

# PSO optimized reduced order PID Controller design

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**Abstract**— A novel algorithm is proposed to obtain a reduced model for stable linear time invariant continuous system. A PSO PID controller is designed for the reduced order model to meet the desired performance specifications by using PSO Optimization method. This controller is designed with the reduced order model and closed loop response is observed. The parameters of the controller are tuned to obtain a response with desired performance specifications. The results satisfy our design criteria. The same PID controller parameter is applied to original higher order system and the closed loop response is observed for to stabilize system.

**Index Terms**— Reduced Order Modeling, PID Controller Tuning, PSO, Optimization Technique.

## 1. INTRODUCTION

The modeling of complex engineering system is one of the most important in control system. In control engineering field, model reduction techniques are fundamental for the design of controllers where particular complex procedures are involved. This would provide the designer to design with low order controllers that have less hardware requirements and low cost with same performance. Efforts towards obtaining low-order models from high-degree systems are related to the aims of deriving stable reduced-order models from stable original ones and assuring that the reduced-order model matches similar characteristics of original higher order system. The emerging general strategy for generating low-order models is a mixed method of modal reduction. First, a general but non-optimal technique is used to reduce a large model to a medium-sized model. Then a reduction method with an optimization technique is used to generate a very efficient low order model from the medium-sized modal. **Order reduction not goal in itself it is a part of design**. In this paper we reduced the system order by combined approach of Balanced Truncation and PSO optimization.

### 2. Model reduction by balanced truncation

#### For Linear models

Let us first outline the idea of model reduction: we consider a linear dynamical system of the standard (vectorial) form

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t), \quad t > 0, x(0) = x_0 \\ y(t) &= C x(t) + D u(t), \quad t \geq 0. \end{aligned} \quad [1]$$

The system comprises  $n$  state variables  $x_i$ , which are controlled by  $m$  input variables  $u_k$  and can be observed via the  $p$  output variables  $y$ . We postulate that for  $u = 0$ , the system has a steady state at  $x = 0$ . For fixed initial conditions, any time course  $u(\cdot)$  of the controlling variables leads to a time course  $y(\cdot)$  of the observables. The same input output relation can be exactly represented by a system with transformed variables  $\hat{x}$ . If  $T$  is an invertible  $n \times n$  matrix, we can apply the transformation without changing the input/output relation between  $u(\cdot)$  and  $y(\cdot)$ .

$$\begin{aligned} x &\rightarrow \hat{x} = T x \\ A &\rightarrow \hat{A} = T A T^{-1} \\ B &\rightarrow \hat{B} = T B \\ C &\rightarrow \hat{C} = C T^{-1} \end{aligned} \quad [2]$$

## 3. PSO

### A. Overview of pso algorithm

The Particle Swarm Optimization (PSO) is a population-based optimization method first proposed by Kennedy and Eberhart. Some of the attractive features of the PSO include the ease of implementation and the fact that no gradient information is required. It can be used to solve a wide array of different optimisation problems, including most of the problems that can be solved using Genetic Algorithms; some example applications include neural network training and function minimization. [1]

The PSO, similarly to the algorithms belonging to the Evolutionary Algorithm family, is a stochastic algorithm that does not need gradient information derived from the error function. This allows the PSO to be used on functions where the gradient is either unavailable or computationally expensive to obtain. PSO is optimization algorithm based on evolutionary computation technique. The basic PSO is developed from research on swarm such as fish schooling and bird flocking. After it was firstly introduced in 1995, a modified PSO was then introduced in 1998 to improve the performance of the original PSO. A new parameter called inertia weight is added. This is a commonly used PSO where inertia weight is linearly decreasing during iteration in addition to another common type of PSO which is reported by Clerc. In PSO, instead of using genetic operators, individuals called as particles are “evolved” by cooperation and competition among themselves through generations. A particle represents a potential solution to a problem. Each particle adjusts its flying according to its own flying

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experience and its companion flying experience. Each particle is treated as a point in a D-dimensional space. The *i*th particle is represented as  $X_i=(x_{i1},x_{i2},\dots,x_{iD})$ . The best previous position (giving the minimum fitness value) of any particle is recorded and represented as  $P_i=(p_{i1},p_{i2},\dots,p_{iD})$ , this is called *pbest*. The index of the best particle among all particles in the population is represented by the symbol *g*, called as *gbest*. The velocity for the particle *i* is represented as  $V_i=(v_{i1},v_{i2},\dots,v_{iD})$ . The particles are updated according to the following equations:

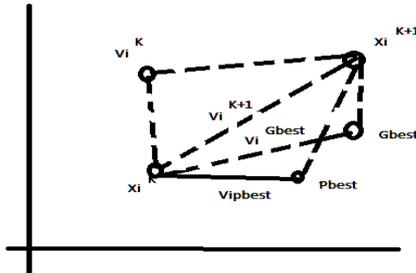


Fig3.1 Concept of modification of a searching point by PSO

$$V_{i,m}^{(t+1)} = W.V_{i,m}^{(t)} + C_1 * rand() * (Pbest_{i,m} - X_{i,m}^{(t)}) + C_2 * rand() * (gbest_m - X_{i,m}^{(t)}) \quad [3]$$

$$X_{i,m}^{(t+1)} = X_{i,m}^{(t)} + V_{i,m}^{(t+1)} \quad [4]$$

where *c1* and *c2* are two positive constant.  $c1=0.12,c2=1.2$ . While *rand()* is random function between 0 and 1, and *n* represents iteration. Eq.[3] is used to calculate particle's new velocity according to its previous velocity and the distances of its current position from its own best experience (position) and the group's best experience. Then the particle flies toward a new position according to Eq.[4]. The performance of each particle is measured according to a pre-defined fitness function (performance index), which is related to the

problem to be solved. Inertia weight, *w* is brought into the equation to balance between the global search and local search capability. It can be a positive constant or even positive linear or nonlinear function of time. We use  $w=0.9$ . It has been also shown that PSO with different number of particles (swarm size) has reasonably similar performance. Swarm size of 10-50 is usually selected. Here, we set 50.

**(B) Flowchart of the PSO-PID Control system**

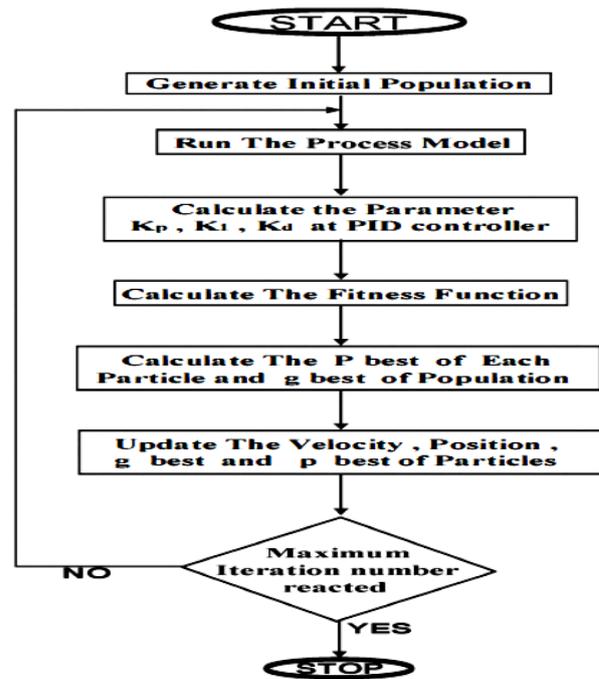


Fig3.2 Flow Chart of PSO Algorithm

**4. PID controller system**

PID controller consists of Proportional, Integral and Derivative gains. The feedback control system is illustrated in Fig. 1 where *r*, *e*, *y* are respectively the reference, error and controlled variables.

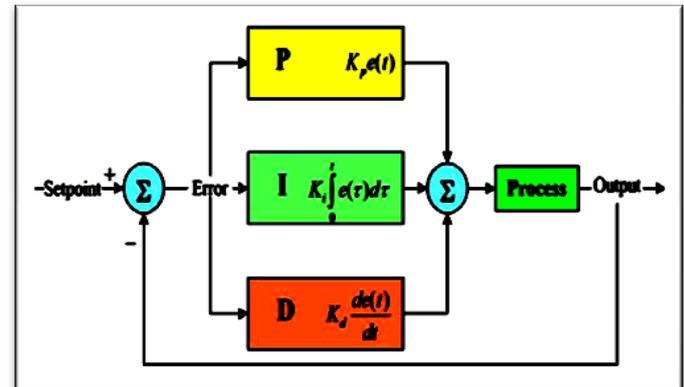


Fig4.1: A common feedback PID control system

In the diagram of Fig.1, *G(s)* is the plant transfer function and *C(s)* is the PID controller transfer function that is given as:

$$C(s) = K_p + \frac{K_i}{s} + K_d \quad [5]$$

Where *Kp*, *Ki*, *Kd* are respectively the proportional, integral, derivative gains/parameters of the PID controllers that are going to be tuned.

**5 Performance evaluation criteria**

Quantification of system performance is achieved through a performance index. The performance selected depends on the process under consideration and is chosen such that emphasis is placed on specific aspects of system performance.

Furthermore, performance index is defined as a quantitative measure to depict the system performance of the

designed PID controller. Using this technique an ‘optimum system’ can often be designed and a set of PID parameters in the system can be adjusted to meet the required specification. For a PID- controlled system, there are often four indices to depict the system performance: ISE, IAE, ITAE and ITSE. They are defined as follows:

ISE Index:

$$ISE = \int_0^{\infty} e^2(t) dt \quad [6]$$

IAE Index:

$$IAE = \int_0^{\infty} |e(t)| dt \quad [7]$$

ITAE Index:

$$ITAE = \int_0^{\infty} t|e(t)| dt \quad [8]$$

ITSE Index:

$$ITSE = \int_0^{\infty} te^2(t) dt \quad [9]$$

Performance index (F1)

$$f = \sum \text{Max error}^2 - \sum \text{Current error}^2$$

$$F1 = 1/f \quad [10]$$

The above performance index is used to minimize the error between step response of higher and reduce order system.

Performance index (F2)

The following performance index F2 is used to minimize the overshoot, settling time, steady state error and reference tracking error for pso pid controller system.

$$F2 = e2 * \beta + \text{sys\_overshoot} * \alpha \quad [11]$$

Where  $\alpha$  and  $\beta$  are constant. Therefore, for the PSO-based PID tuning, these performance indexes will be used as the objective function. In other word, the objective in the PSO-based optimization is to seek a set of PID parameters such that the feedback control system has minimum performance index.

## 6. SIMULATION AND RESULTS

### Modal Reduction with Balance Truncation

24<sup>th</sup> order Transfer function selected for modal reduction [10]:

$$[-1.1 s^{23} - 12.44 s^{22} + 2893 s^{21} + 2764 s^{20} + 1212 s^{19} + 324.2 s^{18} + 59.29 s^{17} + 7.864 s^{16} + 0.7827 s^{15} + 0.05961 s^{14} + 0.003508 s^{13} + 0.00016 s^{12} + 5.623e-006 s^{11} + 1.505e-007 s^{10} + 3e-009 s^9 + 4.293e-011 s^8 + 4.155e-013 s^7 + 2.445e-015 s^6 + 6.972e-018 s^5 + 4.046e-021 s^4 + 8.036e-024 s^3 - 1.177e-026 s^2 - 5.48e-030 s + 8.511e-034]$$

$$[s^{24} + 35.38 s^{23} + 78.31 s^{22} + 68.51 s^{21} + 32.81 s^{20} + 9.998 s^{19} + 2.106 s^{18} + 0.3225 s^{17} + 0.03703 s^{16} + 0.00325 s^{15} + 0.0002203 s^{14} + 1.156e-005 s^{13} + 4.687e-007 s^{12} + 1.45e-008 s^{11} + 3.36e-010 s^{10} + 5.64e-012 s^9 + 6.52e-014 s^8 + 4.785e-016 s^7 + 1.95e-018 s^6 + 8.99e-021 s^5 + 4.505e-024 s^4 + 2.982e-024 s^3 + 1.148e-030 s^2 + 2.389e-034 s + 2.078e-038] \quad [12]$$

Then we reduce the above higher order transfer function via reduce order modeling. We use balanced Truncation method for modal reduction purpose.

Resulted reduced 4th Order system

$$2.295 s^3 - 0.01173 s^2 - 1.642e-005 s + 4.785e-010$$

$$s^4 - 0.01114 s^3 + 5.288e-005 s^2 - 9.232e-010 s + 2.06e-014 \quad [13]$$

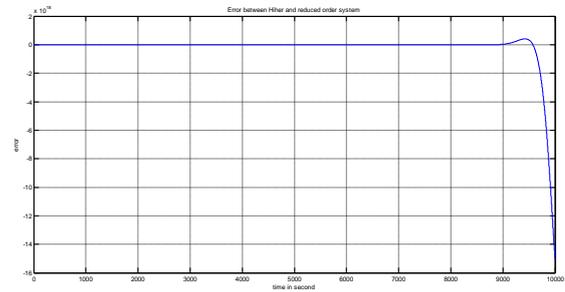


Fig6.1 Error between step response of higher and reduced order system

We can see from the above fig. 6.4 that error is acceptable till 9000sec .After this time error becomes unbounded.

### Numerator Parameter tuning for modal reduction using PSO optimization

Then we apply Particle Swarm Optimization Algorithm for reducing error between higher and reduced order system .Since amplitude of step response is changing for higher and reduced order system (obtained by Balanced Truncation) .To match the system Performance we use the following proposed scheme .We set parameter A and B in numerator. Then use PSO Optimization Technique to tune parameters A,B for reducing steady state error between step response of higher order and reduced order system.

$$2.295 s^3 - 0.01173 s^2 + (- 1.642e-005+A) s + (4.785e-010+B)$$

$$s^4 - 0.01114 s^3 + 5.288e-005 s^2 - 9.232e-010 s + 2.06e-014 \quad [14]$$

We use the following fitness function

$$f = \text{Max error}^2 - \text{Current error}^2$$

$$F = 1/f$$

We use the following PSO parameters to match the both systems.

no of birds = 100, Maximum number of birds steps=50,

dimension = 2, c2 = 1.2, c1 = 0.12 and

w = 0.8; w is inertia weight.

We obtained the following reduced 4<sup>th</sup> order transfer function:

$$2.295 s^3 - 0.01173 s^2 - 1.639e-005 s + 7.368e-009$$

$$s^4 - 0.01114 s^3 + 5.288e-005 s^2 + 9.232e-010 s + 2.06e-014 \quad [15]$$

Fig.6.5 shows that step response of higher order ,reduced order system (BT combined with PSO).

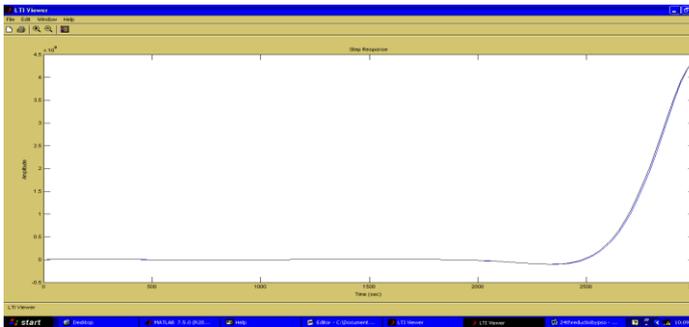


Fig 6.2 Step response of Higher Order, Reduced order system (by BT and PSO).

**PID Controller tuning using PSO Algorithms**

Now we tune PID parameters  $K_p$ ,  $K_i$ ,  $K_d$  for above reduced order system by using PSO Algorithm .

We use the following PSO parameter to tune the PID controller system.

Size of the swarm no of birds = 50;      Maximum number of birds steps=50;

Dimension of the problem = 3;

$c_2 = 1.2$ ;     $c_1 = 0.12$ ;

$w = 0.9$ ;

We obtain the following PID parameters by tuning GA-PSO system for GA reduced order system.

$K_p = 0.6914, K_i = 0.1446, K_d = 1.5615$

Table 6.1

Parameters	Reduced Order system	Higher Order system
RiseTime	0.4986	0.4397
SettlingTime	5.1744	1.1545
Overshoot	9.7429	5.6364
PeakTime	1.3544	0.5756

Fig.6.4 shows that step response of reduced order PSO PID controller system with actuator and without actuator. Now we apply the same PID parameters to the original higher PID controller system system. Fig.8.8 shows the step response of higher order PID controller system

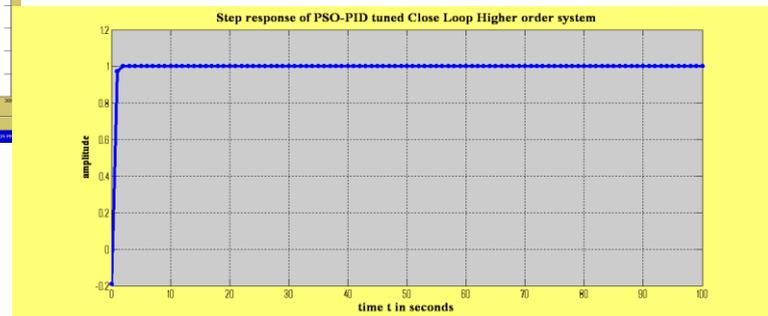


Fig. 6.5 Step response of Original higher order PSO PID cotroller system

**Conclusion**

We can see from the results that there is error between step response of higher and reduced order system. To reduce error we apply the PSO optimization algorithms. We design the PID controller parameters for reduced order system (BT and GA ) using PSO technique. We observe the system performance for reduced order system. Then we apply the same PID parameters to the higher order controller system. We get the maximum overshoot 5.6364 percent and settling time 1.1545seconds for higher order system.

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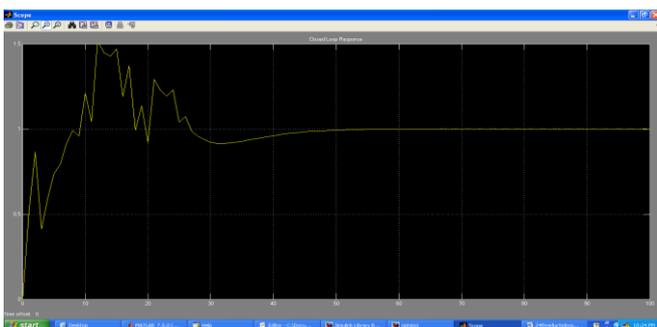


Fig.6.3 Step response of reduced 4<sup>th</sup> order system with actuator.

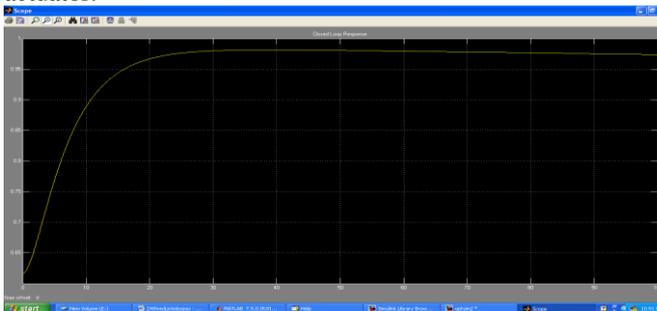


Fig.6.4 Step response of reduced 4<sup>th</sup> order system without actuator