BF-PSO optimized PID Controller design using ISE, IAE, IATE and MSE error criteria

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Abstract — A PID controller is designed using ISE, IAE, ITAE and MSE error criteria for stable linear time invariant continuous system. A BF-PSO PID controller is designed for the plant to meet the desired performance specifications by using BF-PSO optimization algorithm. PID controller gain parameters Kp,Ki,Kd are designed and applied to the PID controller system .The PID controller closed loop response is observed for ISE, IAE, IATE and MSE error criteria. A comparison of system performance observed for all four criteria.

Keywords — PID Controller Tuning, BF-PSO, Optimization technique, Close loop feedback.

1. PID controller system

PID controller consists of Proportional, Integral and Derivative gains. The PID feedback control system is illustrated in Fig. 1 where r, e, y are respectively the reference, error and controlled variables. Where Kp is proportional gain ,Ki is integral gain and Kd is derivative gain.

\[ C(s) = K_p + \frac{K_i}{s} + K_d \]  

Where Kp ,Ki, Kd parameters of the PID controllers that are going to be tuned using BF-PSO.

2 Performance evaluation criteria

Quantification of system performance is achieved through a performance index. The performance selected depends on the process under consideration and is chosen such that emphasis is placed on specific aspects of system performance. Furthermore, performance index is defined as a quantitative measure to depict the system performance of the designed PID controller. Using this technique an ‘optimum system’ can often be designed and a set of PID parameters in the system can be adjusted to meet the required specification. For a PID- controlled system, there are often four indices to depict the system performance ISE, IAE, IATE and MSE. They are defined as follows:

ISE Index:  
\[ ISE = \int_0^\infty e^2(t) \, dt \]  

IAE Index:  
\[ IAE = \int_0^\infty |e(t)| \, dt \]  

IATE Index:  
\[ IATE = \int_0^\infty t|e(t)| \, dt \]  

MSE Index:  
\[ MSE = \frac{1}{t} \int_0^\infty te^2(t) \, dt \]  

The following performance indexes are used to minimize the overshoot, settling time, steady state error and reference tracking error for PSO-PID controller system. Therefore, for the SA-based PID tuning, these performance indexes are used as the objective function. In other word, the objective in the SA-based optimization is to seek a set of PID parameters such that the feedback control system has minimum performance index.

3. Overview of PSO Algorithm

PSO is optimization algorithm based on evolutionary computation technique. The basic PSO is developed from research on swarm such as fish schooling and bird flocking. After it was firstly introduced in 1995, a modified PSO was then introduced in 1998 to improve the performance of the original PSO. A new parameter called inertia weight is added. This is a commonly used PSO where inertia weight is linearly decreasing during iteration in addition to another common type of PSO which is reported by Clerc. The later is...
the one used in this paper. In PSO, instead of using genetic operators, individuals called as particles are “evolved” by cooperation and competition among themselves through generations. A particle represents a potential solution to a problem. Each particle adjusts its flying according to its own flying experience and its companion flying experience. Each particle is treated as a point in a D-dimensional space. The ith particle is represented as \( X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \). The best previous position (giving the minimum fitness value) of any particle is recorded and represented as \( P_i = (p_{i1}, p_{i2}, \ldots, p_{iD}) \), this is called pbest. The index of the best particle among all particles in the population is represented by the symbol g, called as gbest. The velocity for the particle \( i \) is represented as \( V_i = (v_{i1}, v_{i2}, \ldots, v_{iD}) \). The particles are updated according to the following equations:

\[
V_{i,m}^{(t+1)} = \omega \cdot V_{i,m}^{(t)} + C_1 \cdot \text{rand}(t) \cdot (P_{\text{pbest},m} - X_{i,m}^{(t)}) + C_2 \cdot \text{rand}(t) \cdot (G_{\text{gbest},m} - X_{i,m}^{(t)})
\]

\[
X_{i,m}^{(t+1)} = X_{i,m}^{(t)} + V_{i,m}^{(t+1)}
\]

where \( c1 \) and \( c2 \) are two positive constant. As recommended in Clerc’s PSO, the constants are \( c1=1.2, c2=0.5 \). While \( \text{rand}(t) \) is random function between 0 and 1, and \( m \) represents iteration. Eq.7 is used to calculate particle’s new velocity according to its previous velocity and the distances of its current position from its own best experience (position) and the group’s best experience. Then the particle flies toward a new position according to Eq.8. The performance of each particle is measured according to a pre-defined fitness function (performance index), which is related to the problem to be solved. Inertia weight, \( w \) is brought into the equation to balance between the global search and local search capability. It can be a positive constant or even positive linear or nonlinear function of time. A guaranteed convergence of PSO proposed by Clerc set \( w=0.9 \). It has been also shown that PSO with different number of particles (swarm size) has reasonably similar performance. Swarm size of 10-50 is usually selected. Here, we set 40.

**4. BACTERIAL FORAGING OPTIMIZATION**

Introduction Based on the research of foraging behaviour of E.coli bacteria Kevin M.Passino and Liu exploited a variety of bacterial foraging and swarming behaviour, discussing how to connect social foraging process with distributed non-gradient optimization. In the bacterial foraging optimization process four motile behaviours are mimicked:-

1) **CHEMOTAXIS:**
   A chemotactic step can be defined as a tumble followed by a tumble or a tumble followed by a run lifetime.To represent a tumble a unit length random direction, \( (j) \), is generated; this will be used to define the direction of movement after a tumble. In particular
   \[
i(j+1,k,l) = i(j,k,l) + C(i)* (j)
   \]
   Where \( i(j,k,l) \) represents the \( i \)th bacterium at \( j \)th chemotactic, \( k \)th reproductive and \( l \)th elimination and dispersal step.Cl(i) is the size of the step taken in the random direction specified by a tumble(run length unit).

2) **SWARMING:**
   E.Coli cells can cooperatively self organize into highly structured colonies with elevated environmental adaptability using an intricate communication mechanism. Overall, cells provide an attraction signal to each other so they swarm together. The mathematical representation for swarming can be represented by
   \[
   J_{\text{cc}}(\theta, P(j,k,l)) = J_{\text{icc}}(\theta, \theta_i(j,k,l)) = \sum \{ \text{Dattract} \cdot \exp(-\text{Wattract} \cdot \sum (\theta_m - \theta_i m_2)) + \sum [\text{Hrepellant} \cdot \exp(-\text{Wrepellant} \cdot \sum (0m - \theta_i m_2))] \}
   \]
   Where \( J_{\text{cc}}(\theta, P(j,k,l)) \) is the cost function value to be added to the actual cost function to be minimized to present a time varying cost function. \( S \) is the total number of bacteria. \( P \) is the number of parameters to be optimized which are present in each bacterium and Dattract, Wattract, Hrepellant are different coefficients that should be properly choosen.

3) **REPRODUCTION:**

   The least healthiest bacteria die and the other each healthier bacteria split into two new bacteria each placed in the same location.

4) **ELIMINATION AND DISPERSAL:**

   It is possible that in the local environment, the lives of a population of bacteria changes either gradually (eg, via consumption of nutrients) or suddenly due to some other influence. Events can occur that all the bacteria in a region are killed or a group is dispersed into a new part of the environment. They have the effect of possibly destroying the chemotactic progress, but they have also the effect of assisting the chemotactic process, since dispersal may place bacteria near good food sources. From a board perspective, elimination and disposal are parts of the population level long distance motile behavior.

**5.COMBINED PARTICLE SWARM OPTIMIZATION (BF-PSO)**

BF-PSO algorithm combines both BFO and PSO. The aim is to make PSO ability to exchange social information and BF ability in finding new solution by elimination and dispersal, a unit length direction of tumble behavior is randomly generated. Random direction may lead to delay in reaching the global solution. In “BF-PSO” algorithm the unit length random direction of tumble behavior can be decided by the global best position and the best position of each bacterium. During the chemotaxis loop tumble direction is updated by:

\[
\phi(j+1) = \omega * \phi(j + c1 * \text{rand}(p\text{best} - p\text{current}) + c2 * \text{rand}(g\text{best} - p\text{current}))
\]

Where \( \phi \) is the best position of each bacterium and \( g\text{best} \) is the global best bacterium. The brief pseudo-code of BF-PSO has been provided below. Algorithm to find optimal parameters using BFO for the objective function ITAE is described below.

**FLOWCHART OF THE BF-PSO algorithm**
The transfer function (G) for the process is

\[ 2.295 s^3 - 0.01173 s^2 - 1.547 \cdot 10^{-5} s + 6.115 \cdot 10^{-7} \]

\[ \frac{1}{s^4 - 0.01114 s^3 + 5.288 \cdot 10^{-5} s^2 + 9.232 \cdot 10^{-4} s + 2.066 \cdot 10^{-6}} \]

**6. SIMULATION AND RESULTS**

The step response of the open loop plant is shown in Figure 6.1.

**PID Controller tuning using BF-PSO algorithm** [1]

Now we tune PID parameters Kp, Ki, Kd for above reduced order system by using hybrid BF-PSO algorithm. Fitness Functions are used for designing PID controller ISE, IAE, ITAE and ITSE. We set the following parameters:
- Dimension of search space = 3;
- Number of bacteria = 10;
- Number of chemotactic steps = 10;
- Limits the length of a swim = 4;
- Number of reproduction steps = 4;
- Number of elimination-dispersal events = 2;
- Number of bacteria reproductions (splits) per generation = 3;
- The probability that each bacteria will be eliminated/dispersed = 0.25;
c(:,1)=0.5*ones(s,1); the run length.

We use the following PSO parameters:
- C1 = 1.2;
- C2 = 0.5;
- W = 0.9;

**5.1 BF-PSO-PID Controller tuning using ISE**

We apply the BF-PSO algorithm to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Integral of Squared Error. We obtain

\[ Kp = 362.0052, Ki = -18.5505 \text{ and } Kd = 24.7977 \]

The step response of the BF-PSO-PID controller system is shown in Figure 6.2.

**5.2 BF-PSO-PID Controller tuning using IATE**

We apply the BF-PSO to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Integral of absolute time error. We obtain

\[ Kp = 4.6879 \cdot 10^5, Ki = 385.5821 \text{ and } Kd = 848.8285 \]

The step response of the BF-PSO PID controller system is shown in Figure 6.3.
5.3 BF-PSO - PID Controller tuning using IAE

We apply the BFO-PSO to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Integral of absolute error. We obtain the kp = 12.3353, ki = 0.3128 and kd = 2.3478e+005;

The step response of the BF-PSO PID controller system is shown in figure 6.7.

![Step Response of BF-PSO PID Controller for IAE](image)

Fig. 6.7 The step response of the BF-PSO PID controller system for IAE.

5.4 BF-PSO - PID Controller tuning using MSE

We apply the BF-PSO to tune the PID parameters Kp, Ki and Kd to meet the system performance criteria. We use the Mean Squire Error. We obtain the kp = -0.0938, ki = 1.5905 and kd = 1.2114e+004

The step response of the BF-PSO PID controller system is shown in figure 6.8.

![Step Response of BF-PSO PID Controller for MSE](image)

Fig. 6.8 The step response of the BF-PSO PID controller system for MSE.

### Table 1.1 Comparision of BF-PSO PID Controller system for Different Error Criterias

<table>
<thead>
<tr>
<th>Error criteria</th>
<th>ISE</th>
<th>IATE</th>
<th>IAE</th>
<th>MSE</th>
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<tr>
<td>Parameters</td>
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<tr>
<td>Kp</td>
<td>362.0052</td>
<td>4.6879e+005</td>
<td>12.3353; 0.0938</td>
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<tr>
<td>Ki</td>
<td>-18.5505</td>
<td>385.5821</td>
<td>0.3128; 1.5905</td>
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<tr>
<td>Kd</td>
<td>24.7977</td>
<td>848.8285</td>
<td>2.3478e+005; 1.2114e+004</td>
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</tr>
<tr>
<td>RiseTime</td>
<td>0.0829</td>
<td>7.2113e-004</td>
<td>4.0551e-006; 0.4605</td>
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</tr>
</tbody>
</table>

### 7. Conclusion

We can see from the results that open loop plant is highly unstable system. Then we design the PID controller system using BF-PSO algorithm. From the Table 1.1 we can see that overshoot, rise time and settling time is minimum for IAE criteria. For the above BF-PSO PID controller system with IAE criteria undershoot 0 %, overshoot 0 % and settling time 7.2172e-006 seconds and rise time 4.0551e-006 seconds.

### References