

# Implementation on FPGA for Tuned Low Complexity Modified Curve Fitting Algorithm

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**Abstract**— An effort is made to minimize the arithmetic intricacy of signal by using the VHDL module on FPGA. A curve fitting algorithm is used for finding the fundamental frequency of a given signal. Modification of algorithm is reducing the arithmetic complex-city become extinct the cubic solution. This is mature a fast approach for the signal processing and reduced the resultant error .This is made a comparison between basic CFA and modified CFA. Result of this to find out a best fit of a curve .A modified curve fitting algorithm is used for finding the optimum signal quality parameter for further application of signal processing.

**Keywords**— *fundamental Frequency, VHDL CFA, etc.*

## I. INTRODUCTION

The key goal of this study and work is to implement a system in VHDL module for drawing out frequency of signal. This frequency use to evaluate different signal properties. The CFA is processed for measuring the frequency and short out the variation of a demanding signal of required function, in electrical and electronic environment.

This algorithm estimate a frequency which is widely used to examine closely because of their transversal presentation .Different fracas detection algorithms are based on fundamental frequency of a signal[1].

The CFA permits a small number of samples of a signal comparison then other like CFA with STFT, Kalman, Sine fitting. Due to few samples it takes less computational time with a conventional accurateness [1, 2].

CFA allowed extract the quality parameter of signal like harmonic for phase of a fixed length (0, T) window. It is evident the hole system accuracy is link to fundamental frequency evaluation of window(0,T) ,another block are in system use that information to process the sample to evaluate quality parameter [1,2,3].The study and realization of this type of algorithm is basically implemented with FPGA. The superior programmable circuit with enhance quality and higher integration density, is made better choice of implementation on FPGA. Development of custom design, in different level, with

the headwear description language like very high speed integrated circuit language (VHDL), Verilog [4].

FPGA programming was done in VHDL code .to generate .VHDL code use the System Generator of Xilinx ISE 13.1 environment.

## II. CURVE FITTING ALGORITHM

Wherever the curve fitting is specified, to drive mathematical equation of curve is define for data set. This process to select best fit of a curve that approximates a same data set. It show relationship between two variables may exist [5].CFA as regression analysis, to determine the ‘best fit’ curve or line for data sets [3].

Curve fitting find the best fit of a curve to a waveform to get error performance .This is then calculate the samples residual values between the fitted curve and the waveform. The sum of their square values decides the size of residuals. It reduced to calculate least square error, and the phase and amplitude of fitted curve obtain [2].

Curve fitting algorithm is realized for a parametric equation contains shaping parameters to adjust the shape of fitted curve. This flexibility of curve fitting is accepted to produce a curve which is capable of following any set of sampled data points. A modified parametric equation is developed to modify curve allows control over shape of fitted curve by introducing shaping parameter. As a curve fitting method, it can be used on any discrete set of data points to produce a highly accurate shape curve .Curve fitting also used in modeling in complex surface geometry in engineering application. This curve is help in modeling like computer graphics, data structure modeling, and orthogonal distance fitting, computer aided geometry[6][7][8].

The effort is made to modified curve fitting algorithm .That is to accurate compare basic algorithm.

## III. BASIC CURVE- FITTING ALGORITHM (CFA)

The basic algorithm (CFA) is method to get fundamental frequency of a particular signal. It’s developed from Lest Squares method and is tuned to lock and extract the fundamental signal frequency. This algorithm finds the difference between ideal signal sample and the input signal

sample of a given frequency. Define an error function by which is sum of the square of the difference of many samples. The fundamental frequency is found minimize the error function.

As mention in [2] basic CFA algorithm allows find out the difference between  $\Delta\omega$  from the ideal signal frequency, following equation:

$$a_3(\Delta\omega)^3 + a_2(\Delta\omega)^2 + a_1(\Delta\omega) + a_0$$

Where we put the values,

$$Num = \left(\int_0^T m(t) \cos \omega t dt\right)^2 - \left(\int_0^T m(t) \sin \omega t dt\right)^2$$

$$Den = \left(\int_0^T m(t) t \cos \omega t dt\right) \times \left(\int_0^T m(t) \sin \omega t dt\right) -$$

$$\left(\int_0^T m(t) t \sin \omega t dt\right) \times \int_0^T m(t) \cos \omega t dt$$

The values of “a” (coefficient’s) using Taylor series stopped at the second power are:

$$a_0 = 2\omega Den - Num$$

$$a_1 = 2\omega \frac{\partial Den}{\partial \omega} + 2Den - \frac{\partial Num}{\partial \omega}$$

$$a_2 = \omega \frac{\partial^2 Den}{\partial \omega^2} + 2 \frac{\partial Den}{\partial \omega} - \frac{1}{2} \frac{\partial^2 Num}{\partial \omega^2}$$

$$a_3 = \frac{\partial^2 Den}{\partial \omega^2}$$

Find  $\Delta\omega$  we solve a third order equation, and used Girolamo Cadiano method in different case determined by the discriminator ( $\Delta$ ) sign [9,10].The discriminator is defined as:

$$\Delta = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2$$

Where

$$p = \frac{3b - a^2}{3}$$

$$q = \frac{2a^3 - 9ab + 27c}{27}$$

Case I: for ( $\Delta < 0$ ) we solve:

$$x_1 = 2r^{\left(\frac{1}{3}\right)} \cos\left(\frac{\phi}{3}\right) - \frac{a}{3}$$

$$x_2 = 2r^{\left(\frac{1}{3}\right)} \cos\left(\frac{\phi + 4\pi}{3}\right) - \frac{a}{3}$$

$$x_3 = 2r^{\left(\frac{1}{3}\right)} \cos\left(\frac{\phi + 2\pi}{3}\right) - \frac{a}{3}$$

We get three real roots.

Where real part are magnitude and phase of the complex number.

$$R = -\frac{q}{2} + \sqrt{(-\Delta)i}$$

While case II for ( $\Delta \geq 0$ )

$$x_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} - \sqrt[3]{\frac{q}{2} + \sqrt{\Delta}} - \frac{a}{3}$$

$$x_2 = -\frac{1}{2} \left( \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} - \sqrt[3]{\frac{q}{2} + \sqrt{\Delta}} \right) + \left( \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{\frac{q}{2} + \sqrt{\Delta}} \right) i - \frac{a}{3}$$

$$x_3 = -\frac{1}{2} \left( \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} - \sqrt[3]{\frac{q}{2} + \sqrt{\Delta}} \right) - \left( \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{\frac{q}{2} + \sqrt{\Delta}} \right) i - \frac{a}{3}$$

We suppose that  $\Delta\omega$  is reasonably small in magnitude so we take as possible the solution smallest magnitude so take the solution with the smallest magnitude.

Compared with the other method for frequency evaluation (as seen in algorithm [1]).modified CFA algorithm gives a reliable precision with smaller evaluation times we see in the next table:

TABLE I. ALGORITHM COMPEARATION

Actual freq.	Meas. By CFA	Meas. By mod. CFA	Error by CFA	Error by mod. CFA
49	49.22379342	48.998641	-0.223793424	0.001358998
49.1	49.28567097	49.09858625	-0.18567097	0.001413748
49.2	49.35032923	49.1986105	-0.150329229	0.001389502
49.3	49.41798611	49.29870011	-0.117986113	0.001299894
49.4	49.48888354	49.39884145	-0.088883536	0.001158553
49.5	49.56329112	49.4990209	-0.063291119	0.000979101
49.6	49.64151066	49.59922485	-0.04151066	0.000775147
49.7	49.72388159	49.69943972	-0.023881588	0.000560284
49.8	49.81078766	49.79965192	-0.010787664	0.000348083
49.9	49.90266533	49.89984791	-0.002665326	0.000152091
50	50.00001419	50.00001417	-1.41884E-05	-1.41711E-05
50.1	50.10341046	50.10013722	-0.003410461	-0.000137218
50.2	50.21352435	50.2002036	-0.013524346	-0.000203601
50.3	50.33114302	50.30019991	-0.031143024	-0.00019991
50.4	50.45720165	50.40011278	-0.057201646	-0.00011278
50.5	50.59282614	50.4999289	-0.092826143	7.11034E-05
50.6	50.73939405	50.599635	-0.139394054	0.000365005
50.7	50.89862391	50.69921787	-0.198623911	0.000782131
50.8	51.07271204	50.79866437	-0.272712041	0.00133563
50.9	51.26455268	50.89796142	-0.364552681	0.002038585
51	51.47811524	50.99709599	-0.47811524	0.00290401

Table1.1 The proposed method comparison with CFA method

In table 1.1 the comparison was made for CFA and modified CFA without noise. And again the newly compared between two different signals SNR20 and SNR10.

Actual freq.	Meas. By CFA	Meas. By mod. CFA	Error by CFA	Error by mod. CFA
49	49.22046	48.99669	-0.22046	0.003314
49.1	49.28301	49.09496	-0.18301	0.005037
49.2	49.26373	49.07147	-0.06373	0.128529
49.3	49.49645	49.39473	-0.19645	-0.09473
49.4	49.57686	49.51468	-0.17686	-0.11468
49.5	49.59851	49.54185	-0.09851	-0.04185
49.6	49.63216	49.58852	-0.03216	0.011481
49.7	49.70713	49.68179	-0.00713	0.018214
49.8	49.83615	49.82769	-0.03615	-0.02769
49.9	49.87547	49.87202	0.024531	0.027983
50	50.00899	50.00898	-0.00899	-0.00898
50.1	50.10916	50.10569	-0.00916	-0.00569
50.2	50.27709	50.25571	-0.07709	-0.05571
50.3	50.34182	50.30899	-0.04182	-0.00899
50.4	50.5036	50.43429	-0.1036	-0.03429
50.5	50.64709	50.54046	-0.14709	-0.04046
50.6	50.59141	50.49236	0.008587	0.107635
50.7	50.82951	50.65228	-0.12951	0.047723
50.8	50.98683	50.74868	-0.18683	0.051316
50.9	51.31151	50.91367	-0.41151	-0.01367
51	51.63235	51.07324	-0.63235	-0.07324

Table1.2 result with 20 SNR

Actual freq.	Meas. By CFA	Meas. By mod. CFA	Error by CFA	Error by mod. CFA
49	49.38414154	49.21106989	-0.384141544	0.203568853
49.1	49.43174073	49.27696238	-0.33174073	0.203568853
49.2	49.29412131	49.12185903	-0.09412131	0.203568853
49.3	49.57949709	49.50527008	-0.279497091	0.203568853
49.4	49.50335045	49.41653689	-0.103350449	0.203568853
49.5	49.39653572	49.29643115	0.103464275	0.203568853
49.6	49.63226794	49.59426071	-0.032267943	0.203568853
49.7	49.62911321	49.59659334	0.070886786	0.203568853
49.8	49.60104927	49.57187788	0.198950731	0.203568853
49.9	49.74531001	49.72670245	0.154689989	0.203568853
50	49.87583593	49.86951842	0.12416407	0.203568853
50.1	50.11427745	50.10902853	-0.014277447	0.203568853
50.2	50.16635989	50.15499069	0.033640108	0.203568853
50.3	50.12590354	50.11669664	0.174096459	0.203568853
50.4	50.59804399	50.51921071	-0.198043992	0.203568853
50.5	50.67867488	50.55077997	-0.178674885	0.203568853
50.6	50.94803423	50.73301195	-0.348034228	0.203568853
50.7	50.47517858	50.4107571	0.224821416	0.203568853
50.8	50.63566435	50.50896696	0.164335646	0.203568853
50.9	51.67515411	51.07407841	-0.775154113	0.203568853
51	51.87833809	51.18404824	-0.878338087	0.203568853

Table1.3 result with SNR10

This comparison shows that CFA accuracy is compared with modified CFA and show less computational time and robustness to noise. The signal used test the algorithm 50Hz Sinusoid sampled at 128 kHz.

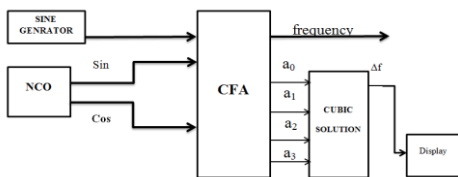


Figure 1. Basic blocks for algorithm (CFA) development.

Because of enormous workload we evaluate the algorithm to find a means to simplify it we found the real block is solving of the third order equation. The basic CFA implemented the

complete Cardano method and put allowed the selection of correct processing branch, after that to select the solution. The algorithm test with the Matlab for frequency range from 49Hz to 51Hz and phase, found that DELTA was always negative, and  $\Delta \omega$  was always found using 'x<sub>2</sub>' expression we before gave in different cases[1].

IV. MODIFIED CURVE-FITTING ALGORITHM

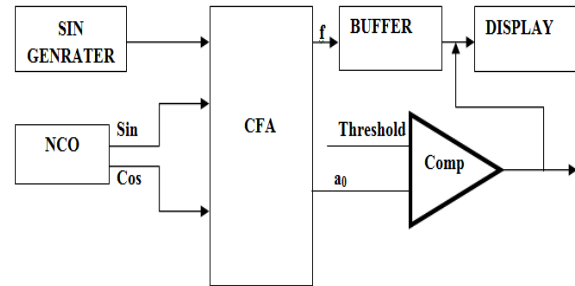


Figure 2. Blocks for modified algorithm (CFA) development.

In modified curve fitting algorithm we have completely disappear the difficult operation like cube roots in different case.

This modification in algorithm for best fit of a curve is made by the following equation:

$$f(x) = ax$$

This equation is allowed to fast operation with considerable goodness of fit a curve and reduces error performance for signal for predefined frequency.

V. IMPLEMENTATION OF THE CFA ON FPGA.

The generation of VHDL programming for implementation of CFA on FPGA following flow is shown in figure 4. After design a block diagram in Simulink and the HDL code generator used to generate VHDL code. This code is complete code for all subsystem in cfa design.

This procedures Simulink build the basic algorithm structure and generate HDL code after then Xilinx ISE 13.1 map the code to FPGA resources a balanced area and speed, finely testing on fpga gives feedback for additive tunings [1].



Figure 3. RTL view for algorithm (CFA) development.



Figure 4. Flow of VHDL code generation for algorithm (CFA) Development

## VI. SIMULATION RESULT

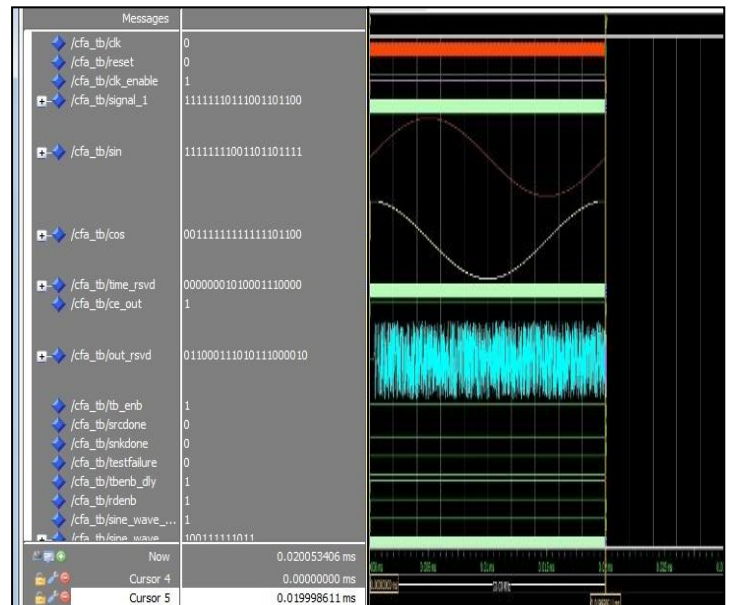


Figure 5. output waveform view for algorithm (CFA) .

## VII. CONCLUSION

In this paper, we proposed the simple and powerful curve fitting algorithms by using iterative error minimization .To define the error between the input curve and the synthesized curve, tune for low complexity. The modified curve fitting algorithm give an appreciable result of error minimization .Curve fitting algorithm calculate less number of sample comparison then other algorithm and saving much computational time with acceptable level of accuracy.

The first experimental result obtain in a measure bench confirm the accurateness of the implementation. The CFA permit to fast development on FPGA and further processing for ASIC technology.

## REFERENCES

- [1] M. Caciotta, S. Giarnetti, F. Leccese, E. Pedruzzi “Curve Fitting Algorithm FPGA implementation” 978-1-4244-8782-0/11/\$26.00 ©2011 IEEE
- [2] M Caciotta, F. Lecense ,T Trifirò: “Curve Fitting Algorithm (CFA) AsPower Quality Basic Algorithm”presented to XVIII IMEKO WORLDCONGRESS, Metrology for a Sustainable Development September, 17– 22, 2006, Rio de Janeiro, Brazil, CD-proceedings.
- [3] F. Leccese, S. Sangiovanni, “Study and realization of an instrumentFPGA based to implement Curve-Fitting Algorithm (CFA),”presented toInternational Telecommunications Energy Conference INTELEC 2007,September 30 – October 4 2007, Rome, Italy, Proceedings pp. 909-913,ISBN: 978-1-4244-1628-8, IEEE Catalog Number: CFP07INTC,Library of Congress: 88-656128
- [4] J. Viejo, M.J. Bellido, A. Millan, E. Ostua, J. Juan, P. Ruiz-e-Clavijo,D. Guerrero, “Efficient Design and Implementation on FPGA of aMicroBlaze Peripheral for Processing Direct Electrical Networks Measurements”, Industrial Embedded Systems, 2006. IES '06.International Symposium on, Antibes Juan-Les-Pins, France.
- [5] B V Ramana, “Higher Engineering mathematics” Tata McGraw Hill Education Private Limited,ISBN-13:978-0-07-063419-0.

- [6] Mohammad Asif Zaman, Shuvro Chowdhury, "Modified Be'zier Curves with Shape-Preserving Characteristics using Differential Evolution Optimization Algorithm" Manuscript Submitted to Journal: Advances in Numerical Analysis, Date of Submission: 27th October, 2012. DOI: 10.5923/j.ijee.20120202.04.
- [7] SOO-CHANG PEI, JI-HWEI HORNG, "OPTIMUM APPROXIMATION OF DIGITAL PLANAR CURVES USING CIRCULAR ARCS" Pattern Recognition, Vol. 29, No. 3, pp. 383-388, 1996, Elsevier Science Ltd. Copyright © 1996 Pattern Recognition Society. Printed in Great Britain. All rights reserved. 0031-3203/96 \$15.00+.00.
- [8] Yang Liu, Wenping Wang, "A Revisit to Least Squares Orthogonal Distance Fitting of Parametric Curves and Surfaces," F. Chen and B. Juttler (Eds.): GMP 2008, LNCS 4975, pp. 384-397, 2008. © Springer-Verlag Berlin Heidelberg 2008.
- [9] Thomas J. Osler, "AN EASY LOOK AT THE CUBIC FORMULA", Mathematics Department, Rowan University, Glassboro NJ 08028.
- [10] Osler, Thomas J., "Cardan polynomials and the reduction of radicals," Mathematics Magazine, Vol 47, No. 1, (2001), pp. 26-32.
- [11] Stephen Brown and Jonathan Rose, "Architecture of FPGAs and CPLDs: A Tutorial" Stephen Brown and Jonathan Rose
- [12] Philip H.W. Leong, Ivan K.H. Leung, "A Microcoded Elliptic Curve Processor Using FPGA Technology" IEEE TRANSACTIONS ON VERY LARGE SCALE INTEGRATION (VLSI) SYSTEMS, VOL. 10, NO. 5, OCTOBER 2002
- [13] Kuon, R. Tessier and J. Rose "FPGA Architecture: survey and Challenges", Foundations and Trends® in Electronic Design Automation Vol. 2, No. 2 (2007) 135-253 ©2008
- [14] Lanping Deng, Kanwaldeep Sobti, Yuanrui Zhang\*, Chaitali Chakrabarti "ACCURATE AREA, TIME AND POWER MODELS FOR FPGA-BASED IMPLEMENTATIONS" This paper is an extension of the ICASSP'08 paper "Accurate Models for Estimating Area and Power of FPGA Implementations".
- [15] By the staff of Berkeley Design Technology, Inc. "An Independent Analysis of Altera's FPGA Floating-point DSP Design Flow", © 2011 Berkeley Design Technology, Inc
- [16] M. A. Farahat<sup>1</sup>, M. Talaat<sup>1,2,\*</sup>, "Short-Term Load Forecasting Using Curve Fitting Prediction Optimized by Genetic Algorithms", International Journal of Energy Engineering 2012, 2(2): 23-28
- [17] Deming Chen, Jason Cong, Peichen Pan "FPGA Design Automation: A Survey". Foundations and Trends® in Electronic Design Automation © Vol. 1, No 3 (November 2006) 195-330, November 2006 DOI: 10.1561/1000000003
- [18] Muhammad Sarfraz "Fitting Curve to Planar Digital Data" Proceedings of the Sixth International Conference on Information Visualization (IV'02) 1093-9547/02 \$17.00 © 2002 IEEE