Spectrum Sensing Based on Poynting Vector Theorem for Cognitive Radio Networks

Amit Talreja
Masters of Engineering
JEC, Jabalpur

Prof Prabhat Patel
HoD ECE
JEC Jabalpur

Abstract: This paper has examined the problem of dynamic spectrum access in the presence of a licensed signal, when unlicensed communicating devices have intelligent radios capable of sensing and reacting to their environment. Our studies have discovered considerable under-utilization of the allocated spectrum by the licensed users. This suggests that the solution to the problem is a transition from static spectrum allocation policies to dynamic spectrum access methodologies. This can be accomplished through the use of Cognitive Radio technology. Cognitive Radio is considered as an intelligent radio which is capable of altering its transmission or reception parameters in accordance to the radio environment and the network state to use the available spectrum optimally.

Spectrum sensing is a technique used to detect the presence of primary users in the licensed spectrum. Various techniques exist for performing spectrum sensing. In addition to primary user detection, spectrum sensing can also be employed for secondary detection and co-existence, interference analysis in multi-radio environments etc.

This paper proposes a new technique to sense, with as low probability of error as possible, weather a frequency spectrum contains a signal or not. For this, POYNTING VECTOR THEOREM is used which state that “the vector product P =E X H at any point is a measure of rate of energy flow per unit area at that point [1]”. Thus the cross multiplication of measured Electric and Magnetic fields containing noise with them will give power flow in that region. The Noise is considered to be white Gaussian in nature. Since nearly same white noise is added to both Electric and Magnetic fields, there is cross multiplication of two similar noise vectors which is always zero. Thus a major portion of noise is automatically eliminated. Results generated by the computer simulation show that the proposed method is more efficient in sensing the vacant spectrum as compared to the existing technique in literature.

Index Terms—Cognitive Radio, Spectrum Sensing, Poynting Vector, EM Waves.

(1) Introduction

Measurements conducted by the Office of Communications (Ofcom) in UK and the Spectrum Policy Task Force (SPTF) in USA indicate that a major of the licensed spectrum is only partially used for significant periods of time [2,3]. Cognitive Radio (CR) is regarded as the technology which will increase the spectrum utilization significantly in the next generation wireless communication systems by implementing opportunistic spectrum sharing. A CR is a radio frequency transmitter/receiver that is designed to intelligently detect whether a particular segment of the radio spectrum is currently in use, and to jump into the temporarily-unused spectrum very rapidly, without interfering with the transmissions of other authorized users [4]. Main functions of CR are [5]: Spectrum sensing, spectrum management, spectrum mobility and spectrum sharing. Spectrum sensing detects the unused spectrum and shares it without harmful interference with other licensed users. Spectrum Management is the task of selecting the best available spectrum to meet user communication requirements. Spectrum Mobility is defined as the process when a cognitive radio user exchanges its frequency of operation. Spectrum Sharing decides which secondary user (SU) can use the spectrum hole at some particular time. One of the major challenges in open spectrum usage is the spectrum sharing, which is also known as Dynamic Spectrum Sharing problem.

Spectrum sensing is one of the most crucial task in a cognitive radio network; it allows the unauthorized users, called SUs, to detect unused portions of the spectrum called “spectrum holes” and opportunistically utilize these spectrum holes without causing harmful interference to the Primary user (PU). To date many spectrum sensing algorithms have been proposed, including e.g., energy detection (ED) [6,7], matched filtering [7], cyclostationary detection [7] eigenvalue-based detection [9] and Polarization Vector Orientation [10]. In order to evaluate the performance of spectrum sensing, two metrics are of great
interest: probability of detection $P_d$ and the probability of false alarm $P_f$. Probability of detection, $P_d$, determines the level of interference-protection provided to the PU while probability of false alarm, $P_f$, indicates percentage of spectrum holes falsely declared as occupied [11]. In the context of spectrum access $P_f$ must be higher than some predefined threshold while $P_f$ should be lower than some desired criteria or as minimum as possible.

In this paper, we consider a spectrum sensing method based on the cross Product of Electric field Intensity (E) and Magnetic Field Intensity (H) at a given point in space over a certain band of frequency. Its mathematical ground lies in the fact that the cross product of two similar vectors is zero.

The rest of this paper is organized as follows. In section 2, statistical model for Cognitive radio Network is presented. Then local Spectrum sensing technique using pointing vector is developed in section 3.

(II) Statistical Model for Cognitive Radio Network

Assume that each CR is equipped with an energy detector that is capable of measuring E (Electric field Intensity (V/m)) and M (Magnetic Field Intensity (A/m)) and is able to perform local spectrum sensing independently. Each CR makes its own observation based on the received signal, that is, information plus noise or noise only. Hence the spectrum sensing problem can be considered as a binary hypothesis testing problem with two possible hypothesis $H_0$ (Presence of only Noise (Hypothesis)) and $H_1$ (Presence of Signal + Noise (Hypothesis)) defined as

$$x(t) = \begin{cases} n(t) & H0 \\ \alpha s(t) + n(t) & H1 \end{cases}$$

where $s(t)$ is the PU signal and is assumed to be an identical and independent random process (i.i.d.). For the $i^{th}$ SU, the receiver noise is modeled as $n(t)$ which is also assumed to be an i.i.d. random process with zero mean, variance $\sigma_n^2$ and power spectral Density (PSD) $\frac{N_0}{2}$ and $\alpha$ is the reduction factor at the receiver due to channel and is assumed to be binomially distributed. Further, it is assumed that the signal and the noise are independent of each other. The ratio of received power to the power of noise is defined as the SNR at the SU receiver. The received SNR at the the SU can be more precisely defined as

$$\frac{E_x}{N_0} = \frac{E[\alpha^2 s(t)]}{\gamma^2 n} = \frac{E[\alpha^2] E[s(t)]}{\sigma^2 n}$$

Since $\alpha$ is a binomially distributed random variable the estimated mean can be assumed as a constant [12]

(III) Local Spectrum Sensing Based on Poynting Vector

Consider a receiver model involving a linear time invariant filter. This filter measures the electric and magnetic component of signal corrupted by additive channel noise $n(t)$ at an interval of $T$ sec apart. Let the electric field is given by

$$E(t) = E_0(t)a_x + n(t)$$

assuming the electric field in x direction and noise present in all direction. Further assuming that magnetic field is in y direction only.

$$M(t) = M_0(t)a_y + n(t)$$

Now since white noise consist of infinite dimension, hence

$$n(t) = \sum_{n=1}^{\infty} \eta_n(t)a_n$$

Since we have assumed that the signal is 3 dimensional $x, y, z$ and the noise is infinite dimensional it can shown that out of all the components of noise only 3 component of noise will affect the signal being transmitted and rest all can be ignored [11]. Thus, we can write

$$n(t) = \sum_{n=x,y,z} \eta_n(t)a_n$$

or

$$n(t) = n_x(t)a_x + n_y(t)a_y + n_z(t)a_z$$

From Poynting vector theorem, Complex Power is given by

$$P = \frac{1}{2} (E(t) \times M^*(t))$$

From Maxwell’s first equation for Time varying field

$$\Delta \times M = \frac{dD}{dt} + J$$

where $\Delta = \frac{d}{dx} a_x + \frac{d}{dy} a_y + \frac{d}{dz} a_z$ is a differential operator.
where \( \varepsilon \) is the permittivity of free space (Farad/Meter) and \( \sigma \) is the conductivity of free space which is zero, hence

\[
\Delta \times M = \frac{\partial \varepsilon}{\partial t}
\]

From Maxwell’s second Equation for Time varying field

\[
\Delta \times E = -\mu \frac{\partial M}{\partial t}
\]

Where \( \mu \) is the permeability of free space (Henry/ meter).

Now in Phasor form these equation may be written as

\[
\Delta \times M = j \omega \varepsilon \mathbf{E}
\]

where \( \omega \) is the frequency of operation

Or \( \Delta \times M^* = j \omega \varepsilon \mathbf{E}^* \) ................................................................. (6)

And

\[
\Delta \times E = -j \omega \mu \mathbf{M}
\]

The divergence of complex Poynting vector specified above is

\[
\nabla \cdot \mathbf{P} = \frac{1}{2} \nabla \cdot (\mathbf{E} \times \mathbf{M}^*)
\]

\[
\nabla \cdot \mathbf{P} = \frac{1}{2} (\mathbf{M} \cdot \nabla \mathbf{E} - \mathbf{E} \cdot \nabla \mathbf{M}^*)
\]

Now putting the value of \( \Delta \times M^* \) and \( \Delta \times E \) from equation (6) and (7) in above equation, we get

\[
\nabla \cdot \mathbf{P} = \frac{1}{2} (\mathbf{M}^* \cdot (-j \omega \mu \mathbf{M}) - E \cdot (j \omega \varepsilon \mathbf{E}^*))
\]

Or

\[
\nabla \cdot \mathbf{P} = \frac{1}{2} (-j \omega \mu \mathbf{M}^* - j \omega \varepsilon \mathbf{E}^*)
\]

Taking the volume integral of above written equation

\[
\iiint \nabla \cdot \mathbf{P} \, dv = (-j \omega) / 2 \iiint (\mu \mathbf{M}^* + \varepsilon \mathbf{E}^*) \, dv \] ........................................... (8)

But the divergence theorem states that a volume integral can be transformed to a surface integral by applying a simple operation i.e.

\[
\oint \mathbf{P} \cdot d\mathbf{s} = \iiint \nabla \cdot \mathbf{P} \, dv
\]

Thus

\[
\oint \mathbf{P} \cdot d\mathbf{s} = (-j \omega) / 2 \iiint (\mu \mathbf{M}^* + \varepsilon \mathbf{E}^*) \, dv \] ........................................... (9)

Again in equation (5), if we put the value of \( E(t) \) and \( M(t) \) from equation (1) and (2) we get

\[
P = \frac{1}{2} \{ (E_0(t) \cdot a_x + n(t)) \times (M_0(t) \cdot a_y + n(t)^*) \}
\]

Or

\[
P = \frac{1}{2} \{ (E_0(t) \cdot a_x + n(t)) \times (M_0(t) \cdot a_y + n(t)^*) \}
\]

but as already stated above that the noise assumed is real and gaussian in nature, hence \( n(t)^* = n(t) \) therefore above equation can be written as

\[
P = \frac{1}{2} \{ (E_0(t) \cdot a_x + n(t)) \times (M_0(t) \cdot a_y + n(t)) \}
\]

Thus

\[
P = \frac{1}{2} \{ E_0(t) M_0(t) a_x + E_0(t) a_x X n(t) + n(t) X M_0(t) a_y + n(t) X n(t) \}
\]

From the vector law we know that cross product of two similar vector is zero.

Hence

\[
P = \frac{1}{2} \{ E_0(t) M_0(t) a_x + E_0(t) a_x X n(t) + n(t) X M_0(t) a_y \}
\]

Now distributing the noise components and cross multiplying it with \( a_x \) and \( a_y \)

\[
\text{OR} \quad a_x X n(t) = a_x (n_x(t) a_x + n_y(t) a_y + n_z(t) a_z)
\]

\[
= n_x(t) a_x - n_y(t) a_y \quad \text{.................................(11)}
\]

\[
\text{OR} \quad n(t) a_y = [n_x(t) a_x + n_y(t) a_y + n_z(t) a_z] X a_y
\]

\[
= n_y(t) a_x - n_z(t) a_z \quad \text{.................................(12)}
\]

Now from equation (8) it is clear that Power transferred has no real component, hence

\[
\frac{1}{2} \text{Re} \{ E_0(t) M_0(t)^* a_x a_x + E_0(t) a_x X n(t) + n(t) X M_0(t) a_y \} = 0
\]

Or

\[
\frac{1}{2} \text{Re} \{ E_0(t) M_0(t)^* a_x a_x + E_0(t) a_x X n(t) + n(t) X M_0(t) a_y \} = 0
\]

Or

\[
E_0(t) M_0(t)^* a_x a_x - E_0(t) n_x(t) a_y + E_0(t) n_y(t) a_x + M_0(t) n_x(t) a_x - M_0(t) n_x(t) a_y = 0
\]

Or

\[
M_0(t)^* n_x(t) a_x - E_0(t) n_x(t) a_y + E_0(t) n_y(t) a_x + M_0(t) n_x(t) a_x - M_0(t) n_x(t) a_y = 0
\]

Since the above written term is distributed in three different component, we can say that in each direction there is no real power is flowing and hence

\[
E_0(t) M_0(t)^* - E_0(t) n_y(t) = 0 \quad \text{.................................(14a)}
\]

\[
M_0(t)^* n_x(t) = 0 \quad \text{.................................(14b)}
\]

\[
E_0(t) n_x(t) = 0 \quad \text{.................................(14c)}
\]
Also as we know for free space \( |E|/|M| = \sqrt{\mu/\epsilon} = 120\pi \)
Or
\[ M_0(t) = \frac{E_0(t)}{120\pi} \] .............................. (15)

Putting this value of \( M_0(t) \) in equation (14a), we get
\[ \frac{P^2(t)}{120\pi} = \frac{E_0(t)n_x(t)}{120\pi} - E_0(t)n_y(t) \]
\[ E_0(t) = n_x - 120\pi * n_y \] ................................................. (16a)

And
\[ M_0^*(t) = \frac{1}{120\pi} n_x - n_y \] ......................................................... (16b)

Also since \( E_0(t) \) and \( M_0(t) \) must have some value i.e. cannot be zero thus we conclude from equation (14b) and (14c) that \( n_x \) component of noise must be zero.

Thus
\[ E(t) = [(2n_x - 120\pi * n_y) a_x + n_y a_y] \] ...................................................... (17a)

and
\[ M(t) = n_x a_x + \left( \frac{1}{120\pi} n_x - 2n_y \right) a_y \] ......................................................... (17b)

Thus
\[ P(t) = \frac{1}{2} (E(t) X M^*(t)) \] from equation 5th we get
\[ P(t) = \frac{1}{2} \{ (120\pi * n_y + 2n_x) a_x + n_y a_y \} X \{ n_x a_x + \{ 2n_x + (\frac{1}{120\pi}) n_x \} a_y \} \]
\[ P(t) = \frac{1}{2} \{ (753.6 * n_x^2) + 4 * n_x n_y + 0.006 n_x^2 \} a_x \] ...................................................... (18)

Taking \( n_y = k n_x \) ...................................................... (19)

and solving \( P(t) \)
\[ P(t) = (376.8 \, k^2 + 2 \, k + 0.003) \, n_x^2 \]

Similarly noise power can be given as
\[ |n(t)| = \sqrt{(nx^2 + ny^2)} \]
again using equation (19) we can replace noise power as
\[ |n(t)| = \sqrt{1 + k^2 \, n_x^2} \]

Thus SNR is given as
\[ \frac{P(t)}{E|n(t)|} = \frac{P(t)}{\sqrt{|n(t)|^2}} \]
\[ = (368.8 \, k^2 + 2 \, k + 0.003) |n_x|^2 / \sqrt{(k^2 + 1)} \, |n_x|^2 \]
\[ = (368.8 \, k^2 + 2 \, k + 0.003) / (k^2 + 1) \] ...................................................... (20)

Similarly from equation (13) \( n_y(t) = 0 \) as Electric and Magnetic field component cannot be equal. Also at the start we have taken noise to be white gaussian type having PSD as \( \frac{N_0}{2} \), we can plot the graph for detected noise \( n(t) = n_x a_x + n_y a_y \) vs SNR.

(IV) Numerical Analysis

A. Probability of False Alarm

The probability distribution function of a random variable \( X \) with \( 2N \) degrees of freedom is given by [12]
\[ f_x(x) = \frac{x^{N-1} e^{-x/2}}{2^N \Gamma(N)} \] ...................................................... (24)
where \( \Gamma(.) \) is the gamma function.
\[ \Gamma(u) = \int_0^\infty \theta^{u-1} e^{-\theta} \, d\theta \]

Now for a given threshold \( \lambda \) the probability of false alarm under hypothesis \( H_0 \) (as defined in (1)) can be computed as
\[ P_f = \text{Prob}[X > \lambda \mid H_0] \]
\[ = \int_\lambda^\infty f_x(x) \, dx \] ...................................................... (25)
\[ = \int_\lambda^\infty \frac{x^{N-1} e^{-x/2}}{2^N \Gamma(N)} \, dx \]

Let \( x = 2u \); so,
\[ P_f = \frac{1}{2^N \Gamma(N)} \int_0^\infty 2^{(N-1)} u^{(N-1)} e^{-u} \, 2 \, du \]
\[ = \frac{1}{\Gamma(N)} \int_0^\infty u^{N-1} e^{-u} \, du \] ...................................................... (26)

From the definition of incomplete gamma function
\[ \Gamma(x, s) = \int_s^\infty t^{x-1} e^{-t} \, dt \]

Hence,
\[ P_f = \frac{\Gamma(N\lambda/2)}{\Gamma(N)} \] ...................................................... (27)

B. Probability of Detection

Probability density function of noncentral chi-square random variable \( x \) with \( 2N \) degrees of freedom and noncentrality parameter of \( 2N\gamma \) is given by
\[ f_x(x) = \frac{1}{2^N \Gamma(N)} \Gamma(N/2, N\gamma/2) \exp(-x/2) I_{N-1} \sqrt{2N\gamma x} \] ...................................................... (28)
So for the threshold $\lambda$, probability of detection, that is, probability that $X > \lambda$ under $H_1$, is given as

$$P_d = \Pr[X > \lambda | H_1] = \int_{\lambda}^{\infty} f_X(x) \, dx$$

$$= \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi}N} e^{-\frac{x^2}{2}} \, dx$$

$$= \frac{1}{\sqrt{2\pi}N} \left[ -x e^{-\frac{x^2}{2}} \right]_{\lambda}^{\infty} = \frac{\lambda e^{-\frac{\lambda^2}{2}}}{\sqrt{2\pi}N}$$

$$= F_{\text{nc}}(\lambda)$$

$$(29)$$

Assume $x = z^2$ then,

$$P_d = \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi}N} \left( \frac{z^2}{2} \right)^{N-1/2} e^{-\frac{z^2}{2}} \, 2z \, dz$$

$$= \frac{1}{\sqrt{2\pi}N} \left[ -z^2 e^{-\frac{z^2}{2}} \right]_{\lambda}^{\infty} = \frac{\lambda e^{-\frac{\lambda^2}{2}}}{\sqrt{2\pi}N}$$

Using definition of generalized Marcum $Q$-function,

$$Q_m(\alpha, \beta) = \frac{1}{2} e^{-\frac{\alpha^2}{2}} \int_{\frac{\alpha}{\beta}}^{\infty} x^m e^{-\frac{x^2}{2}} \, dx$$

$$= \frac{1}{\sqrt{2\pi}N} \left[ -z^2 e^{-\frac{z^2}{2}} \right]_{\lambda}^{\infty} = \frac{\lambda e^{-\frac{\lambda^2}{2}}}{\sqrt{2\pi}N}$$

$$P_d = Q_{\sqrt{2N\gamma}}(\sqrt{\lambda})$$

$$= Q_{\sqrt{2N\gamma}}(\sqrt{\lambda})$$

$$= Q_{\sqrt{2N\gamma}}(\sqrt{\lambda})$$

$$= Q_{\sqrt{2N\gamma}}(\sqrt{\lambda})$$

$$(30)$$

$$(31)$$

$$(32)$$

(V) Simulation Result

The ROC curve are obtained by theoretical distribution of the $H_0$ and $H_1$ hypothesis. The result generated as per simulation shows a close resemblance to the curve obtained from the equation (26) and (32). The ROC plot between Probability of miss $v/s$ Probability of false alarm for a threshold SNR of 3db and 5db (for higher SNR the curve does not show any difference). Figure 1 shows a theoretical $v/s$ simulated curves (obtained by using proposed Pointing Vector approach) showing a close resemblance between the 5db curve using an already defined approach [12] and the pointing vector approach. Here the curve is inspected to classify the performance more closely.

Figure 2 shows the Power spectral density at a mean SNR of $\gamma = -1$ dbm, -2 dbm, -4 dbm, -8 dbm and the local detection is made with energy detection after observing the signal for 1000 samples.

![Figure 2(a): Power Spectrum Density v/s frequency in MHz for stated spectrum sensing under AWGN channel for an SNR of 1dbm and 2dbm.](image)

![Figure 2(b): Power Spectrum Density v/s frequency in MHz for stated spectrum sensing under AWGN channel for an SNR of 4dbm and 8dbm.](image)

![Figure 3: Probability of Miss v/s SNR in fading due to 3 kinds of channel for $P_f = 10^{-1}$ and one is the simulated curve for proposed method which shows a close resemblance to the AWGN channel.](image)
Figure 3: Probability of Miss v/s SNR in fading due to 3 kinds of channel for $P_t = 10^{-1}$ and No. of user=1000

Figure 4: shows PSD of the received signal for 3 different spectrum sensing method

As it is clear from figure 4 showing the PSD of the received signal strength showing that received signal strength is best for Matched filter after which Poynting vector theorem can plays a dominant role in spectrum sensing. But as we know that Matched filter technique require the shape of the signal to be known but in this method the prior knowledge about the shape is not compulsory. Hence this method proves to be more beneficial in terms of prior requirements. Below we have given some advantages and disadvantages of Matched filtering technique:

- Advantage of matched filter is that it needs less time to achieve high processing gain and probability of false alarm and missed detection due to coherent detection [13].
- Disadvantage of matched filter is that it would require a dedicated sensing receiver for all primary user signal types.
- It requires the prior information of primary user signal which is very difficult to be available at the CRs.
- Another disadvantage of matched filtering is large power consumption as various receiver algorithms need to be executed for detection.

Here assumption has been made for receiver having noise figure (NF) of 11dB (as NF more than this will cause a serious distortion to the signal which could hamper our analysis very much).

At last figure 4 shows probability of miss v/s SNR for different kinds of Channel is shown in figure. Figure shows 3 kinds of channel carrying the signal out of which AWGN shows the best graph in terms of probability of miss v/s SNR.

Figure 4: Probability of Miss v/s SNR for Different Spectrum Sensing Technique

Under Rayleigh fading, the signal amplitude follows a Rayleigh distribution. In this case, the SNR follows an exponential PDF,

$$f(y) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

Hence simulation of Rayleigh distribution is given at the end. Where as two plots are given below one showing magnitude response of Rayleigh Fading Channel and other showing the PDF simulated and theoretical Rayleigh PDF.
Spectrum is a scarce resource and it has been a major focus of research over the last several decades. Cognitive radio technology, which is one of the promising approaches to utilize radio spectrum efficiently, has become an attractive option. Deployment of cognitive radio networks mainly depends on the ability of cognitive devices to detect licensed or primary users accurately and hence minimize interference to the licensed users. Spectrum sensing has been identified as a key functionality of a cognitive radio.

In this paper Numerical Analysis for Spectrum Sensing based on Poynting vector has been done, which give us the relation between electric field and noise. Hence if know the value of transmitted power you can calculate the values of noise component. Thus this method could be proved quite efficient while estimating the power transmitted.

(VII) Conclusion

Figure 5: Shows the probability of false alarm v/s SNR for AWGN channel and Rayleigh fading Channel

(VIII) References


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