

An effective Golden Section Search Approach based Distribution Load Flow

Naveen Jain and Jai Kumar Maherchandani

Abstract—This paper presents an effective Golden Section Search approach based Distribution Load Flow (DLF) for planning of distributed generators as PQ and/or PV node. The practical distribution system may have different types of loads. The proposed DLF method can also handle all kinds of voltage dependent load models. The incorporation of PV bus in the DLF is based on the simple concept and can easily be implemented with any other classical optimization methods as well as evolutionary techniques. This load flow method can be suitable for small-, medium- and large-scale distribution systems. The proposed load flow algorithm is tested on distribution systems with fixed standard size capacitor and/or DG for various load models to show its effectiveness.

Index Terms- Distributed generation, distribution system, Golden Section Search approach.

I. INTRODUCTION

Demand of electricity is growing rapidly across the world. It forces power system utilities to take appropriate action before the situation gets worsened. Hence, the utilities have been taking several actions such as demand side management, increasing contribution from Distributed Generation (DG), etc. Thus, Distribution System (DS) along with the DG is getting attention across the globe. The DG is generally a small size electric power source, directly connected to the DS and may be a feasible alternative for new capacity addition as it offers many benefits such as system loss reduction, voltage profile improvement, pollutants reduction, short start-up time, low investment risk, etc [1], [2]. The DG can use either renewable energy resources or non-renewable energy resources [3] as a primary source.

The DS is usually radial network with high line resistance to reactance ratio (R/X). It is well known to power system researchers that the Load Flow (LF), used in analyzing a power system, based on Newton-Raphson and Gauss-Seidel methods, may not be able to solve a large level Radial Distribution System (RDS) with high value of ($R/X \geq 5$). Therefore, the importance of Distribution Load Flow (DLF) becomes more being a main tool to calculate the DS parameters.

In literature, several load flow approaches for transmission- and distribution-system have been suggested due to their key features in power system. Several impressive approaches have been suggested for distribution load flow using Forward/Backward Sweep (F/BS) method given as [4]- [28].

In [12], [14], [18], [19]-[21], the suggested load flow approach included DG as PQ & PV node. A table for DG models to be used in DLF study was also presented in [18]. Since, inclusion of the DG as PV/PQ node is an issue depends upon type of the DG as well as on converter scheme. Eminoglu and Hocaoglu [29] presented a review of various F/BS-based distribution systems' power flow algorithms, which shows that very less work has been addressed to incorporate the DG as a PV bus in radial DS. These approaches involved different expressions and techniques, to determine the quantity of reactive power injection, such as secant technique, Thevenin method, sensitivity matrix approach, etc. If, the system has more than one PV node, it is difficult to apply Thevenin method [20] and similar problems may also be observed with other techniques.

In [30], Acharjee and Goswami proposed a chaotic particle swarm optimization based load flow. Although, it was tested well on transmission networks and claimed to be fast for online application but not tested on radial DS.

Milano presented Power System Analysis Tool (PSAT) in [31] as an open source and compared its features against other similar open source tool box. In [32], Zimmerman *et al.* also presented various features of their simulator (MATPOWER) which is also an open source power system simulator. The load flow technique in PSAT, MATPOWER and some other open source tool boxes used either Newton-Raphson or Gauss-Seidel method for network analysis. The load flow techniques used in these open source tool boxes are good tool for transmission system analysis but may not be as appropriate tool for DS analysis due to some typical characteristics of the DS [16], [31], [32].

Several plannings have been carried out using Newton-Raphson or Gauss-Seidel based load flow methods but those approach may not converge for typical practical DS [16], [22]. However, it appears in the literature that very less work incorporated the DG in DLF as PV model but these approaches involved several complications which are already discussed in this paper. Therefore, it is little bit difficult to incorporate DG node as PV model in the DLF study. To ease of such burden a Golden Section Search (GSS) approach based DLF is proposed in this paper and the best of the authors' knowledge, this is the first work reported which uses GSS method for F/BS method utilizing the network topology in the DLF.

The proposed approach gives an easy way to incorporate PV node. This can easily be implemented with other optimization methods due to its simplicity. The Proposed Method (PM) is very suitable in planning and online application due to its accuracy, robustness. It can be used with any small-, medium- and large-size DS. The accuracy and

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effectiveness of the proposed load flow approach is demonstrated on 12-bus and 85-bus distribution systems [15], [28], [33], [34].

The paper is organized in five sections. In section II, formulation of the problem is presented. The algorithm of Golden Section Search (GSS) approach based Distribution Load Flow (DLF) is given in section III. In section IV, simulation results are presented, compared and discussed. Finally, the paper is concluded in section V.

II. PROBLEM FORMULATION

The paper presents development of GSS based DLF which can handle the DG as either PQ or PV model.

A three phase balanced network with n_b nodes is considered in this study. The Sub-Station (SS) bus has been considered as slack bus. The DG is considered as either PQ or PV node model.

A. Golden Section Search Optimization Technique

The Golden Section Search (GSS) algorithm is one of the classical optimization techniques. The GSS method guaranteed gives solution if the solution lies in the bracket of the two limits.

The GSS method has few standard steps as following.

1. It uses a factor, $\chi = \frac{(3-\sqrt{5})}{2}$, which requires in the algorithm.
2. Compute $\delta = (a-b)$, $a_1 = (a+\chi\delta)$, $b_1 = (b-\chi\delta)$ and evaluating the fitness of the objective function at a_1 and b_1 . Here, a is the lower bound of the search value and b is the upper bound of the search value.
3. If $F(a_1) \leq F(b_1)$ go to next step. Else keep $b = a_1$ and $a_1 = b_1$ solving for fitness of the function at this value. Compute $b_1 = a - \delta(a-b)$, then fitness of the function and go to step-5.
4. Take $a = b_1$ and evaluate the fitness. Set $b_1 = a_1$ and evaluated the fitness of the function. Compute Else keep $b = a_1$ and solving for fitness of the function at this value. Compute $a_1 = b + \chi(a-b)$, then fitness of the function.
5. Check $(a-b) \geq \epsilon$, if yes, go to Step-2. Otherwise set optimal variable value as a and compute corresponding optimal value of the function [35].

B. Load Model

The DS has various types of loads viz. constant power, constant current, constant impedance, small/large industrial, domestic, commercial, etc. The typical load characteristics have significant impact on the load flow solutions. The active and reactive powers are generally expressed in polynomial or exponential form. It can be expressed as [23], [34], [35],

$$P_{li} = P_{lo(i)} (V_i / V_0)^p \quad (1)$$

$$Q_{li} = Q_{lo(i)} (V_i / V_0)^q \quad (2)$$

where, P_{li} and Q_{li} are the active and the reactive power load demand at the bus- i bus for bus voltage (V_i) at bus- i with respect to the nominal voltage (V_0) whereas p and q are the

exponents for the voltage dependent loads, whereas $P_{lo(i)}$ and $Q_{lo(i)}$ are constant real power and reactive power load demand at bus- i . The values of these exponents are given in [23], [36].

C. Distribution Load Flow including DG [34]

The FBS based DLF method can broadly be classified into two parts based on- (i) Kirchoff's formulation [4], [22], [24], [25] (ii) Bi-quadratic equation algorithms [23], [26], [27]. In some work, the bi-quadratic method is used in modified way as per the need arises viz. computational time, simplicity of the algorithm, weekly mesh network, etc [27]. The convergence criteria were used in the literature as tolerance in active power/reactive power/voltage/current mismatch at each node irrespective of the nature of the load either balance or unbalanced [29].

At bus- i , the complex load demand can be expressed as,

$$S_{li} = P_{li} + jQ_{li}; \forall i = 2, 3 \dots n_b \quad (3)$$

The equivalent current injection is given as,

$$I_{li} = \left(\frac{P_{li} + jQ_{li}}{V_i} \right)^* ; \forall i = 2, 3, \dots n_b \quad (4)$$

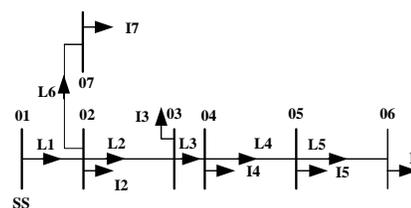


Fig.1. Radial Distributed System (RDS) [34].

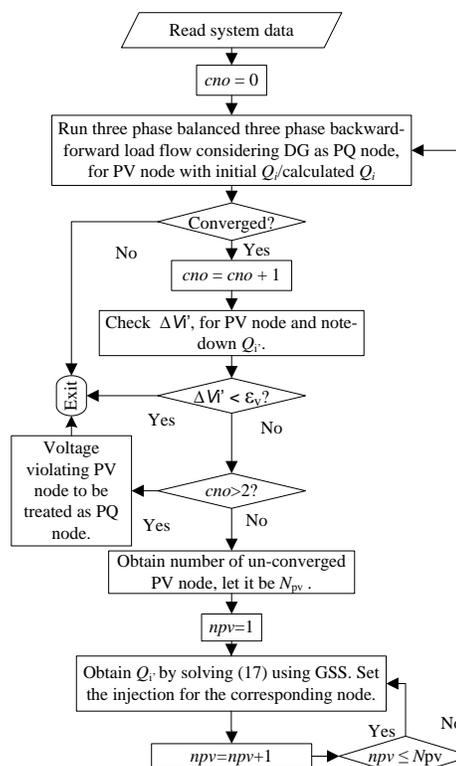


Fig. 2. Algorithm for GSS based load flow [34].

In this work, the network topological based approach of DLF is referred to [22], [33]. The bus-injection to branch-

current matrix, the branch current to bus-voltage matrix and equivalent current injections are formed similarly.

A small DS of seven buses as shown in Fig. 1 is considered for the sake of simplicity. The relationship between the bus current injection and the branch current is expressed as,

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} \quad (5)$$

$$[M]=[BIBC][I] \quad (6)$$

where, $[M]$ and $[I]$ are branch current and bus injection matrices, respectively. The network topological matrix $BIBC$ is the bus injection to line current matrix. Following expression in (7) is written using Kirchoff's voltage law in Fig. 1,

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & Z_{56} & 0 \\ Z_{12} & 0 & 0 & 0 & 0 & Z_{27} \end{bmatrix} \cdot \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{bmatrix} \quad (7)$$

Above matrix expression can be written as,

$$[\Delta V]=[BCBV].[M] \quad (8)$$

$$\text{where, } [BCBV] = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & Z_{56} & 0 \\ Z_{12} & 0 & 0 & 0 & 0 & Z_{27} \end{bmatrix}$$

$$[\Delta V]=[BCBV].[BIBC].[I] \quad (9)$$

$$[\Delta V]=[LFM].[I] \quad (10)$$

where, $BCBV$ is the branch current to bus voltage matrix. The equation (4) is used to determine current vector $[I]$ at every bus in K^{th} iteration and it is used in (8) to determine $[\Delta V]$ vector for K^{th} iteration. The following expression will be used to determine voltage in $(K+1)^{\text{th}}$ iteration.

$$[V^{K+1}]=[V^0]+[\Delta V^{K+1}] \quad (11)$$

The algorithm of the proposed GSS-DLF is shown in Fig. 2 and discussed in following steps-

1. Input the system topology and prepare $[BIBC]$ matrix which will remain constant throughout the iterations.
2. Initialize all node voltages.
3. Input the load with its type, considering the DG as negative load.
4. Calculate equivalent current injection at every node using (4).
5. Prepare the $[M]$ matrix using (5).

6. Calculate the branch voltage drop using (7).
7. Now, update the bus voltage matrix using (11).
8. The convergence criterion in this DLF is change in bus voltage in subsequent iterations. If, it is less than set error tolerance ϵ_v , process will terminate. Else, go to step-4 and the process continues till convergence criterion achieved.

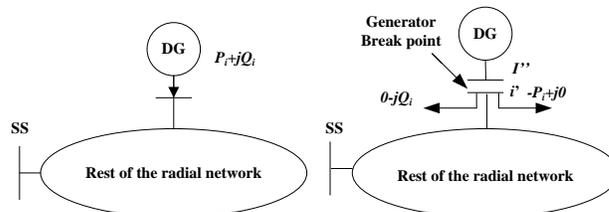


Fig.3. Treatment of the PV node in the radial distribution system [34].

The above steps are carried out considering the DG as negative load. In case, the DG is incorporated as PV node(s), following steps are to be followed.

9. The DG node is to be broken, $i''-i''$, [21] as shown in Fig. 3. The power injection at fictitious bus i' is maintained as shown in Fig. 3, viz. it is to be treated as negative load.
10. The DLF steps 1-8 are to be executed and if the system converged with PV node voltage within the specified limit, $\Delta V_{i'} = |V_{spec(i')}| - |V_{i'}|, i' \in \{\text{set of PV nodes}\}$. The algorithm will stop and the results are displayed.
11. If the PV bus voltage is out of the bound then reactive power compensation is required. The procedure to obtain reactive power injection is as following.

At PV bus, the GSS based optimization is to be carried out on following objective function.

$$\min \left[F_2 = \sum_{i'}^{N_{PV}} (|V_{spec(i')}| - |V_{i'}|) \right] \quad \forall i' \in S_{PV} \quad (12)$$

where, $V_{spec(i')}$, $V_{i'}$ and S_{PV} are specified PV bus voltage at i'^{th} bus, voltage at i'^{th} bus and a set of PV buses, respectively.

The equation (12) is to be minimized by the reactive power injection, subject to the reactive power handling capability of the DG. It is important to address that in this method whatever reactive power injection obtained is to be used without any further changes as the GSS searches the solution only in the range of the reactive power handling capacity defined in GSS algorithm. If the bus voltage is found within the limit, the bus will be said as PV bus else it is to be treated as PQ, with reactive power injection provided by the GSS algorithm and it will be corresponding to the either boundary value of the reactive power limits.

12. To calculate the amount of reactive power compensation, following steps are adopted as:

12.1 Input Data-

- GSS-data, bracketing the solution between minimum and maximum reactive power limit of the DG at i'^{th} bus, $[Q_{\min}, Q_{\max}]$.
- System constraints, various coefficients, etc.
- DG real power injection.

- 12.2 Applying the GSS to minimize the objective function (12).

12.3 Terminate on condition if satisfied. Store the value of the injected reactive power Q_i' . Else go to step 12.2.

13. The DLF steps 1-8 are to be executed with obtained Q_i' and if the PV node voltage is same as the specified voltage the node will remain PV, else it is to be treated as PQ node with the obtained reactive power, Q_i' , as it is corresponding to either the minimum reactive power or maximum reactive power limit of the DG [34].

Since, the solution techniques described, the LU decomposition and Jacobian matrix or the admittance matrix are no longer necessary for the GSS based DLF. Only the LFM matrix given in (10) is necessary to solve the load flow of the system. Thus, the PM can save considerable computational time. It can be used in planning as well as online application due to converge for any DS irrespective of the size. It is simple and can be implemented with other optimization method just replacing GSS with desirable optimization method. It may give new dimensions in implementing the algorithm including DG as a PV node model and will open new opportunity to beginners.

III. SIMULATION RESULTS

The PM of the GSS-DLF is tested on 12-bus [15], [17], [33] and 85-bus [28] DSs for the base case and the planning of the capacitor and the DG. In literature, these systems were usually modeled with constant power load model. The 12-bus and 85-bus DSs are taken with constant power load. The 85-bus system is also taken with mixed load models. The allocation of the mixed loads in 85-bus DS is shown in Table-I. The DLF is prepared with the accuracy of 10^{-5} p.u. in convergence on MATLAB platform.

TABLE-I
LOAD ALLOCATION

Load component	85-bus system	
Small industrial motors	7,32-33	Industrial 8
Commercial	10-31	Residential 34-41
Constant power	2-6,9,42-84	Constant current 85

The proposed GSS-DLF is compared with the results of [17] and DIgSILENT Power Factory software (version-14.0) [37] to confirm the accuracy. In Table III, difference in voltage is zero except at node-7, which is also negligible. Also, the similar results are obtained in [15], [33]. In Table IV, the voltage results and calculated reactive power requirement by the PM is in close proximate with that of DIgSILENT Power Factory-14.0. A DG of 0.28 MW (PV bus) is placed at node-9 (may not be optimal) with reactive power handling limit assumed corresponding to ± 0.85 power factor limits of the DG for the sake of the simplicity, the PM provides the load flow solution in 0.2-second. In 85-bus system with constant power load model, voltage profile at 83 buses are same up to four decimal places as in [28].

In the same 85-bus system with mixed load model, one DG (PV type) for 0.85 power factor (lead/lag) at bus-8 is considered for placement. In addition, few capacitors are also placed as given in Table V. The computed reactive power is also given in Table V. However, the open source tool box has not converged for this typical radial distribution system.

TABLE-II

LOAD SCALING FACTOR FOR HOURLY VARYING LOAD IN DS

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8
LSF	0.65	0.61	0.57	0.52	0.5	0.53	0.6	0.68
Hour	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16
LSF	0.8	0.85	0.87	0.89	0.83	0.85	0.88	0.98
Hour	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
LSF	0.99	0.98	1	0.95	0.88	0.83	0.7	0.6

TABLE-III

LOAD FLOW VOLTAGE SOLUTION (PU) FOR 12-BUS SYSTEM WITH NO DG

Volt.(bus) [17]	Volt. (bus) (*PM)	Volt. (bus) [17]	Volt. (bus) (*PM)
1.0 (1)	1.0 (1)	0.9637 (7)	0.9638 (7)
0.9943 (2)	0.9943 (2)	0.9553 (8)	0.9553 (8)
0.9890 (3)	0.9890 (3)	0.9473 (9)	0.9473 (9)
0.9806 (4)	0.9806 (4)	0.9445 (10)	0.9445 (10)
0.9698 (5)	0.9698 (5)	0.9436 (11)	0.9436 (11)
0.9665 (6)	0.9665 (6)	0.9434 (12)	0.9434 (12)

Total loss =0.02071+j 0.0081 MVA same as in [17]. *PM-Proposed Method.

TABLE-IV

LOAD FLOW VOLTAGE SOLUTION (PU) FOR 12-BUS SYSTEM WITH DG AT BUS-9 OF 0.28 MW AS PV MODEL

Volt. (bus) [37]	Volt. (bus) (PM)	Volt. (bus) [37]	Volt. (bus) pu (PM)
1.0 (1)	1.0 (1)	0.9889 (7)	0.9903 (7)
0.9971 (2)	0.9973 (2)	0.9926 (8)	0.9934 (8)
0.9947 (3)	0.9953 (3)	1.0 (9)	1.0 (9)
0.9908 (4)	0.9926 (4)	0.9973 (10)	0.9970 (10)
0.9889 (5)	0.9906 (5)	0.9965 (11)	0.9965 (11)
0.9887 (6)	0.9904 (6)	0.9963 (12)	0.9963 (12)

[Total loss 0.0061+j 0.0023 MVA, Injected reactive power $Q_i=-0.0882$ Mvar (PM)] [Total loss 0.0060+j 0.0022 MVA, Injected reactive power $Q_i=-0.0885$ Mvar [37].

TABLE-V

IMPACT RESULTS OF DG (PV MODEL) AND CAPACITORS IN 85 BUS DS

Capacitor/DG size (bus no.)	Line loss		Min.Voltage	
	MW	Mvar	p.u.	Bus no.
Base case (-)	0.2536	0.1585	0.8829	54
0.15(42,43,55,71,77,84), 0.3(51),0.45(47,54), 0.60(75) Mvar	0.2111	0.1196	0.9525	56
2.3107 MW (8)	0.0907	0.0392	0.9892	56

With $Q_i = -0.8121$ Mvar the voltage at bus-8 is 1.0 pu.

IV. CONCLUSIONS

This paper has proposed a Golden Section Search approach based distribution load flow to incorporate DG as PV node. The important feature of the proposed load flow is simple algorithm with high accuracy to handle DG as PQ/PV bus without facing any convergence problem. Due to simplicity, it can easily be implemented with other optimization algorithm as well. The effectiveness of the proposed algorithm in solving base case, multiple capacitors or/and multiple DGs allocation problem have been demonstrated on 12-bus and 85-bus distribution systems with constant power load/mixed load

model. The DG placement results on various test cases reveal that the proposed method utilizing GSS-DLF performs similar in comparison to commercial/open source software results.

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