big(2e(n)x(n) - x(n)e^*(n)\big) \qquad (29)$$

Again, the non commutativity of the quaternion products must be taken into account during the derivation of the update (29).Finally, (27) and (29) can be combined into a compact "augmented" form as

$$h^a(n) = [w^T(n)g^T(n)\,]^T \qquad (30)$$

and the weight update of the augmented QLMS (AQLMS), can be expressed as

$$h^a(n+1) = h^a(n) + \mu\,[2e^a(n)x^{a*}(n) - x^{a*}(n)e^{a*}(n)] \quad (31)$$

Where the augmented error and input vector are given by

$$e^a(n) = d(n) - h^{a^T}(n)x^a(n)x^a(n) = [x^T(n)x^H(n)]^T \qquad (32)$$

B.1: Analysis of AQLMS

To investigate statistical properties of AQLMS, define the error e(n) in terms of the "augmented" weight error vectors and to give

$$e(n) = d(n) - [w^T(n)x(n) + g^T(n)x^*(n)]$$

$$= \big[w_{opt}^T\ x(n) + g_{opt}^T x^*(n)\big] - [w^T(n)x(n) + g^T(n)x^*(n)] = -\big[v_w^T(n)x(n) + v_g^T x^*(n)\big] \qquad (33)$$

where and . Based on Property 2, the real/scalar part of the AQLMS update of (27) can now be written as

$$\Re\{w(n+1)\} = \Re\{w(n)\} + \mu\Re\{e(n)[2x^*(n) - x(n)]\} \qquad (34)$$

Replace the error e(n) with its augmented counterpart, to give

$$\Re\{w(n+1)\} = \Re\{w(n)\} - \mu\Re\big\{[\boldsymbol{v}_w^T(n)\,x(n) + \boldsymbol{v}_g^T(n)x^*(n)\big][2x^*(n) - x(n)]\}$$

$$= \Re\{w(n)\} - \mu\Re\big\{[\boldsymbol{v}_w^T(n)\,x(n)[2x^*(n) - x(n)]\} - \mu\Re\{\boldsymbol{v}_g^T(n)x^*(n)[2x^*(n) - x(n)\big]\} \qquad (35)$$

Subtract $w_{opt}(n)$ from both sides of (35) to obtain

$$\boldsymbol{\Re}\{\boldsymbol{v_w}^T(n+1)\} = \boldsymbol{\Re}\{\boldsymbol{v_w}^T(n)[I - 2\mu\boldsymbol{x}(n)\boldsymbol{x}^H(n) - \mu\boldsymbol{x}(n)\boldsymbol{x}^T(n)]\} - \mu\boldsymbol{\Re}\{\boldsymbol{v_g}^T(n)[2\boldsymbol{x}^*(n)\boldsymbol{x}^H(n) - \boldsymbol{x}^*(n)\boldsymbol{x}^T(n)]\} \qquad (36)$$

$$\boldsymbol{\Re}\{\boldsymbol{v}_g^T(n+1)\} = \boldsymbol{\Re}\{\boldsymbol{v_g}^T(n)[I - 2\mu\boldsymbol{x}^*(n)\boldsymbol{x}^T(n) - \mu\boldsymbol{x}^*(n)\boldsymbol{x}^H(n)]\} \quad (37) \quad - \mu\boldsymbol{\Re}\{\boldsymbol{v_W}^T(n)[2\boldsymbol{x}(n)\boldsymbol{x}^T(n) - \boldsymbol{x}(n)\boldsymbol{x}^H(n)]\}$$

$$\Im\{\boldsymbol{v}_w^T(n+1)\} = \Im\{\boldsymbol{v_w}^T(n)[\boldsymbol{I} - 2\mu\boldsymbol{x}(n)\boldsymbol{x}^H(n) - \mu\boldsymbol{x}(n)\boldsymbol{x}^T(n)]\} - \mu\boldsymbol{\Im}\{\boldsymbol{v_g}^T(n)[2\boldsymbol{x}^*(n)\boldsymbol{x}^T(n) + \boldsymbol{x}^*(n)\boldsymbol{x}^H(n)]\} \qquad (38)$$

$$\boldsymbol{\Im}\{\boldsymbol{v}_g^T(n+1)\} = \boldsymbol{\Im}\{\boldsymbol{v_g}^T(n)[I - 2\mu\boldsymbol{x}^*(n)\boldsymbol{x}^T(n) - \mu\boldsymbol{x}^*(n)\boldsymbol{x}^H(n)]\} \quad (39) \quad - \mu\boldsymbol{\Im}\{\boldsymbol{v_W}^T(n)[2\boldsymbol{x}(n)\boldsymbol{x}^T(n) + \boldsymbol{x}(n)\boldsymbol{x}^H(n)]\}$$

IV THE QUATERNION WIDELY LINEAR MODEL

To account for the complete second order statistics of quaternion valued signals in mean-squared error (MSE) estimation, we need to introduce a filtering model corresponding to the widely linear model in the complex case [4]. Consider the MSE estimator of a signal y in terms of another observation x, that is, $\hat{y} = E\left[\frac{y}{x}\right]$. For zero mean,jointly normal real y and x , the solution is

$$\hat{y} = h^T x \qquad (40)$$

In the quaternion domain, however, the real estimator (40) applies to each component (the real and the three imaginary parts) of quaternion variables, that is

$$\hat{\boldsymbol{y}}_\gamma = E\big[\boldsymbol{y}_\gamma | x_a, x_b, x_c, x_d\big], \quad \gamma \in \{a, b, c, d\}$$

and thus

$$\hat{y} = E[\boldsymbol{y}_a|x_a, x_b, x_c, x_d] + iE[\boldsymbol{y}_b|x_a, x_b, x_c, x_d] + jE[\boldsymbol{y}_c|x_a, x_b, x_c, x_d] + kE[\boldsymbol{y}_d|x_a, x_b, x_c, x_d] \qquad (41)$$

Upon employing the identities (5), it is clear that the quaternion estimator can also be expressed as

$$\hat{\boldsymbol{y}} = E[y|x, x^*, x^{i*}, x^{j*}] + iE[y^i|x, x^*, x^{i*}, x^{j*}] + jE[y^j|x, x^*, x^{i*}, x^{j*}] + kE[y^k|x, x^*, x^{i*}, x^{j*}] \qquad (42)$$

and thus arrive at the widely linear estimator for general quaternion signals

$$y = h^H x + g^H x^* + U^H x^* + V^H x^{j*} = w^{aH} x^a \qquad (43)$$

where $w^a = [h^T g^T u^T v^T]^T$ and $x^a = [x\, x^H x^{iH} x^{jH}]^T$

Following on the proposed quaternion widely linear model, the Wienersolution is now derived as the optimal MSE estimator. Consider thestandard real valued quadratic cost function, that is,

$$\mathcal{J} = E\{ee^*\} = E\{(d-y)(d^* - y^*)\} = E\{dd^*\} + E\{yy^*\} - E\{dy^*\} - E\{yd^*\} \qquad (44)$$

The derivative of the cost function (44) can be expressed as

ISSN: 2278 – 1323

$$\nabla_{w^a} \mathcal{J} = E\{(\nabla_{w^a} y)y^* + y(\nabla_{w^a} y^*) - (\nabla_{w^a} y)d^* - d(\nabla_{w^a} y^*) \quad (45)$$

$$= \underbrace{E\{4[x^a y^* - x^a d^*]\}}_{I} + \underbrace{E\{2[x^a d(n) - y x^a]\}}_{II}$$

To obtain the Wiener solution, the expectations of I and II in (44) are set to zero. In the complex domain, we can sum up the terms I and II in(45); however, due to the non commutativity of the quaternion product, we need to consider the terms in (45) individually, giving the solution

$$\text{I:} \ w_0 = E\{x^a \ x^{aH}\}^{-1} E\{x^a \ d^*\} \quad (46)$$

$$\text{II:} \ w_0 = E\{x^{a*} x^{aH}\}^{-1} E\{x^{a*} d^*\} \quad (47)$$

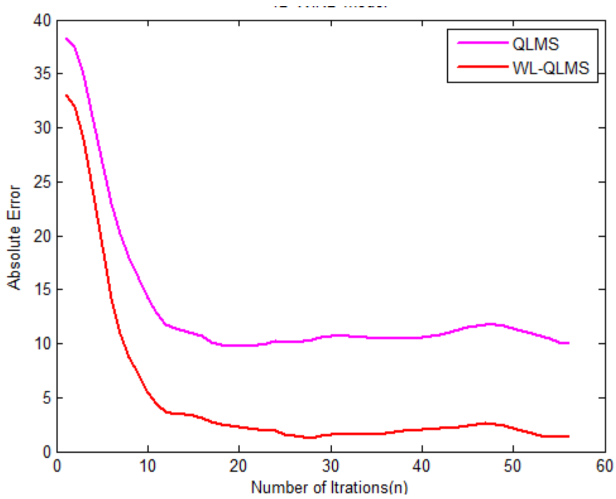

Fig 2 : comparison Between 3D WIND QLMS and WL - QLMS

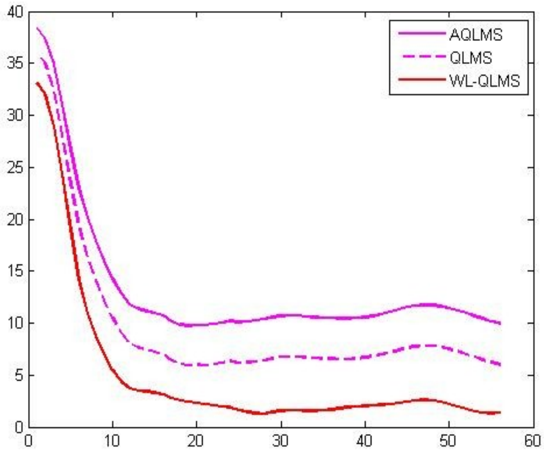

Fig 3: comparison Between3D WIND QLMS, AQLMS and WL – QLMS

The first condition for the Wiener solution (46) requires the inversion of the augmented covariance $C_x^a = E\{x^a x^{aH}\}$. On the other hand, the second condition (47) also depends on the conjugate of pseudo covariance matrix of the augmented vector x^a, which conforms with theobservation in [3] that the quaternion domain accounts inherently forthe information contained in pseudocovariance.Fig 2 and Fig 3 shows comparison Between *3D WIND* QLMS, AQLMS and WL – QLMS algorithms

V. THE WIDELY LINEAR QUATERNION LEAST MEAN-SQUARE ALGORITHM

We now extend some recent results in quaternion adaptive filtering [3], and employ the quaternion widely linear model (43) within the stochastic gradient adaptive filtering framework in$\mathbb{H}$, to propose the widely linear quaternion least mean-square (WL-QLMS) adaptive filtering algorithm. Within the stochastic gradient descent optimization, the gradient of the instantaneous cost function (45) is

$$\nabla_{w^a} \mathcal{J}(n) = e(n)(\nabla_{w^a} e^*(n)) + e^*(n)(\nabla_{w^a} e(n)) \quad (48)$$
$$= 2e(n)x^a(n) - 4x^a(n)e^*(n)$$

Notice that due to the non-commutativity of the quaternion product, the two error gradient terms in (48) need to be treated separately. Based on the generic stochastic gradient update $\Delta w^a = -\mu \nabla_{w^a} \mathcal{J}(n)$the updateof the weight vector of the WL-QLMS algorithm can be obtainedas

V SIMULATION RESULTS

Since prediction is at the core of adaptive filtering, our simulation was conducted in the prediction setting, for -step ahead prediction. For a quantitative assessment of the prediction performance, we employ the prediction gain$\boldsymbol{R_p}$ [45], given by

$$\boldsymbol{R_p} = 10log\,\frac{\sigma_x^2}{\sigma_e^2}(dB) \quad (49)$$

VII CONCLUSION

This contribution has discussed quaternion widely linear model (QWL) quaternion least mean square (QLMS) algorithms for enhanced second order estimation of general quaternion signal. Finally, we have demonstrated the efficacy of the QWL model, by incorporating it into the quaternion least mean square algorithm(QLMS).to yield the widely linear QLMS (WL-QLMS) algorithm and the argument QLMS(AQLMS) has also been derived by taking into account the so called augmented second order statistics. For rigor, the convergence analysis includes the step size bound and learning curves for both second order circular and noncircular signal and finally compares the result for both methods, absolute error is low in (WL-QLMS)when number of iterations increases

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ISSN: 2278 – 1323