

# Steering Vector and Interference Cancellation of Two-Dimensional Arrays

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**Abstract**— In this paper, the two-dimensional array geometry is demonstrated and the corresponding two-dimensional steering vector is deduced which is necessary in the array processing and beamforming applications. The array performance is demonstrated using this steering vector representation for calculating the array gain and controlling the power pattern. The array is also checked for the capability of null steering and the related gain equations are deduced. The analysis has shown that the maximum number of nulls that the array can provide is given by the multiplication of the two array dimensions or the array size minus 1 which represents the array degree of freedom.

**Index Terms**— Beamforming, Two-dimensional arrays, steering vector.

## I. INTRODUCTION

Antenna arrays are very important in many applications including radar, sonar, mobile communications, satellite communications, terrestrial broadcasting, and even in medical and industrial applications [1-4]. There are many forms of antenna arrays including one-dimensional, two-dimensional, circular and concentric circular [5-11]. The array may be formed by using omnidirectional antennas as in some broadcasting stations or use directional antennas that can be in the form of multiple horn antennas as in satellite communications. The main processing area dealing with antenna array is called array processing in which the antenna elements are usually modeled as isotropic sources. This means that the radiation pattern of the single element is unity in all directions. If the antenna element has some directivity pattern, it will be the same processing techniques that can be used with isotropic elements but adding the effect of element directivity. The overall radiation pattern of the array is then will be multiplied with the element pattern. The widespread use of antenna arrays necessitates the analysis of the array geometry in depth to find the proper representation of the signals collected from the array. Linear one-dimensional arrays has the capability of beamforming in only one plane which is the plane that making angles with the array line which around this array the pattern will be omnidirectional. Some other recent research utilize this feature of the linear one-dimensional arrays in forming conical beams [13] to cover ring-shaped areas from High-Altitude Platforms (HAPs) [14-21].

The circular arrays are also having widespread use especially in Direction-of-Arrival applications (DOA) but suffer from the higher sidelobe levels. Recently, the concentric circular arrays (CCA) have provided superior performance in sidelobe levels and azimuth independent beamforming [5]. The main difficulty in CCA construction is the feeding method of the array which is difficult compared to the two-dimensional planer arrays. Therefore, in this paper, the array structure, beamformer and steering vector, and null capabilities of this array will be demonstrated.

We will start with defining the array geometry and its related equations. The signal impinging on the array will induce some signals at each element where the overall signal vectors is also determined. Then, the weighting vector is introduced to formulate a general equation which is very important for the two-dimensional beamforming applications. Finally, the null steering of the two-dimensional array is demonstrated for the applications where the interfering signal direction is known and need to be cancelled.

The paper is arranged as follows; section II demonstrates the two-dimensional array structure, section III investigates the two-dimensional array beamformer. In section IV, the null steering capability of the array is developed and the array conventional beamformer is shown in section V. Finally, section VI concludes the paper.

## II. TWO-DIMENSIONAL ARRAYS STRUCTURE

Dealing with spatial array processing it is needed to find a definition of the array steering vector, which in our case corresponds to the two-dimensional arrays. This vector is essential in determining the array radiation pattern for different weight vectors for beamforming applications. Now, consider a two-dimensional array resides in the x-y plane as shown in Fig. 1. Assume that the number of elements is  $M$  and  $N$  with inter-element spacing  $d_x$  and  $d_y$  in the x- and y-directions respectively. If a source located at a distance  $r$  from the origin, it will radiate a plane wave which is received at the array at a direction of  $(\theta, \phi)$ . This plane wave will generate a corresponding signal at each antenna element in the array that will differ in phase according to its direction. The array steering or response vector can be derived from the array geometry and the signal direction. Assume that the array beamformer network shown in Fig. 2 is used to generate the desired radiation pattern.

The signal induced at the first element (i.e.  $m = 1, n = 1$ ) will be:

$$x(t) e^{j2\pi f_c(t-\tau)} \quad (1)$$

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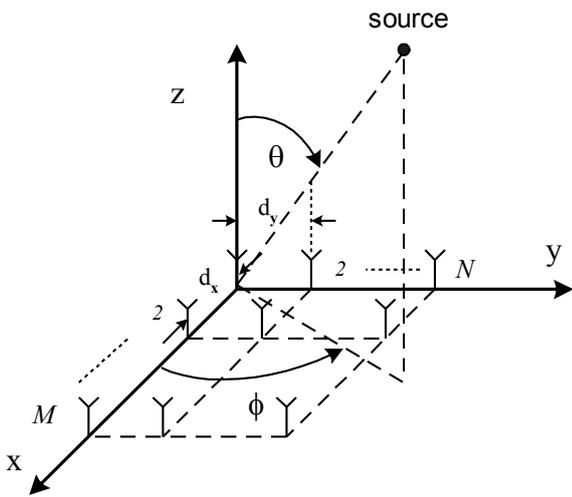


Fig. 1 Two-dimensional array of  $M \times N$  elements and lies in the  $x$ - $y$  plane

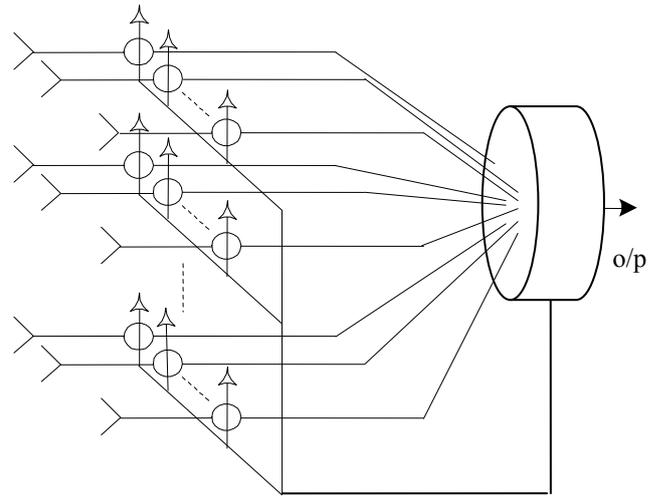


Fig. 1: Two-dimensional arrays beamforming network

where  $x(t)$  is the baseband signal (envelope),  $f_c$  is the carrier frequency, and  $\tau$  is the propagation time from the source to that element. Assuming that the baseband signal is narrowband, we can rewrite the induced signals at the elements ( $m=2, n=1$ ), ( $m=1, n=2$ ), and ( $m=2, n=2$ ) respectively as follows:

$$x(t) e^{j\left(2\pi f_c(t-\tau) + \frac{2\pi}{\lambda} d_x \sin(\theta) \cos(\phi)\right)} \quad (2)$$

$$x(t) e^{j\left(2\pi f_c(t-\tau) + \frac{2\pi}{\lambda} d_y \sin(\theta) \sin(\phi)\right)} \quad (3)$$

$$x(t) e^{j\left(2\pi f_c(t-\tau) + \frac{2\pi}{\lambda} \sqrt{d_x^2 + d_y^2} \sin(\theta) \cos\left(\phi - \tan^{-1}\left(\frac{d_y}{d_x}\right)\right)\right)} \quad (4)$$

The results can be extended the general induced signal at element ( $m=l, n=k$ )

$$x(t) e^{j\left(2\pi f_c(t-\tau) + 2\pi d_{l,k} \sin(\theta) \cos(\phi_{l,k})\right)} \quad (5)$$

where  $d_{l,k}$  and  $\phi_{l,k}$  are given by [20]

$$d_{l,k} = \sqrt{\left((l-1)\frac{d_x}{\lambda}\right)^2 + \left((k-1)\frac{d_y}{\lambda}\right)^2} \quad (6)$$

$$\phi_{l,k} = \phi - \tan^{-1}\left(\frac{(k-1)d_y}{(l-1)d_x}\right) \quad (7)$$

Now, we may write the received signal matrix  $X(t)$  as

$$X(t) = x(t) e^{j2\pi f_c(t-\tau)} A(\theta, \phi) \quad (8)$$

where

$$A(\theta, \phi) = (S_{M1}^T(\theta, \phi), S_{M2}^T(\theta, \phi), \dots, S_{MN}^T(\theta, \phi)) \quad (9)$$

where

$$S_{Mi}(\theta, \phi) = \begin{bmatrix} e^{j(2\pi d_{1,i} \sin(\theta) \cos(\phi_{1,i}))} \\ e^{j(2\pi d_{2,i} \sin(\theta) \cos(\phi_{2,i}))} \\ \cdot \\ \cdot \\ e^{j(2\pi d_{M,i} \sin(\theta) \cos(\phi_{M,i}))} \end{bmatrix}^T \quad (10)$$

where  $S_{Mi}(\theta, \phi)$  is the  $i$ th linear one-dimensional array steering vector.

The two-dimensional array steering vector  $S(\theta, \phi)$  for that array is thus given by

$$S(\theta, \phi) = [S_{M1}(\theta, \phi), S_{M2}(\theta, \phi), \dots, S_{MN}(\theta, \phi)]^T \quad (11)$$

### III. TWO-DIMENSIONAL ARRAYS BEAMFORMER

In the beamformer depicted in Fig. 2, if each induced signal is weighted and then summed to form the beamformer output, then this output may be written as

$$y(t) = \sum_{m=1}^M \sum_{n=1}^N w_{m,n}^* X_{m,n}(t) \quad (12)$$

or

$$y(t) = x(t) e^{j2\pi f_c(t-\tau)} W^H S(\theta, \phi) \quad (13)$$

where \* and H denote the complex conjugate and complex conjugate transpose (Hermitian) operators of the weight value  $w_{m,n}$  and weight vector  $W$  respectively.  $X_{m,n}(t)$  is the received signal at the  $mn^{\text{th}}$  element.

The weight vector is thus written as

$$W = [ w_{1,1} \ w_{2,1} \ \dots \ w_{M,1} \ w_{1,2} \ w_{2,2} \ \dots \ w_{M,2} \ w_{1,3} \ w_{2,3} \ \dots \ w_{M,N} ]^T \quad (14)$$

In practical case, there will be a background noise generated at each antenna element known as thermal noise. This kind of noise is white Gaussian noise and has the Gaussian distribution. So the final output may be written as

$$y(t) = x(t) e^{j2\pi f_c(t-\tau)} W^H S(\theta, \phi) + N(t) \quad (15)$$

$$N(t) = \sum_{m=1}^M \sum_{n=1}^N w_{m,n}^* n_{m,n}(t) \quad (16)$$

where  $N(t)$  is the total background noise at the output of the beamformer and  $n_{m,n}(t)$  is the white noise generated at the  $mn^{\text{th}}$  element. The process can be extended to find the output resulting from several sources or due to several copies of the same source. Each of these signal arrivals has its own corresponding steering vector. Assuming  $L$  different impinging signals on the array, the  $i^{\text{th}}$  corresponding output will be

$$y_i(t) = x_i(t) e^{j2\pi f_c(t-\tau_i)} W^H S(\theta_i, \phi_i) \quad (17)$$

Therefore the output will be

$$y(t) = \sum_{i=1}^L y_i(t) \quad (18)$$

#### IV. NULL STEERING USING TWO-DIMENSIONAL ARRAYS

Another form of (18) can be used as follows:

$$y(t) = W^H G_T X(t) \quad (19)$$

where the signal vector  $X(t)$  is given by

$$X(t) = [ x_1(t) e^{j2\pi f_c(t-\tau_1)} \ x_2(t) e^{j2\pi f_c(t-\tau_2)} \ \dots \ x_L(t) e^{j2\pi f_c(t-\tau_L)} ]^T \quad (20)$$

and the matrix  $G_T$  is given by

$$G_T = [ S(\theta_1, \phi_1), S(\theta_2, \phi_2), \dots, S(\theta_L, \phi_L) ] \quad (21)$$

This is the null steering matrix which can be used to determine the weights that accept signals from certain directions while null the others. The number of received signals may be equal to the number of array elements in order

to obtain the inverse of  $G_T$ , so the weight vector can be written as:

$$W^H = D^T G_T^{-1} \quad (22)$$

where  $D$  is a  $1 \times MN$  vector that determines the desired and undesired components or source arrival directions. If only the first signal is desired while others are to be cancelled, this vector is given by

$$D = [1 \ 0 \ 0 \ \dots \ 0]^T \quad (23)$$

The maximum number of nulls is  $MN - 1$  which determines the degree of freedom of the array.

#### V. CONVENTIONAL BEAMFORMING FOR TWO-DIMENSIONAL ARRAYS

The two-dimensional arrays can be used in several scenarios where it can be simply used as a phased array to a fully adaptive antenna concept. Track each source with a separate beam while monitoring the other co-channel interference sources and null out these interfering signals is a big benefit of adaptive antennas. To demonstrate the array gain by using the steering vector we will concentrate on the conventional beamforming technique [1], which is used in phased arrays. This technique can be modified to suit the coverage requirements. The main advantage of conventional beamforming is that it gives the narrowest beamwidth, largest directivity, and easier hardware implementation. The later advantage arises from the fact that it is only implemented with phase shifters and attenuators. All the magnitudes of the weights are equal, but suffer from the relatively higher sidelobes.

The weight vector in this technique is chosen equal to the steering vector at the desired main lobe direction, i.e.

$$W = S(\theta_o, \phi_o) \quad (24)$$

where  $(\theta_o, \phi_o)$  is the desired main lobe direction.

Assuming narrow band signals, the output can be considered as the signal impinging on the first element multiplied by the array sensitivity  $G(\theta, \phi)$ , i.e.

$$y(t) = x(t) e^{j2\pi f_c(t-\tau)} G(\theta, \phi) + N_b(t) \quad (25)$$

where

$$G(\theta, \phi) = W^H S(\theta, \phi) \quad (26)$$

and in this case is given by

$$G(\theta, \phi) = S^H(\theta_o, \phi_o) S(\theta, \phi) \quad (27)$$

It is clear that the maximum value of the array sensitivity,  $G(\theta, \phi)$ , occurs at  $\theta = \theta_o, \phi = \phi_o$  and equals  $MN$ , so it may be appropriate to normalize the weight vector by  $MN$  to have unity response at the desired main lobe direction, i.e.

$$G_n(\theta, \phi) = \frac{1}{MN} S^H(\theta_o, \phi_o) S(\theta, \phi) \quad (28)$$

The power pattern of the array  $P(\theta, \phi)$  is given by the square of the sensitivity magnitude

$$P(\theta, \phi) = |S(\theta_o, \phi_o)^H S(\theta, \phi)|^2 \quad (29)$$

and the normalized power pattern is given by

$$P_n(\theta, \phi) = \frac{1}{(MN)^2} |S(\theta_o, \phi_o)^H S(\theta, \phi)|^2 \quad (30)$$

Figure 4 depicts a normalized sensitivity pattern of an array of  $10 \times 10$  elements and the main direction is toward the broadside of the array (i.e.  $\theta_o = 0, \phi_o = 0$ ).

The first side lobe level is about 0.2 of the main lobe or it is lower by  $-13.6$  dB in the power pattern, this signifies the higher side lobe level for that arrays. Another point is that there is a back lobe as well as the forward one, which arises from the array symmetry. We utilize only one of them and a ground plane absorbs the other.

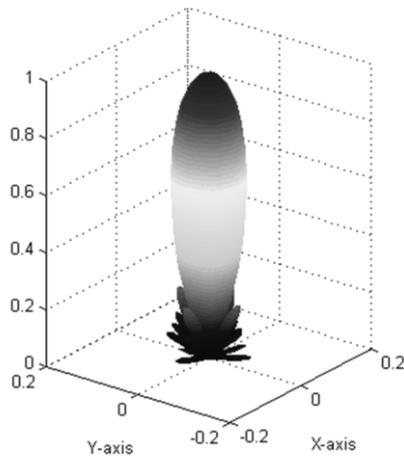


Fig. 4: Normalized array factor for 10x10 array

## VI. CONCLUSION

In this paper, we have demonstrated in analytical fashion the two-dimensional arrays which have many widespread applications. The array structure and the beamforming network for controlling beam directions are investigated where the steering vector is deduced. The array is also analyzed for the null steering of jamming or co-channel interfering signal by pre-information about the interference direction-of-arrival. The reviewed equations in this paper provide a good basis for array processing of two-dimensional arrays.

## REFERENCES

- [1] L. Godara, "Application of antenna arrays to mobile communications, Part II: beam-forming and direction of arrival considerations", *Proceeding of IEEE*, Vol. 85, No. 8, pp. 1195-1245, Aug. 1997.
- [2] Y. Albagory and A. Ashour, "MUSIC 2D-DOA Estimation using Split Vertical Linear and Circular Arrays," *International Journal of Computer Network and Information Security (IJCNIS)*, vol. 5, p. 12, 2013.
- [3] A. Ashour and Y. Albagory, "LPA 2D-DOA Estimation for Fast Nonstationary Sources Using New Array Geometry Configuration," *International Journal of Computer Network & Information Security*, vol. 5, No. 11, pp. 1-8, 2013.
- [4] Y. A. Albagory, "Performance of 2-D DOA Estimation for Stratospheric Platforms Communications," *Progress In Electromagnetics Research M*, vol. 36, pp. 109-116, 2014.
- [5] M. I. Dessouky, H. A. Sharshar, and Y. A. Albagory, "Efficient sidelobe reduction technique for small-sized concentric circular arrays," *Progress In Electromagnetics Research*, vol. 65, pp. 187-200, 2006.
- [6] Yasser Albagory, Omar Said, "Optimizing Concentric Circular Antenna Arrays for High-Altitude Platforms Wireless Sensor Networks" , *International Journal of Computer Network and Information Security*, Vol. 6 No. 5, pp. 1-8, 2014.
- [7] Y. A. Albagory, M. Dessouky, and H. Sharshar, "An approach for low sidelobe beamforming in uniform concentric circular arrays," *Wireless Personal Communications*, vol. 43, pp. 1363-1368, Dec 2007.
- [8] M. Dessouky, H. Sharshar, and Y. Albagory, "An approach for Dolph-Chebyshev uniform concentric circular arrays," *Journal of Electromagnetic Waves and Applications*, vol. 21, pp. 781-794, 2007.
- [9] M. Dessouky, H. Sharshar, and Y. Albagory, "Optimum normalized-gaussian tapering window for side lobe reduction in uniform concentric circular arrays," *Progress in Electromagnetics Research-Pier*, vol. 69, pp. 35-46, 2007.
- [10] M. Dessouky, H. Sharshar, and Y. Albagory, "A novel tapered beamforming window for uniform concentric circular arrays," *Journal of Electromagnetic Waves and Applications*, vol. 20, pp. 2077-2089, 2006.
- [11] M. I. Dessouky, H. A. Sharshar, and Y. A. Albagory, "Optimum normalized-Gaussian tapering window for side lobe reduction in uniform concentric circular arrays," *Progress In Electromagnetics Research*, vol. 69, pp. 35-46, 2007.
- [12] E. I. Elsaidy, M. I. Dessouky, S. Khamis, and Y. A. Albagory, "Concentric circular antenna array synthesis using comprehensive learning particle swarm optimizer," *Progress in Electromagnetics Research Letters*, vol. 29, pp. 1-13, 2012.
- [13] Y. Albagory, "An Approach for Designing Flat-Top High-Altitude Platforms Ring-Shaped Cells Using Vertical Linear Array," *Wireless Personal Communications*, vol. 77, pp. 661-673, Jul 2014.
- [14] Y. Albagory, "Sectorized Hamming Concentric Circular Arrays for Stratospheric Platforms Cellular Design," *International Journal of Computer Network and Information Security (IJCNIS)*, Vol. 5 No. 9, pp. 21-27, 2013.
- [15] M. Dessouky, H. Sharshar, and Y. Albagory, "Design of high altitude platforms cellular communications," *Progress in Electromagnetics Research*, vol. 67, pp. 251-261, 2007.
- [16] M. Dessouky, H. Sharshar, and Y. Albagory, "Geometrical analysis of high altitude platforms cellular footprint," *Progress in Electromagnetics Research*, vol. 67, pp. 263-274, 2007.
- [17] Y. Albagory and A. E. Abbas, "Smart cell design for high altitude platforms communication," *Aeu-International Journal of Electronics and Communications*, vol. 67, pp. 780-786, 2013.
- [18] Y. Albagory and F. A. Raddady, "High-Altitude Platforms Cellular System for Sparsely Populated Areas," *International Journal of Computer Network and Information Security (IJCNIS)*, vol. 6, pp. 13-20, 2014.
- [19] B. M. Eldowek, E. T. Michailidis, Y. A. Albagory, M. Abd-Elnaby, E.-S. M. El-Rabaie, M. I. Dessouky, A.-A. T. Shalaby, B. M. Sallam, F. E. Abd El-Samie, and A. G. Kanatas, "Complex Envelope Second-Order Statistics in High-Altitude Platforms Communication Channels," *Wireless Personal Communications*, vol. 77, pp. 2517-2535, Aug 2014.
- [20] Yasser Albagory, Fahad Alradday, "High-Altitude Platforms Cellular System for Sparsely Populated Areas", *International Journal of Computer Network and Information Security*, Vol. 6 No. 4, pp. 13-20, 2014.
- [21] Yasser Albagory, "Flat-Top Ring-Shaped Cell Design for High-Altitude Platform Communications", *International Journal of Computer Network and Information Security*, Vol. 5 No. 7, pp. 51-57, 2013.