Abstract— A PID controller is designed using ISE, IAE, ITAE and MSE error criteria for stable linear time invariant continuous system. A Simulated Annealing PID controller is designed for the plant to meet the desired performance specifications by using SA optimization method. PID controller gain parameters $K_p, K_i, K_d$ are designed and applied to the PID controller system. The PID controller closed loop response is observed for ISE, IAE, IATE and MSE error criteria. A comparison of system performance observed for all four criteria.

Keywords— PID Controller Tuning, Simulated annealing, Optimization technique, Close loop feedback.

1. PID controller system

PID controller consists of Proportional, Integral and Derivative gains. The PID feedback control system is illustrated in Fig. 1 where $r$, $e$, $y$ are respectively the reference, error and controlled variables. Where $K_p$ is proportional gain ,$K_i$ is integral gain and $K_d$ is derivative gain.

In the diagram of Fig.1, $G(s)$ is the plant transfer function and $C(s)$ is the PID controller transfer function that is given as:

$$C(s) = K_p + \frac{K_i}{s} + K_d$$

Where $K_p, K_i, K_d$ parameters of the PID controllers that are going to be tuned using Simulated annealing.

2 Performance evaluation criteria

Quantification of system performance is achieved through a performance index. The performance selected depends on the process under consideration and is chosen such that emphasis is placed on specific aspects of system performance. Furthermore, performance index is defined as a quantitative measure to depict the system performance of the designed PID controller. Using this technique an ‘optimum system’ can often be designed and a set of PID parameters in the system can be adjusted to meet the required specification. For a PID-controlled system, there are often four indices to depict the system performance ISE, IAE, IATE and MSE. They are defined as follows:

- **ISE Index:**

  $$ISE = \int_0^\infty e^2(t) \, dt$$

- **IAE Index:**

  $$IAE = \int_0^\infty |e(t)| \, dt$$

- **IATE Index:**

  $$IATE = \int_0^\infty t|e(t)| \, dt$$

- **MSE Index:**

  $$MSE = \frac{1}{t} \int_0^\infty t|e(t)| \, dt$$

The following performance indexes are used to minimize the overshoot, settling time, steady state error and reference tracking error for PSO-PID controller system. Therefore, for the SA-based PID tuning, these performance indexes are used as the objective function. In other word, the objective in the SA-based optimization is to seek a set of PID parameters such that the feedback control system has minimum performance index.
3. Simulated Annealing algorithms [1]

3.1 Simulated Annealing

Step 1: Initialize – Start with a random initial placement. Initialize a very high “temperature”.

Step 2: Move – Perturb the placement through a defined move.

Step 3: Calculate score – calculate the change in the score due to the move made.

Step 4: Choose – Depending on the change in score, accept or reject the move. The probability of acceptance depending on the current “temperature”.

Step 5: Update and repeat – Update the temperature value by lowering the temperature. Go back to Step 2. The process is done until “Freezing Point” is reached.

The simulated annealing is based on the following characteristics. Annealing in metals, heat the solid state metal to a high temperature, cool it down very slowly according to a specific schedule. If the heating temperature is sufficiently high to ensure random state and the cooling process is slow enough to ensure thermal equilibrium, then the atoms will place themselves in a pattern that corresponds to the global energy minimum of a perfect crystal.

Simulated annealing (SA) is a random-search technique which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system; it forms the basis of an optimization technique for combinatorial and other problems. Simulated annealing was developed in 1983 to deal with highly nonlinear problems. SA approaches the global maximization problem similarly to using a bouncing ball that can bounce over mountains from valley to valley. It begins at a high “temperature” which enables the ball to make very high bounces, which enables it to bounce over any mountain to access any valley, given enough bounces. As the temperature declines the ball cannot bounce so high, and it can also settle to become trapped in relatively small ranges of valleys. A generating distribution generates possible valleys or states to be explored. An acceptance distribution is also defined, which depends on the difference between the function value of the present generated valley to be explored and the last saved lowest valley. The acceptance distribution decides probabilistically whether to stay in a new lower valley or to bounce out of it. All the generating and acceptance distributions depend on the temperature. It has been proved that by carefully controlling the rate of cooling of the temperature, SA can find the global optimum. However, this requires infinite time. Fast annealing and very fast simulated reannealing (VFSR) or adaptive simulated annealing (ASA) are each in turn exponentially faster and overcome this problem.

3.2 Strengths of SA

Simulated annealing can deal with highly nonlinear models, chaotic and noisy data and many constraints. It is a robust and general technique. Its main advantages over other local search methods are its flexibility and its ability to approach global optimality. The algorithm is quite versatile since it does not rely on any restrictive properties of the model. SA methods are easily “tuned”. For any reasonably difficult nonlinear or stochastic system, a given optimization algorithm can be tuned to enhance its performance and since it takes time and effort to become familiar with a given code, the ability to tune a given algorithm for use in more than one problem should be considered an important feature of an algorithm.

3.3 Weaknesses of SA

Since SA is a metaheuristic, a lot of choices are required to turn it into an actual algorithm. There is a clear tradeoff between the quality of the solutions and the time required to compute them. The tailoring work required to account for different classes of constraints and to fine-tune the parameters of the algorithm can be rather delicate. The precision of the numbers used in implementation is of SA can have a significant effect upon the quality of the outcome.

4. Flow chart of Simulated Annealing [1]

Fig. 2 Flow chart of simulated annealing algorithms
5. SIMULATION AND RESULTS

The transfer function (G) for the process is:

\[ G(s) = \frac{2.295 s^3 - 0.01173 s^2 - 1.639 \times 10^{-5} s + 7.368 \times 10^{-9}}{s^4 - 0.01114 s^3 + 5.288 \times 10^{-5} s^2 + 9.232 \times 10^{-10} s + 2.06 \times 10^{-14}} \]

PID Controller tuning using Simulated Annealing algorithms [1]

Now we tune PID parameters Kp, Ki, Kd for above reduced order system by using simulated annealing Algorithm. We set Initial Temperature 100. Fitness Function is used for designing PID controller ISE, IAE, ITAE and MSE.

5.1 SA-PID Controller tuning using ISE

We apply the simulated annealing to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Integral of Squired Error. We obtain the kp = 1.0735e+003, ki = 9.6417e+003 and kd = 1.0021e+003.

5.2 SA-PID Controller tuning using IATE

We apply the simulated annealing to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Integral of absolute time error. We obtain the kp = 1.0182e+003, ki = 1.0102e+003 and kd = 3.7867e+004.

5.3 SA-PID Controller tuning using IAE

We apply the simulated annealing to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Integral of absolute error. We obtain the kp = 1.0000e+003, ki = 1.0152e+003 and kd = 3.1448e+004.

The step response of the SA-PID controller system is shown in figure 5.
5.4 SA-PID Controller tuning using MSE

We apply the simulated annealing to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Mean Square Error. We obtain the kp = 1.0000e+003, ki = 1.0152e+003 and kd = 3.1448e+004.

The step response of the SA-PID controller system is shown in figure 8.

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<tr>
<th>Parameters</th>
<th>ISE</th>
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### Conclusion

We can see from the results that open loop plant is highly unstable system. Then we design the PID controller system using simulated annealing. From the Table 1.1 we can see that fval is minimum for IATE criteria. For the above SA-PID controller system with IATE criteria undershoot 0%, overshoot 0%, and settling time 2.2641e-005 seconds.

### References

[1] Franco Busetti “Simulated annealing overview”.