Steady State Behavior Of a Network Queue Model Comprised of Two Bi-serial Channels Linked with a Common Server

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Abstract—This paper is an attempt to study the steady-state behavior of a network queuing model in which two bi-serial channels are linked with a common server. The arrivals of service pattern follow Poisson Law. The various queue characteristics such as mean queue length, variance of the queue length have been obtained explicitly by applying Generating function technique and laws of calculus. The model finds an application in decision making in the process industries, in banking and in many administrative steps.

Keywords—Poisson Law, Steady State Behavior, Mean Queue Length, Variance and Bi-serial channels.

I. INTRODUCTION


The present queue model differ the study made by Gupta Deepak, Singh T.P. et.al. in the sense that in this model the first and second both the systems consist of biserial channels and both the systems are linked with a common server. The differential difference equations formed in the model have been solved by using generating function technique, laws of calculus and statistical tools in order to find various queue characteristics such as mean queue length, variance of queue etc.

II. PRACTICAL SITUATION

Many practical situations of the model arise in industries, administrative setup, banking system, computer networks, office management, super-market etc. For example: In a mall, there are three sections one is drink section second is for movie and third is food section. Drink section consists of two subsections one is for soft-drink and other is for hot drink. A person may take the soft drink and directly go to watch the movie . Or the person may also take the hot drink and then go to see the movie. After watching the movie the person may go to the food section to eat food. Similarly Food section consists of two subsections one is Indian food sections and other is Chinese food section . After movie, a person may either go to the Indian food section and leave the queue, or the person may go to the Chinese food section.

Further this type of queue network is widely used for modeling and performance prediction in diverse areas as communication networks and teletraffic, multiprogramming systems, maintenance and repair facilities, production and assembly lines, steel making, air traffic control, urban transportation systems etc.

III. THE PROBLEM

The entire queue model is comprised of three service channels $S_1, S_2$ and $S_3$.

Where subsystem $S_1$ consists of two biserial service channels $s_{11}$ and $s_{12}$ and also $S_3$ contains two biserial channels $s_{31}$ and $s_{32}$. $S_1$ and $S_3$ are linked with a common server $S_2$ in between both. The service time at $S_{ij}$ are distributed exponentially. For convenience, we assume the service rate $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$ for $\mu_{ij}$ at $S_{ij}$ respectively. The customers initially join $S_{ij}$ under poisson assumptions and the mean rate $\lambda_1, \lambda_2$ respectively. Queues are formed in the front of the service channels $S_{11}$ and $S_{12}$ if they are busy. Customers coming at the rate $\lambda_1$ after completion of phase service at $S_{11}$ will join $S_{12}$ or $S_2$ i.e either they may go to the network of server $S_{12}$ with the probability $p_{12}$ or go to the server $S_2$ with the probability $p_{13}$ such that $p_{12} + p_{13} = 1$. And the customers coming at the rate $\lambda_1$ after completion of phase service at $S_{12}$ will join $S_{11}$ or $S_2$ i.e either they may go to the network of server $S_{11}$ with the probability $p_{21}$ or go to the server $S_2$ with the probability $p_{23}$ such that $p_{21} + p_{23} = 1$. After that customer will go from the server $S_2$ either to the server $S_{21}$ with the probability $p_{34}$ or go to the server $S_{22}$ with the probability $p_{35}$ such that $p_{34} + p_{35} = 1$.

![Diagram](image.png)

Fig: 1

IV. MATHEMATICAL ANALYSIS
Define \( P_{n_1,n_2,n_3,n_4,n_5} \) the probability, there are \( n_1 \) units waiting in the queue \( Q_1 \) in front of \( S_{11} \), \( n_2 \) units waiting in the queue \( Q_2 \) in front of \( S_{12} \), \( n_3 \) units waiting in the queue \( Q_3 \) in front of \( S_2 \), and \( n_4 \) units waiting in the queue \( Q_4 \) in front of \( S_{21} \), \( n_5 \) units waiting in the queue \( Q_5 \) in front of \( S_{22} \). In each case the waiting includes a unit in service, if any (\( \lambda, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \)).

Differential Difference Equation in Transient form for the model is:

\[
P_{n_1,n_2,n_3,n_4,n_5} = -(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5) P_{n_1,n_2,n_3,n_4,n_5} + \lambda_1 P_{n_1-1,n_2,n_3,n_4,n_5} + \lambda_2 P_{n_1,n_2-1,n_3,n_4,n_5} + \mu_1 P_{1,n_2,n_3,n_4,n_5} + \mu_2 P_{1,n_1+n_2-1,n_3,n_4,n_5} + \mu_3 P_{n_1,n_2,n_3-1,n_4,n_5} + \mu_4 P_{n_1,n_2,n_3,n_4-1,n_5} + \mu_5 P_{n_1,n_2,n_3,n_4,n_5-1} \]

\[ (n_1, n_2, n_3, n_4, n_5 > 0) \] (1)

Equation in the steady state form:

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5) \sum_{n_1,n_2,n_3,n_4,n_5} P_{n_1,n_2,n_3,n_4,n_5} = 0 \]

\[ (n_1 = 0, n_2, n_3, n_4, n_5 > 0) \] (2)

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4 + \mu_5) \sum_{n_1,n_2,n_3,n_4,n_5} P_{n_1,n_2,n_3,n_4,n_5} = 0 \]

\[ (n_2 = 0, n_1, n_3, n_4, n_5 > 0) \] (3)

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4 + \mu_5) \sum_{n_1,n_2,n_3,n_4,n_5} P_{n_1,n_2,n_3,n_4,n_5} = 0 \]

\[ (n_3 = 0, n_1, n_2, n_4, n_5 > 0) \] (4)

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_5) \sum_{n_1,n_2,n_3,n_4,n_5} P_{n_1,n_2,n_3,n_4,n_5} = 0 \]

\[ (n_4 = 0, n_1, n_2, n_3, n_5 > 0) \] (5)

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\[ \lambda_1 P_{n_1-1,n_2,n_3,n_4,0} + \mu_2 P_{n_1,n_2-1,n_3,n_4,0} + \mu_1 P_{12} P_{n_1+1,n_2-1,n_3,n_4,0} + \mu_2 P_{13} P_{n_1+1,n_2,n_3-1,n_4,0} + \mu_2 P_{23} P_{n_1,n_2+1,n_3,n_4,0} + \mu_2 P_{34} P_{n_1,n_2,n_3+1,n_4,0} + \mu_3 P_{46} P_{n_1,n_2,n_3,n_4+1,0} + \mu_3 P_{54} P_{n_1,n_2,n_3,n_4+1,1} + \mu_3 P_{57} P_{n_1,n_2,n_3,n_5,1} \]

\[ (n_5 = 0, n_1, n_2, n_3, n_4, > 0) \] (6)

\[ (\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \mu_5) P_{0,0,0,0,0,0} \]

\[ = \mu_1 P_{13} P_{1,0,n_3-1,n_4,n_5,0} + \mu_2 P_{23} P_{0,1,n_3-1,n_4,n_5,0} + \mu_3 P_{34} P_{0,0,n_3+1,n_4,n_5,0} + \mu_3 P_{35} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{45} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5+1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

\[ (n_1, n_2 = 0, n_3, n_4, n_5 > 0) \] (7)

\[ (\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \mu_5) P_{0,0,0,0,0,0} \]

\[ = \mu_2 P_{23} P_{0,n_2-1,0,n_4,n_5,0} + \mu_1 P_{12} P_{1,n_2-1,0,n_4,n_5,0} + \mu_2 P_{13} P_{1,n_2,n_3-1,0,n_4,n_5,0} + \mu_2 P_{23} P_{n_1,n_2+1,n_3-1,n_4,n_5,0} + \mu_3 P_{34} P_{n_1,n_2,n_3-1,n_4,n_5-1} + \mu_4 P_{45} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5+1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

\[ (n_1, n_3 = 0, n_2, n_4, n_5 > 0) \] (8)

\[ (\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \mu_5) P_{0,0,0,0,0,0} \]

\[ = \mu_2 P_{23} P_{0,n_2-1,0,n_4,n_5,0} + \mu_1 P_{12} P_{1,n_2-1,0,n_4,n_5,0} + \mu_2 P_{13} P_{1,n_2,n_3-1,0,n_4,n_5,0} + \mu_2 P_{23} P_{n_1,n_2+1,n_3-1,n_4,n_5-1} + \mu_3 P_{34} P_{n_1,n_2,n_3-1,n_4,n_5-1} + \mu_4 P_{45} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5+1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

\[ (n_1, n_4 = 0, n_2, n_3, n_5 > 0) \] (9)

\[ (\lambda_1 + \lambda_2 + \mu_2 + \mu_4 + \mu_5) P_{0,0,0,0,0,0} \]

\[ = \mu_2 P_{23} P_{0,n_2-1,0,n_4,n_5,0} + \mu_1 P_{12} P_{1,n_2-1,0,n_4,n_5,0} + \mu_2 P_{13} P_{1,n_2,n_3-1,0,n_4,n_5,0} + \mu_2 P_{23} P_{0,n_2,n_3+1,n_4-1,0,n_5,0} + \mu_3 P_{34} P_{0,n_2,n_3+1,n_4-1,0,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5-1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

\[ (n_1, n_5 = 0, n_2, n_3, n_4 > 0) \] (10)

\[ (\lambda_1 + \lambda_2 + \mu_2 + \mu_4 + \mu_5) P_{0,0,0,0,0,0} \]

\[ = \mu_2 P_{23} P_{0,n_2-1,0,n_4,n_5,0} + \mu_1 P_{12} P_{1,n_2-1,0,n_4,n_5,0} + \mu_2 P_{13} P_{1,n_2,n_3-1,0,n_4,n_5,0} + \mu_2 P_{23} P_{0,n_2,n_3+1,n_4-1,0,n_5,0} + \mu_3 P_{34} P_{0,n_2,n_3+1,n_4-1,0,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5-1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

\[ (n_2, n_3 = 0, n_1, n_4, n_5 > 0) \] (11)

\[ (\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_5) P_{0,0,0,0,0,0} \]

\[ = \mu_2 P_{23} P_{0,n_2-1,0,n_4,n_5,0} + \mu_1 P_{12} P_{1,n_2-1,0,n_4,n_5,0} + \mu_2 P_{13} P_{1,n_2,n_3-1,0,n_4,n_5,0} + \mu_2 P_{23} P_{0,n_2,n_3+1,n_4-1,0,n_5,0} + \mu_3 P_{34} P_{0,n_2,n_3+1,n_4-1,0,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5-1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

\[ (n_2, n_4 = 0, n_1, n_3, n_5 > 0) \] (12)

\[ (\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_4) P_{0,0,0,0,0,0} \]

\[ = \mu_2 P_{23} P_{0,n_2-1,0,n_4,n_5,0} + \mu_1 P_{12} P_{1,n_2-1,0,n_4,n_5,0} + \mu_2 P_{13} P_{1,n_2,n_3-1,0,n_4,n_5,0} + \mu_2 P_{23} P_{0,n_2,n_3+1,n_4-1,0,n_5,0} + \mu_3 P_{34} P_{0,n_2,n_3+1,n_4-1,0,n_5-1} + \mu_4 P_{46} P_{0,0,n_3,n_4+1,n_5-1} + \mu_4 P_{54} P_{0,0,n_3,n_4+1,n_5-1} + \mu_5 P_{57} P_{0,0,n_3,n_4+1,n_5+1} \]

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\[ (n_2, n_5 = 0, n_1, n_3, n_4 > 0) \quad (13) \]

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_5) \, P_{n_1, n_2, 0, n_3, 0, n_5} \]

\[
= \lambda_1 \, P_{n_1-1, n_2, 0, 0, n_5} + \lambda_2 \, P_{n_1, n_2-1, 0, 0, n_5} + \mu_1 p_{12} \, P_{n_1+1, n_2-1, 0, 0, n_5} + \mu_2 p_{21} \, P_{n_1-1, n_2+1, 0, 0, n_5} + \mu_3 p_{35} \, P_{n_1, n_2-1, 0, 0, n_5} + \mu_4 p_{45} \, P_{n_1, n_2, 0, 1, n_5-1} + \mu_4 p_{46} \, P_{n_1, n_2, 0, 1, n_5} + \mu_5 p_{57} \, P_{n_1, n_2, 0, n_5+1} \]

\[ (n_3, n_4 = 0, n_1, n_2, n_5 > 0) \quad (14) \]

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4) \, P_{n_1, n_2, 0, n_4, 0} \]

\[
= \lambda_1 \, P_{n_1-1, n_2, 0, n_4, 0} + \lambda_2 \, P_{n_1, n_2-1, 0, n_4, 0} + \mu_1 p_{12} \, P_{n_1+1, n_2-1, 0, n_4, 0} + \mu_2 p_{12} \, P_{n_1+1, n_2-1, 0, n_4, 0} + \mu_3 p_{13} \, P_{n_1, n_2, 1, n_4-1, 0} + \mu_4 p_{45} \, P_{n_1, n_2, 0, n_4+1, 0} + \mu_5 p_{54} \, P_{n_1, n_2, 0, n_4-1, 1} + \mu_5 p_{57} \, P_{n_1, n_2, 0, n_4+1} \]

\[ (n_3, n_5 = 0, n_1, n_2, n_4 > 0) \quad (15) \]

\[
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3) \, P_{n_1, n_2, 0, 0, 0, n_5} \]

\[
= \lambda_1 \, P_{n_1-1, n_2, 0, 0, 0, n_5} + \lambda_2 \, P_{n_1, n_2-1, 0, 0, 0, n_5} + \mu_1 p_{12} \, P_{n_1+1, n_2-1, 0, 0, 0, n_5} + \mu_1 p_{13} \, P_{n_1+1, n_2-1, 0, 0, 0, n_5} + \mu_2 p_{21} \, P_{n_1-1, n_2+1, 0, 0, 0, n_5} + \mu_2 p_{12} \, P_{n_1+1, n_2+1, 0, 0, 0, n_5} + \mu_3 p_{13} \, P_{n_1+1, n_2+1, 0, 0, 0, n_5} + \mu_4 p_{45} \, P_{n_1, n_2, 0, n_4-1, 0} + \mu_4 p_{46} \, P_{n_1, n_2, 0, n_4-1, 1} + \mu_4 p_{54} \, P_{n_1, n_2, 0, n_4+1} + \mu_5 p_{57} \, P_{n_1, n_2, 0, n_4+1} \]

\[ (n_4, n_5 = 0, n_1, n_2, n_3 > 0) \quad (16) \]

\[
(\lambda_1 + \lambda_2 + \mu_4 + \mu_5) \, P_{0, 0, 0, n_4, n_5} \]

\[
= \mu_3 p_{34} \, P_{0, 0, 0, n_4-1, n_5} + \mu_3 p_{35} \, P_{0, 0, 0, n_4-1, n_5} + \mu_4 p_{45} \, P_{0, 0, 0, n_4+1, n_5} + \mu_4 p_{46} \, P_{0, 0, n_4-1, n_5} + \mu_5 p_{54} \, P_{0, 0, n_4-1, n_5} + \mu_5 p_{57} \, P_{0, 0, n_4+1, n_5} \]

\[ (n_1, n_2, n_3 = 0 n_4, n_5 > 0) \quad (17) \]

\[
(\lambda_1 + \lambda_2 + \mu_3 + \mu_5) \, P_{0, 0, n_3, n_5} \]

\[
= \mu_1 p_{13} \, P_{0, 0, n_3-1, 0, n_5} + \mu_2 p_{23} \, P_{0, 0, n_3-1, 0, n_5} + \mu_3 p_{35} \, P_{0, 0, n_3+1, 0, n_5} + \mu_4 p_{45} \, P_{0, 0, n_3+1, 0, n_5} + \mu_5 p_{57} \, P_{0, 0, n_3+1, 0, n_5} \]

\[ (n_1, n_2, n_4 = 0 n_3, n_5 > 0) \quad (18) \]

\[
(\lambda_1 + \lambda_2 + \mu_3 + \mu_4) \, P_{0, 0, n_3, 0, n_4} \]

\[
= \mu_1 p_{13} \, P_{0, 0, n_3-1, n_4, 0} + \mu_2 p_{23} \, P_{0, 0, n_3-1, n_4, 0} + \mu_3 p_{34} \, P_{0, 0, n_3+1, n_4, 0} + \mu_4 p_{46} \, P_{0, 0, n_3+1, n_4, 0} + \mu_5 p_{54} \, P_{0, 0, n_3+1, n_4+1} + \mu_5 p_{57} \, P_{0, 0, n_3+1, n_4+1} \]

\[ (n_1, n_2, n_5 = 0 n_3, n_4 > 0) \quad (19) \]

\[
(\lambda_1 + \lambda_2 + \mu_2 + \mu_5) \, P_{0, n_2, 0, 0, n_5} \]

\[
= \mu_2 p_{0, n_2-1, 0, 0, n_5} + \mu_1 p_{12} \, P_{0, n_2-1, 0, 0, n_5} + \mu_3 p_{35} \, P_{0, n_2, 0, n_5-1} + \mu_4 p_{45} \, P_{0, n_2, 0, n_5-1} + \mu_4 p_{46} \, P_{0, n_2, 0, n_5} + \mu_5 p_{57} \, P_{0, n_2, 0, n_5+1} \]

\[ (n_1, n_3, n_4 = 0 n_2, n_5 > 0) \quad (20) \]

\[
(\lambda_1 + \lambda_2 + \mu_2 + \mu_4) \, P_{0, n_2, 0, 0, n_4} \]

\[
= \mu_2 p_{0, n_2-1, 0, n_4, 0} + \mu_1 p_{12} \, P_{0, n_2-1, 0, n_4, 0} + \mu_3 p_{34} \, P_{0, n_2, 0, n_4-1, 0} + \mu_4 p_{46} \, P_{0, n_2, 0, n_4-1, 0} + \mu_5 p_{54} \, P_{0, n_2, 0, n_4-1, 1} + \mu_5 p_{57} \, P_{0, n_2, 0, n_4+1} \]

\[ (n_1, n_3, n_5 = 0 n_2, n_4 > 0) \quad (21) \]
\((\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5) \, P_{n_2,n_3,0,0} \)

\(= \lambda_2 \, P_{1,n_2-1,n_3,0,0} + \mu_1 \, P_{12} \, P_{1,n_2-1,n_3,0,0} + \mu_1 \, P_{13} \, P_{1,n_2,n_3-1,0,0} + \mu_5 \, P_{23} \, P_{0,n_2+1,n_3-1,0,0} + \mu_4 \, P_{46} \, P_{n_1,n_2,n_3,1,0} + \mu_5 \, P_{57} \, P_{0,n_2,n_3,1,0} \)

\((n_1, n_4, n_5 = 0, n_2, n_3 > 0) \quad (22)\)

\((\lambda_1 + \lambda_2 + \mu_1 + \mu_5) \, P_{n_1,0,0,n_5} \)

\(= \lambda_1 \, P_{n_1-1,0,0,n_5} + \mu_2 \, P_{21} \, P_{n_1-1,0,0,n_5} + \mu_3 \, P_{35} \, P_{n_1,0,0,n_5-1} + \mu_4 \, P_{45} \, P_{n_1,0,1,n_5-1} + \mu_5 \, P_{57} \, P_{n_1,0,0,0,n_5+1} \)

\((n_2, n_3, n_4 = 0, n_5 > 0) \quad (23)\)

\((\lambda_1 + \lambda_2 + \mu_1 + \mu_4) \, P_{n_1,0,n_4,0} \)

\(= \lambda_1 \, P_{n_1-1,0,0,n_4,0} + \mu_2 \, P_{21} \, P_{n_1-1,0,0,n_4,0} + \mu_3 \, P_{34} \, P_{n_1,0,1,n_4-1,0} + \mu_4 \, P_{46} \, P_{n_1,0,0,n_4-1,0} + \mu_5 \, P_{57} \, P_{n_1,0,0,n_4,1} \)

\((n_2, n_3, n_5 = 0, n_1 > 0) \quad (24)\)

\((\lambda_1 + \lambda_2 + \mu_1 + \mu_3) \, P_{n_1,n_3,0,0} \)

\(= \lambda_1 \, P_{n_1-1,0,0,n_3,0} + \mu_1 \, P_{12} \, P_{n_1-1,0,0,n_3,0} + \mu_2 \, P_{21} \, P_{n_1-1,1,n_3-1,0,0} + \mu_3 \, P_{35} \, P_{n_1,1,n_3-1,0,0} + \mu_4 \, P_{46} \, P_{n_1,0,n_3,1,0} + \mu_5 \, P_{57} \, P_{n_1,0,n_3,0,1} \)

\((n_2, n_4, n_5 = 0, n_1, n_3 > 0) \quad (25)\)

\((\lambda_1 + \lambda_2 + \mu_1 + \mu_2) \, P_{n_1,n_2,0,0,0} \)

\(= \lambda_1 \, P_{n_1-1,n_2-1,0,0,0} + \mu_2 \, P_{21} \, P_{n_1-1,n_2-1,0,0,0} + \mu_1 \, P_{12} \, P_{n_1,n_2-1,1,0,0,0} + \mu_2 \, P_{21} \, P_{n_1-1,n_2-1,0,0,0} + \mu_4 \, P_{46} \, P_{n_1,1,n_2,n_3,1,0} + \mu_5 \, P_{57} \, P_{n_1,1,n_2,0,0,1} \)

\((n_3, n_4, n_5 = 0, n_1, n_2 > 0) \quad (26)\)

\((\lambda_1 + \lambda_2 + \mu_5) \, P_{n_1,0,0,n_5} \)

\(= \mu_3 \, P_{35} \, P_{0,0,1,n_5-1} + \mu_4 \, P_{45} \, P_{0,0,1,n_5-1} + \mu_4 \, P_{46} \, P_{0,0,0,1,n_5} + \mu_5 \, P_{57} \, P_{0,0,0,0,n_5+1} \)

\((n_1, n_2, n_3, n_4 = 0, n_5 > 0) \quad (27)\)

\((\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5) \, P_{n_1,0,n_4,0} \)

\(= \mu_3 \, P_{34} \, P_{0,0,1,n_4-1,0} + \mu_4 \, P_{46} \, P_{0,0,0,n_4+1,0} + \mu_5 \, P_{54} \, P_{0,0,0,n_4-1,1} + \mu_5 \, P_{57} \, P_{0,0,0,n_4,1} \)

\((n_1, n_2, n_3, n_4 = 0, n_5 > 0) \quad (28)\)

\((\lambda_1 + \lambda_2 + \mu_3) \, P_{n_1,0,n_3,0,0} \)

\(= \mu_1 \, P_{13} \, P_{1,n_3-1,0,0,0} + \mu_2 \, P_{21} \, P_{1,n_3-1,0,0,0} + \mu_4 \, P_{46} \, P_{0,1,n_3,0,0} + \mu_5 \, P_{57} \, P_{0,0,0,n_3,0,1} \)

\((n_1, n_2, n_4, n_5 = 0, n_3 > 0) \quad (29)\)

\((\lambda_1 + \lambda_2 + \mu_2) \, P_{n_1,n_2,0,0,0} \)

\(= \lambda_2 \, P_{0,n_2-1,0,0,0} + \mu_1 \, P_{12} \, P_{0,n_2-1,0,0,0} + \mu_4 \, P_{46} \, P_{0,n_2,0,1,0} + \mu_5 \, P_{57} \, P_{0,n_2,0,0,1} \)

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\begin{align*}
(n_1, n_3, n_4, n_5 &= 0, n_2 > 0) & (30) \\
(\lambda_1 + \lambda_2 + \mu_1) P_{n_1,0,0,0} &= \lambda_1 P_{n_1-1,0,0,0} + \mu_2 p_{21} P_{n_1-1,1,0,0} + \mu_4 p_{46} P_{n_1,0,0,1} + \mu_5 p_{57} P_{n_1,0,0,0} & (31) \\
(\lambda_1 + \lambda_2 + \mu_2) P_{0,0,0,0} &= \mu_4 p_{46} P_{0,0,0,1} + \mu_5 p_{57} P_{0,0,0,0} & (32) \\

\text{Now Define Generating Function as} \\
F(X,Y,Z,R,S) &= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \sum_{n_5=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5} X^{n_1} Y^{n_2} Z^{n_3} R^{n_4} S^{n_5} & (33) \text{ Where } [X], [Y], [Z], [R], [S] = 1 \\

\text{Also define partial generating function as} \\
F_{n_2,n_3,n_4,n_5}(X) &= \sum_{n_1=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5} X^{n_1} & (34) \\
F_{n_3,n_4,n_5}(X,Y) &= \sum_{n_2=0}^{\infty} F_{n_2,n_3,n_4,n_5}(X) Y^{n_2} & (35) \\
F_{n_4,n_5}(X,Y,Z) &= \sum_{n_3=0}^{\infty} F_{n_3,n_4,n_5}(X,Y) Z^{n_3} & (36) \\
F_{n_5}(X,Y,Z,R) &= \sum_{n_4=0}^{\infty} F_{n_4,n_5}(X,Y,Z) R^{n_4} & (37) \\
F(X,Y,Z,R,S) &= \sum_{n_5=0}^{\infty} F_{n_5}(X,Y,Z,R) S^{n_5} & (38) \\

\text{Now proceeding on the lines of Maggu, Singh T.P. et.al. and following the standard technique, which after manipulation gives the final reduced results as:} \\
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5) F(X,Y,Z,R,S) - \mu_2 F(X,Y,Z,R) - \mu_4 F(X,Y,Z,S) - \\
\mu_5 F(X,Y,R,S) - \mu_2 F(X,Z,R,S) - \mu_4 F(Y,Z,R,S) - \lambda_1 X F(X,Y,Z,R,S) + \lambda_2 X F(X,Y,Z,R,S) + \mu_1 p_{11} X [F(X,Y,Z,R,S) - F(Y,Z,R,S)] + \\
\frac{\mu_1 p_{12} X}{F(X,Y,Z,R,S) - F(X,Z,R,S)} + \frac{\mu_2 p_{21} X}{F(X,Y,Z,R,S) - F(X,Z,R,S)} + \\
\frac{\mu_3 p_{34} R}{F(X,Y,Z,R,S) - F(X,Y,R,S)} + \frac{\mu_3 p_{35} S}{F(X,Y,Z,R,S) - F(X,Y,Z,S)} + \\
\frac{\mu_4 p_{46} S}{F(X,Y,Z,R,S) - F(X,Y,Z,S)} + \frac{\mu_5 p_{57} S}{F(X,Y,Z,R,S)} - F(X,Y,Z,S) & (39) \\

(\lambda_1 (1 - X) + \lambda_2 (1 - Y) + \mu_1 (1 - p_{21} \frac{Y}{X} - p_{34} \frac{Z}{X}) + \mu_2 (1 - p_{21} \frac{X}{Y} - p_{23} \frac{Z}{Y}) + \mu_3 (1 - p_{34} \frac{R}{Z} - p_{35} \frac{S}{Z}) + \mu_4 (1 - p_{46} \frac{S}{R} - p_{45} \frac{R}{S} + \mu_5 (1 - p_{54} \frac{R}{S} - p_{57} \frac{S}{R}) F(X,Y,Z,R,S) = \mu_5 F(X,Y,Z,R) (1 - p_{54} \frac{R}{S} - p_{57} \frac{S}{R}) + \mu_4 F(X,Y,Z,S) (1 - p_{34} \frac{S}{R} - p_{35} \frac{S}{Z}) + \mu_2 F(X,Z,R,S) (1 - p_{21} \frac{Y}{X} - p_{23} \frac{Z}{Y}) - p_{21} \frac{Y}{X} - p_{23} \frac{Z}{Y} - F(Y,Z,R,S) & (40) \\

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\end{align*}
\[
\mu_3 F(Y,Z,R) \left( 1 - p_{34} \frac{\lambda_2}{\lambda_1} \right) + \mu_4 F(X,Y,Z) \left( 1 - p_{45} \frac{\lambda_1}{\lambda_2} \right) + \mu_3 F(X,Y,R) \left( 1 - p_{34} \frac{\lambda_3}{\lambda_2} \right) + \mu_1 F(X,Y,Z) \left( 1 - p_{45} \frac{\lambda_2}{\lambda_1} \right)
\]

\[
+ \mu_2 F(X,Z,R) \left( 1 - p_{23} \frac{\lambda_2}{\lambda_1} \right) + \mu_1 F(Y,Z,R) \left( 1 - p_{23} \frac{\lambda_1}{\lambda_2} \right)
\]

\[
\left( \lambda_2 (1-X) + \lambda_3 (1-Y) + \mu_1 (1-p_{12} \frac{\lambda_1}{\lambda_2} + p_{13} \frac{\lambda_2}{\lambda_1}) + \mu_2 (1-p_{12} \frac{\lambda_1}{\lambda_2} + p_{13} \frac{\lambda_2}{\lambda_1}) + \mu_3 (1-p_{34} \frac{\lambda_3}{\lambda_2} + p_{45} (1-p_{45} \frac{\lambda_3}{\lambda_2} + p_{35} (1-p_{35} \frac{\lambda_2}{\lambda_1})))
\]

(41)

Let us denote :-

\[ F(Y,Z,R,S) = F_1 \]

\[ F(X,Z,R,S) = F_2 \]

\[ F(X,Y,R,S) = F_3 \]

\[ F(X,Y,Z,S) = F_4 \]

\[ F(X,Y,Z,R) = F_5 \]

Also \( F(1,1,1,1,1) = 1 \) Total probability.

Let \( X = 1 \) as

\[ 1 = \mu_1 F_1 (p_{12} + p_{13}) + \mu_2 F_2 (p_{21}) \]

\[ -\lambda_1 + \mu_1 - \mu_2 p_{21} = \mu_1 F_1 - \mu_2 F_2 p_{21} \]

where \( p_{13} + p_{12} = 1 \) \( (42) \)

Again differentiating w.r.t. \( Z \) gives \( Y = 1 \)

\[ 1 = \frac{\mu_2 F_2 (p_{23} + p_{21}) + \mu_1 F_1 (p_{12})}{-\lambda_2 + \mu_2 (p_{21} + p_{23}) + \mu_1 (p_{12})} \]

\[ -\lambda_2 = \mu_1 p_{12} + \mu_2 = -\mu_1 F_1 p_{12} + \mu_2 F_2 \]

where \( p_{21} + p_{23} = 1 \)

Again differentiating w.r.t. \( Z \) gives \( Z = 1 \)

\[ 1 = \frac{\mu_3 F_3 (p_{34} + p_{35}) + \mu_2 F_2 (p_{23}) + \mu_1 F_1 (p_{13})}{\lambda_3 (p_{34} + p_{35}) + \mu_2 (p_{23} + p_{21}) + \mu_1 (p_{13})} \]

\[ -\mu_1 p_{13} + \mu_2 p_{23} + \mu_3 = -\mu_1 p_{13} F_1 - \mu_2 p_{23} F_2 + \mu_3 F_3 \]

where \( p_{34} + p_{35} = 1 \)

Again differentiating w.r.t. \( R \) gives \( R = 1 \)

\[ 1 = \frac{\mu_3 F_3 (p_{45} + p_{46}) + \mu_3 F_3 (p_{34} + p_{35} + p_{36})}{\lambda_4 (p_{45} + p_{46}) + \mu_3 (p_{34} + p_{35} + p_{36}) + \mu_4 (p_{34} + p_{35} + p_{36})} \]

\[ -\mu_3 p_{34} + \mu_3 p_{35} + \mu_4 = -\mu_3 p_{34} F_3 - \mu_5 p_{34} F_5 + \mu_4 F_4 \]

where \( p_{45} + p_{46} = 1 \)

Again differentiating w.r.t. \( S \) gives \( S = 1 \)
\[
1 = \mu_5 F_5 (p_{35} + p_{35}) + \mu_4 F_3 (p_{35}) + \mu_4 F_4 (p_{35}) + \mu_4 (p_{35}) + \mu_5
\]

After solving the above for \( F_1, F_2, F_3, F_4, F_5 \), we get

\[
F_1 = 1 - \frac{\lambda_2 + \lambda_1 p_{12}}{(1 - p_{21} p_{12}) \mu_1}
\]

\[
F_2 = 1 - \frac{\lambda_2 + \lambda_1 p_{12}}{(1 - p_{21} p_{12}) \mu_2}
\]

\[
F_3 = 1 - \frac{[(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}]}{(1 - p_{21} p_{12}) \mu_3}
\]

\[
F_4 = 1 - \left( p_{34} + p_{35} p_{54} \right) \left( \frac{\lambda_2 + \lambda_1 p_{12} p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{(1 - p_{21} p_{12}) \mu_4 (1 - p_{45} p_{54})} \right)
\]

\[
F_5 = 1 - \left( p_{34} + p_{35} p_{54} \right) \left( \frac{\lambda_2 + \lambda_1 p_{12} p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{(1 - p_{21} p_{12}) \mu_5} \right)
\]

V. SOLUTION OF MODEL

\[
P_{n_1,n_2,n_3,n_4,n_5} = p_1^{n_1} p_2^{n_2} p_3^{n_3} p_4^{n_4} p_5^{n_5} (1 - p_1) (1 - p_2) (1 - p_3) (1 - p_4) (1 - p_5)
\]

Where

\[
\rho_1 = \frac{\lambda_1 + \lambda_2 p_{21}}{(1 - p_{21} p_{12}) \mu_1}
\]

\[
\rho_2 = \frac{\lambda_2 + \lambda_1 p_{12}}{(1 - p_{21} p_{12}) \mu_2}
\]

\[
\rho_3 = \frac{[(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}]}{(1 - p_{21} p_{12}) \mu_3}
\]

\[
\rho_4 = \left[ p_{34} + p_{35} p_{54} \right] \left( \frac{\lambda_2 + \lambda_1 p_{12} p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{(1 - p_{21} p_{12}) \mu_4 (1 - p_{45} p_{54})} \right)
\]

\[
\rho_5 = \left[ p_{34} + p_{35} p_{54} \right] \left( \frac{\lambda_2 + \lambda_1 p_{12} p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{(1 - p_{21} p_{12}) \mu_5} \right)
\]

The solution in steady state exists, if the conditions \( \rho_1, \rho_2, \rho_3, \rho_4, \rho_5 \leq 1 \) are satisfied.

VI. VARIOUS QUEUE CHARACTERISTIC

A. Average number of customers

\[
L = \sum_{n_1}^{\infty} \sum_{n_2}^{\infty} \sum_{n_3}^{\infty} \sum_{n_4}^{\infty} \sum_{n_5}^{\infty} (n_1 + n_2 + n_3 + n_4 + n_5) P_{n_1,n_2,n_3,n_4,n_5}
\]

On putting the values of \( P_{n_1,n_2,n_3,n_4,n_5} \), \( P_1, P_2, P_3, P_4, P_5 \) and after simplification,

We get

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\[ L = L_1 + L_2 + L_3 + L_4 + L_5 \]

\[
L = \frac{\lambda_1 + \lambda_2 p_{21}}{(1-p_{21} p_{12})(1-\lambda_1 - \lambda_2 p_{21})} + \frac{\lambda_2 + \lambda_1 p_{12}}{(1-p_{21} p_{12})(1-\lambda_2 - \lambda_1 p_{12})} + \frac{(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{(\mu_3 (1-p_{21} p_{12}) - (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}) p_{13}} + \frac{(\mu_4 (1-p_{45} p_{54}) (1-p_{21} p_{12}) - ((P_{34} + P_{35} P_{54}) (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13})}{(\mu_5 (1-p_{45} p_{54}) (1-p_{21} p_{12}) - (P_{45} (P_{34} + P_{35} P_{54}) + P_{35} (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13})}
\]

\[ B. \text{ Variance of queue} \]

\[ V(n_1 + n_2 + n_3 + n_4 + n_5) = \sum_{n_1} \sum_{n_2} \sum_{n_3} \sum_{n_4} \sum_{n_5} (n_1 + n_2 + n_3 + n_4 + n_5)^2 p_{1,n_2,n_3,n_4,n_5} + L^2 \sum_{n_1} \sum_{n_2} \sum_{n_3} \sum_{n_4} \sum_{n_5} p_{1,n_2,n_3,n_4,n_5} - 2L \sum_{n_1} \sum_{n_2} \sum_{n_3} \sum_{n_4} \sum_{n_5} (n_1 + n_2 + n_3 + n_4 + n_5) \]

After substituting the values, the result reduces to

\[ E(W) = \frac{\lambda_1 + \lambda_2 p_{21}}{A (1 - \lambda_1 - \lambda_2 p_{21})} + \frac{\lambda_2 + \lambda_1 p_{12}}{A (1 - \lambda_2 - \lambda_1 p_{12})} + \frac{(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{A (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13}} + \frac{(\mu_4 (1-p_{45} p_{54}) (1-p_{21} p_{12}) - ((P_{34} + P_{35} P_{54}) (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13})}{(\mu_5 (1-p_{45} p_{54}) (1-p_{21} p_{12}) - (P_{45} (P_{34} + P_{35} P_{54}) + P_{35} (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13})}
\]

Where

\[ E(W) = \frac{\lambda_1 + \lambda_2 p_{21}}{A (1 - \lambda_1 - \lambda_2 p_{21})} + \frac{\lambda_2 + \lambda_1 p_{12}}{A (1 - \lambda_2 - \lambda_1 p_{12})} + \frac{(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}}{A (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13}} + \frac{(\mu_4 (1-p_{45} p_{54}) (1-p_{21} p_{12}) - ((P_{34} + P_{35} P_{54}) (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13})}{(\mu_5 (1-p_{45} p_{54}) (1-p_{21} p_{12}) - (P_{45} (P_{34} + P_{35} P_{54}) + P_{35} (\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_1 + \lambda_2 p_{21}) p_{13})}
\]
VII. ALGORITHM

The following algorithm provides the procedure to determine the joint probability and various queue characteristics of above discussed queue model:

Step1. Obtain the number of customers  \( n_1, n_2, n_3, n_4, n_5 \).

Step2. Obtain the values of mean service rate  \( \mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \).

Step3. Obtain the values of mean arrival rate  \( \lambda_1, \lambda_2 \).

Step4. Obtain the values of the probabilities  \( p_{12}, p_{21}, p_{13}, p_{23}, p_{34}, p_{35}, p_{45}, p_{46}, p_{54}, p_{57} \).

Step5. Calculate the value of

(i) \( F_1 = 1 - \frac{\lambda_2 + \lambda_1 p_{12}}{(1-p_{21} p_{12}) \mu_1} \)

(ii) \( F_2 = 1 - \frac{\lambda_2 + \lambda_1 p_{12}}{(1-p_{21} p_{12}) \mu_2} \)

(iii) \( F_3 = 1 - \frac{[\lambda_2 + \lambda_1 p_{12} p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13}]}{(1-p_{21} p_{12}) \mu_3} \)

(iv) \( F_4 = 1 - \left[ \left( p_{34} + p_{35} p_{54} \right) \frac{(\lambda_2 + \lambda_1 p_{12} p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{13})}{(1-p_{21} p_{12}) \mu_4 (1-p_{45} p_{54})} \right] \)

(v) \( F_5 = 1 - \left[ \left( p_{35} + p_{34} p_{46} + p_{35} \right) \frac{p_{45} (p_{34} + p_{35} p_{54}) + p_{35}}{\mu_5 (1-p_{45} p_{54})} \right] \)

Step 6. Calculate

(i) \( \rho_1 = 1 - F_1 \)

(ii) \( \rho_2 = 1 - F_2 \)

(iii) \( \rho_3 = 1 - F_3 \)

(iv) \( \rho_4 = 1 - F_4 \)

(v) \( \rho_5 = 1 - F_5 \)

Step 7. Check \( \rho_1, \rho_2, \rho_3, \rho_4, \rho_5 < 1 \)

If so, then go to step (8) else steady state condition does not holds good.

Step 8. The joint probability

\[ P_{n_1, n_2, n_3, n_4, n_5} = \rho^{n_1} \rho^{n_2} \rho^{n_3} \rho^{n_4} \rho^{n_5} (1-\rho_1) (1-\rho_2) (1-\rho_3) (1-\rho_4) (1-\rho_5) \]

Step 9. Calculate average no. of customers (mean queue length)

\[ L = \frac{\rho_1}{(1-\rho_1)} + \frac{\rho_2}{(1-\rho_2)} + \frac{\rho_3}{(1-\rho_3)} + \frac{\rho_4}{(1-\rho_4)} + \frac{\rho_5}{(1-\rho_5)} \]

Step 10. Calculate variance of queue

\[ V = \frac{\rho_1}{(1-\rho_1)^2} + \frac{\rho_2}{(1-\rho_2)^2} + \frac{\rho_3}{(1-\rho_3)^2} + \frac{\rho_4}{(1-\rho_4)^2} + \frac{\rho_5}{(1-\rho_5)^2} \]

Step 11. Calculate average waiting time for customers
VIII. NUMERICAL ILLUSTRATION

Given customers coming to three servers out of which two servers consists biserial channels and one server is commonly linked in series with each of the two servers in biseries. The number of customers, mean service rate, mean arrival rate and associated probabilities are given as follows:

<table>
<thead>
<tr>
<th>S.No.</th>
<th>No. of Customers</th>
<th>Mean Service Rate</th>
<th>Mean Arrival Rate</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( n_1 = 3 )</td>
<td>( \mu_1 = 9 )</td>
<td>( \lambda_1 = 2 )</td>
<td>( p_{12} = 0.6 )</td>
</tr>
<tr>
<td>2.</td>
<td>( n_2 = 6 )</td>
<td>( \mu_2 = 8 )</td>
<td>( \lambda_2 = 3 )</td>
<td>( p_{13} = 0.4 )</td>
</tr>
<tr>
<td>3.</td>
<td>( n_3 = 7 )</td>
<td>( \mu_3 = 7 )</td>
<td></td>
<td>( p_{21} = 0.7 )</td>
</tr>
<tr>
<td>4.</td>
<td>( n_4 = 4 )</td>
<td>( \mu_4 = 10 )</td>
<td></td>
<td>( p_{23} = 0.3 )</td>
</tr>
<tr>
<td>5.</td>
<td>( n_5 = 5 )</td>
<td>( \mu_5 = 15 )</td>
<td></td>
<td>( p_{45} = 0.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p_{46} = 0.5 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p_{34} = 0.6 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p_{35} = 0.4 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p_{54} = 0.7 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p_{57} = 0.3 )</td>
<td></td>
</tr>
</tbody>
</table>

SOLUTION:

\[
\rho_1 = \frac{\lambda_1 + \lambda_2 p_{21}}{(1 - p_{21} p_{12}) \mu_1} = 0.78
\]

\[
\rho_2 = \frac{\lambda_2 + \lambda_1 p_{12}}{(1 - p_{21} p_{12}) \mu_2} = 0.90
\]

\[
\rho_3 = \frac{[(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{131}]}{(1 - p_{21} p_{12}) \mu_3} = 0.71
\]

\[
\rho_4 = \left[ (p_{34} + p_{35} p_{54}) \left( \frac{(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{131}}{(1 - p_{21} p_{12}) \mu_4 (1 - p_{45} p_{54})} \right) \right] = 0.68
\]

\[
\rho_5 = \left[ \frac{(\lambda_2 + \lambda_1 p_{12}) p_{23} + (\lambda_2 + \lambda_1 p_{12}) p_{131}}{(1 - p_{21} p_{12})} \right] \left[ \frac{p_{45} (p_{34} + p_{35} p_{54}) + p_{45}}{\mu_5 (1 - p_{45} p_{54})} \right] = 0.25
\]

Average no. of customers

\[
L = L_1 + L_2 + L_3 + L_4 + L_5
\]

\[
= 3.66 + 9.54 + 2.5 + 2.50 + 0.8193
\]

\[
= 19.0193
\]
Average waiting time for customers

\[ E(W) = \frac{L}{\lambda} \]

where \( \lambda = \lambda_1 + \lambda_2 \)

\[ \frac{L_1 + L_2 + L_3 + L_4 + L_5}{\lambda} \]

= 0.732 + 1.908 + 0.5 + 0.5 + 0.16386

= 3.80386

Variance

\[ \frac{p_1}{(1-p_1)^2} + \frac{p_2}{(1-p_2)^2} + \frac{p_3}{(1-p_3)^2} + \frac{p_4}{(1-p_4)^2} + \frac{p_5}{(1-p_5)^2} \]

= 0.037752 + 90 + 8.442 + 6.6406 + 0.44

= 105.560352

REFERENCES


