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Determination of optimum coefficients of IIR Digital Butterworth Band-Stop Filter

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***Abstract*— Infinite Impulse Response(IIR) filter is of recursive type i.e. the present output sample depends on the present input samples, past input samples and past output samples. There are a number of methods available for the determination of filter coefficients. This paper is based on the analog to digital mapping technique to determine the IIR digital filter coefficient along with the computer simulation in Matlab 7 on the basis of the proposed algorithm. The result of the simulation is found to be satisfying.**

***Index Terms*— IIR filter, Digital filters, Band Stop Filter, coefficient, analog to digital mapping.**

I. Introduction

Filter is very essential to process different signal. There are two type of filters, the first one is the Ideal filter and the second one is the practical filter[1][2]. In case of the impulse response of the filter, the filter can be categorized as the Infinite Impulse Response(IIR) Filter and the Finite Impulse Response(FIR) Filter[1][2][3]. In analog domain, when the filter, basically the circuit of the filter is constructed, this is known as the analog filter. After the proper analog circuit design of the filter, when the analog to digital mapping technique is applied to it, the generated filter is known to as digital filter[3][4][5].

The impulse response of the IIR filter is of infinite duration but the impulse response of the FIR filter is of finite duration. IIR filter processes some properties such as pass-band width, stop-band width, maximum allowable pass-band ripple and maximum allowable stop-band ripple[4][5][6] . A suitable IIR filter can be designed with help of those properties. There are various methods available for designing the digital IIR filter. In this paper, the digital IIR filter is designed from analog filter by applying the analog to digital mapping technique[1][2][4].

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The digital filter, very essential in Digital Signal Processing(DSP), is employed for signal filtering in time domain[3][6]. The analog filter can be constructed from the analog components such as resistor, capacitor and with or without OpAmp IC. If an active component such as the Voltage source is used to design an analog filter, this is known as active filter[4][7], otherwise it is called the passive filter[2][8]. The digital filters have many features such as high accuracy and reliability, small physical size and reduced sensitivity to component tolerances or drift[4][8][9], and depending upon those features, the analog filter can be replaced by digital filter for better performance in filtering the signal.

Now, when the analog filter is designed, by applying the frequency transformation(Bilinear transform or Impulse invariant method) or analog to digital mapping technique, the proper digital filter can be easily obtained[1][2][4][6].

II. Design of Digital IIR Filter

There are mainly six types of filter and they are,

1. Low pass filter
2. High pass filter
3. Band pass filter
4. Band stop or Notch filter
5. All pass filter
6. Comb filter

The low pass filters filter out the low frequency band, the Highpass filters filter out the high frequency band, the Band pass filter pass a specific band of frequency. The Band stop filter reject a specific band of frequencies and the Notch filter is an extreme form of Band stop filter that is used to reject only one specific frequency. All pass filter pass all the input frequencies but with change in phase. The Comb filter is special type of filter which has multiple pass bands which form a shape looks like the comb[1][2][10][11][12].

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We have concentrated on the study of Band stop filter and the determination of the optimum coefficients of this filter. The circuit of 3rd order Butterworth Band stop filter is shown in fig.1[12][13].

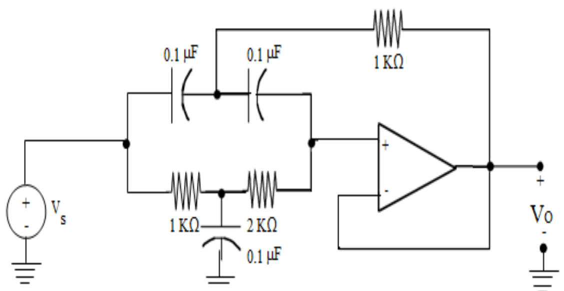

Fig.1 Band Stop Filter

We can obtain the transfer function of the analog filter in the analog domain or s-domain. After that, with the application of analog-to-digital mapping technique, the required digital filter will be suitably designed[1][2].

The impulse response for a realizable filter is,

$$h(n) = 0 \quad \text{for } n \leq 0 \qquad \text{… …(1)}$$

The satisfactory condition for a stable filter is,

$$\sum_{n=0}^{\infty} |\,h(n)\,| < \infty \qquad \text{... …(2)}$$

Now the generalized transfer function[1][2][4][5] of an IIR Digital filter is,

$$H(z) = \frac{\sum_{n=0}^{M} b(n) z^{-n}}{1 + \sum_{n=1}^{N} a(n) z^{-n}} \qquad \text{… …(3)}$$

$$= \frac{B(z)}{A(z)} = \frac{b(0) + b(1)z^{-1} + b(2)z^{-2} + ... + b(M)z^{-M}}{1 + a(1)z^{-1} + a(2)z^{-2} + ... + a(N)z^{-N}} \qquad \text{… …(4)}$$

Where,

b(n) = Numerator coefficient
a(n) = Denominator coefficient

The numerator and denominator coefficients are essential for the filter design. There are various methods available for design and calculation of the filter and filter coefficient. The algorithm proposed in this paper has the eligibility to determine the filter coefficients as well as design of the desired filter efficiently[4][5][6]. Through this algorithm we can also get the transfer function in the digital domain. The algorithm is given in Fig. 2

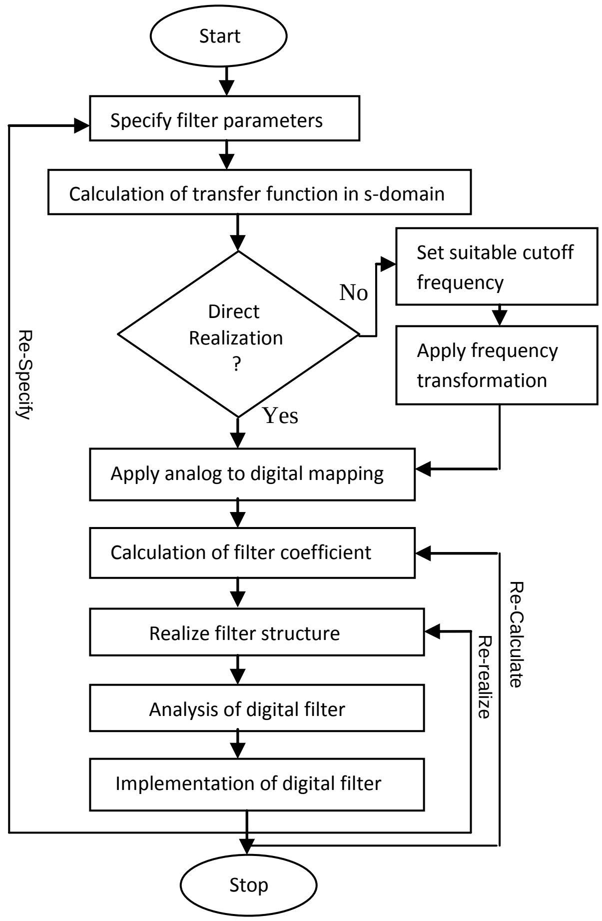

Fig.2 Proposed algorithm

Now, with help of the algorithm and by specifying the required parameters, a required filter can be designed in proper way. This algorithm is also helpful to convert the frequency transformation such as Low pass filter to High pass filter and vice versa.

The algorithm also provides two types of realization, one is the direct realization and another is the indirect realization. In direct realization, the analog to digital mapping technique is applied to map the transfer function from s-domain to z-domain. In direct realization, the suitable cut-off frequency must be predefined. The stability of the system is determined by the pole zero plot. By applying the algorithm, a suitable stable filter can be designed.

III. IIR BAND STOP FILTER

The Band stop filter eliminates a selected band of frequencies. This filter is also known to as Band-reject filter or Band-eliminator filter. If a band stop filter eliminates a single frequency, the filter is known to as the Notch filter. The construction of a Band stop filter is shown in Fig.1. The Band stop filter can be constructed by connecting the low pass filter in parallel with the high pass filter. The circuit for the passive Band stop filter by using inductor and capacitor (T-section) is shown in Fig.3 [13][14][15]

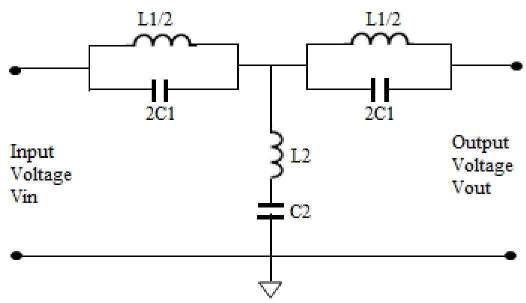

Fig.3 T-section Band stop filter(LC)

The designing equation for the T-section Band stop filter is given below[14][15],

$$f_m = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} \quad \text{.... .(5)}$$

$$R_k = \sqrt{L_1/C_1} = \sqrt{L_2/C_2} \quad \text{....(6)}$$

$$C_1 = \frac{1}{2R_k(\omega_2 - \omega_1)} \quad \text{....(7)}$$

$$L_1 = \frac{2R_k(\omega_2 - \omega_1)}{\omega_2\omega_1} \quad \text{....(8)}$$

$$C_2 = \frac{2}{R_k}\frac{(\omega_2 - \omega_1)}{\omega_2\omega_1} \quad \text{....(9)}$$

$$L_2 = \frac{R_k}{2(\omega_2 - \omega_1)} \quad \text{....(10)}$$

Where,

R_k = Nominal characteristic impedance

f_m = Geometric mean of two cut-off Frequencies

ω_1 = Low cut-off frequency

ω_2 = High cut-off frequency

The band stop filters can be classified into two categories as follows,

1. Wide band stop filter
2. Narrow band stop filter

The narrow band stop filter is typically known to as the Notch filter. The most common narrow band stop filter is twin-T band stop filter, already shown in Fig.3. The network for the twin-T band stop filter shown in Fig.3 is a LC passive filter. The modified passive RC twin-T band stop filter shown in Fig.4[14][15]

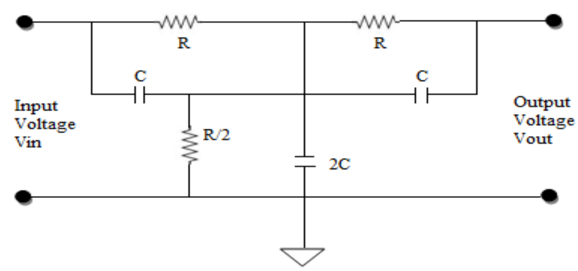

Fig.4 Twin-T passive Band stop filter(RC)

The equivalent band stop filter can be constructed by an op-amp along with resistors, capacitors and a voltage source, i.e. the active filter and this is a typically active IIR Butterworth Band stop filter, as shown in Fig.5[16]

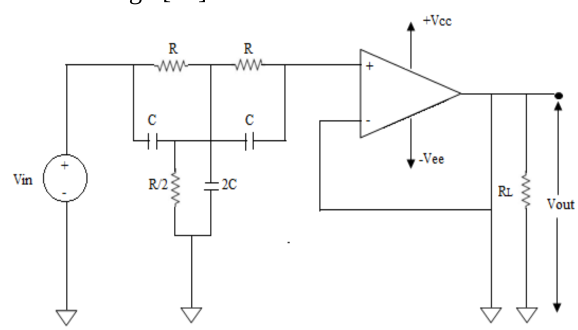

Fig.5 Active IIR Band stop filter

So, till now the matter about the analog IIR Band stop filter is discussed. Now, it is necessary to design the digital IIR Band stop filter from the predesigned analog IIR Band stop filter. In the next section the technique for the analog to digital domain conversion or analog to digital mapping or s-domain to z-domain mapping is introduced.

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IV. ANALOG TO DIGITAL MAPPING TECHNIQUE

The analog to digital mapping technique is applied to transform the transfer function of a circuit from analog or s-domain to digital or z-domain. Now, a filter is called a analog filter if we have the transfer function in s-domain and the filter is called the digital filter if its transfer function is in z-domain.

Let the impulse response of a filter in time domain is h(t). Now by applying the Laplace transform on the impulse function h(t), we can get the transfer function on frequency domain or s-domain[1][2][3][4], i.e.,

$$H(s) = L\{h(t)\} = \int_0^{\infty} h(t).e^{-st}dt \qquad ....(11)$$

Where,

$s = \sigma + j\omega$
= complex variable

Now, if h(t) is continuous in nature, we can get the discrete form of h(t) by substituting

$$t = nT \qquad .....(12)$$

Where,

T = sampling time

So, h(t) becomes h(nT) with the application of equation(12). Now, if T=1 sec, then h(t) becomes,

$$h(t) = h(n) \qquad .....(13)$$

Now, as soon as the h(n) is obtained, the transfer function in z-domain i.e. H(z) can be obtained by applying the Z-transform over h(n) i.e.[1][2],

$$H(z) = Z\{h(n)\} = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \qquad .....(14)$$

So, the transfer function of an IIR Digital filter can be obtained with help of equation(14). The relationship between the z-plane and s-plane can de described by equation(15)[2][3],i.e.

$$z = e^{sT} \qquad ....(15)$$

V. SIMULATION RESULT AND DISCUSSION

The Matlab 7.6.0 (R2008a) version is used to write and simulate the IIR Butterworth Band stop filter designing program. The simulation results for the magnitude response, phase response, impulse response and the pole-zero plot by choosing proper specifications such as passband frequency, passband ripple, stopband frequency, stopband ripple, are shown from fig.4 to fig.10. In this simulation result, the pole zero plot shows that the Band stop filter, that is designed, is stable. So, by applying the algorithm, we can design a stable Band stop filter along with the determination of the filter coefficient.

In this simulation process, the bilinear transform is used for analog to digital mapping technique. The exact Matlab code that is helpful to calculate and determine the filter coefficient is,

```
[bz,az] = bilinear(b,a,fstop);
```

Where,

bz = Numerator coefficient in z-plane
az = Denominator coefficient in z-plane
bilinear = Bilinear transform
b = Numerator coefficient in Bilinear transform
a = Denominator coefficient in Bilinear transform
fstop = stopband frequency

The coefficients for the IIR Butterworth Band stop filter are shown in Table 1.

Table.1 Coefficient of Butterworth Band stop filter

Filter name	Filter order	Numerator coefficient	Denominator coefficient
Butterworth Band stop Filter	4	-0.7683, 0.7753, -0.365, 0.1159	-3.454, 4.585, -2.305, 0.4132
	6	-0.9138, 1.727, -1,812, 1.518, -0.7052, 0.2509	-5.944, 15.4, -20.55, 14.23, -5.043, 0.7679

The simulation results for the magnitude response, phase response, impulse response and the pole zero plot of IIR Butterworth Band stop filter are shown from Fig.6 to Fig.13.

A. IIR Butterworth Band stop filter(Order = 4)

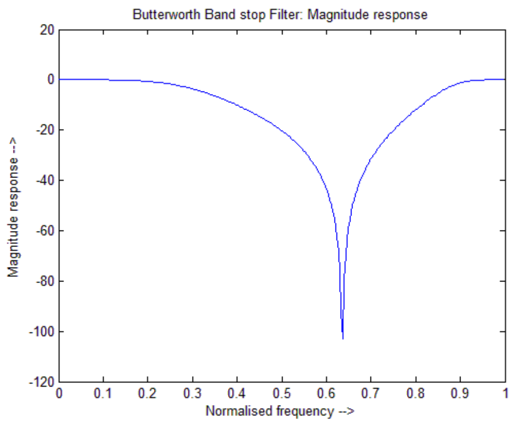

Fig.4 Magnitude response(Order=4)

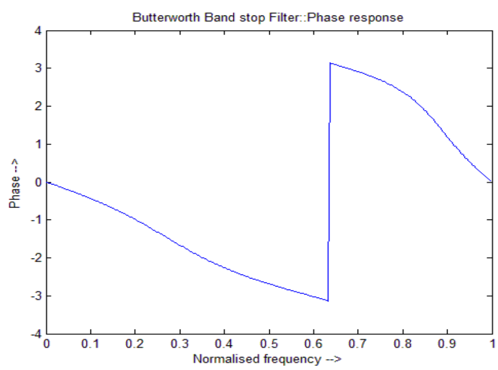

Fig.6 Phase response(Order=4)

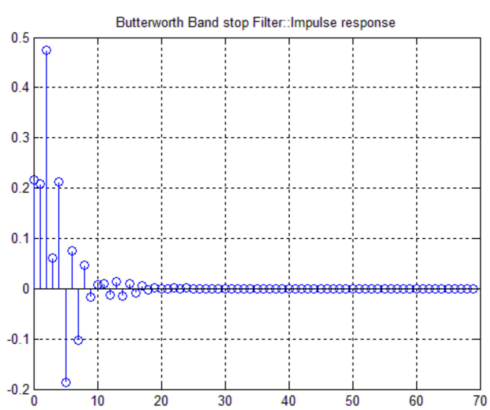

Fig.8 Impulse response(Order=3)

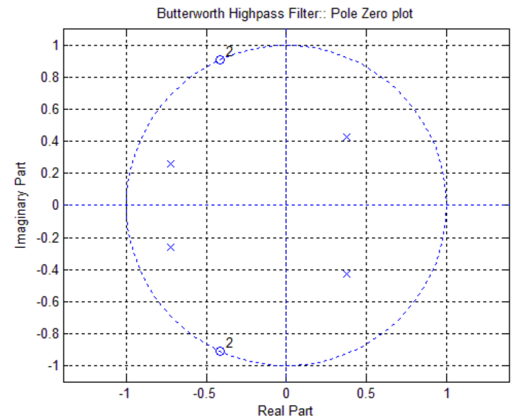

Fig.10 Pole-Zero plot(Order=3)

B. IIR Butterworth Band stop filter(Order = 4)

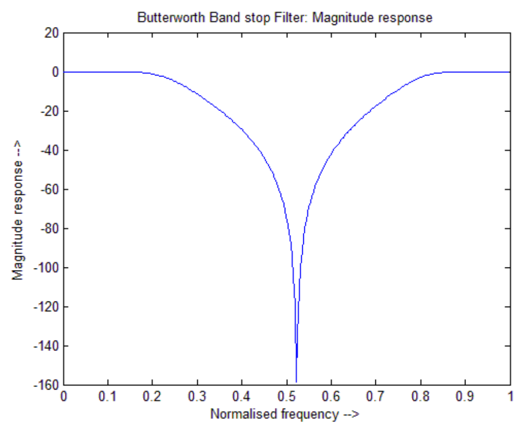

Fig.4 Magnitude response(Order=6)

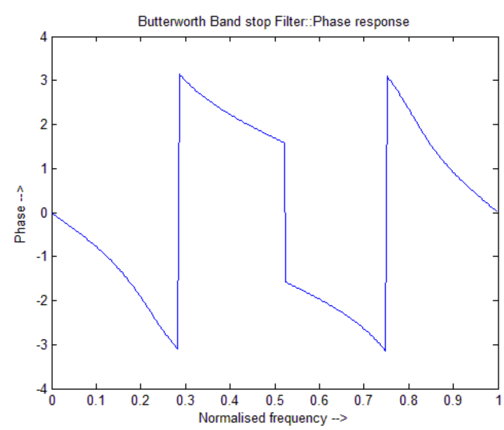

Fig.6 Phase response(Order=6)

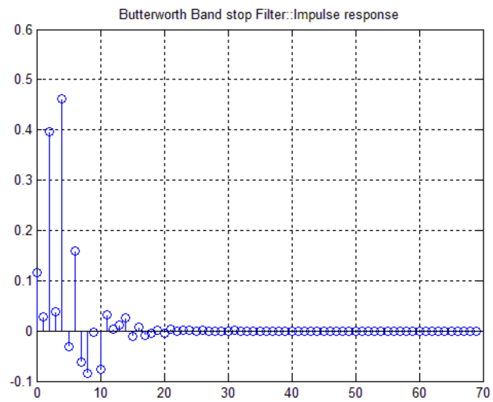

Fig.8 Impulse response(Order=6)

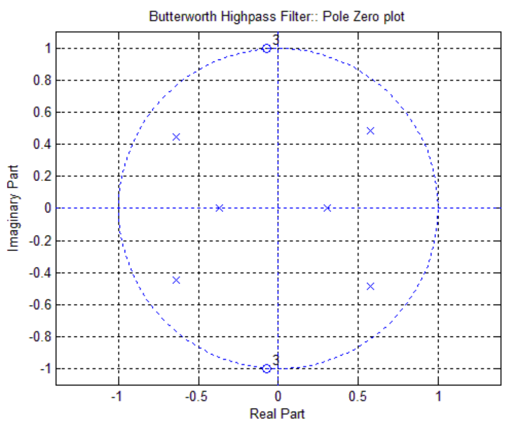

Fig.10 Pole-Zero plot(Order=6)

VI. CONCLUSION

In conclusion, it can be said that after observing the simulation result, the IIR Digital Butterworth Band stop filter can be designed properly with its stability. The proposed algorithm is perfect for the design of the filter which is reflected in the coefficient values and the simulation result particularly the magnitude response and the pole zero plotting based on Matlab 7.6.0. The coefficients that are determined are essentially required for the digital filter design.

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