

# A General Method to Solve All Two-Port Ladder Networks

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**Abstract**— Solving the two-port ladder networks to determine the parameters such as the driving point impedance, the transfer impedance and the voltage transfer ratio requires writing the given network in the form of KVI and KVL equations. Further different methods have been adopted in solving these equations depending upon the problem type. These methods at times are tricky and complicated. Here a general method which has already been applied for one-port network [1] but with an assumption and further with some modifications can be worked out for all kinds of two-port ladder networks is proposed. This general method exactly gives the same value as been calculated by the other methods.

**Index Terms**— Circuit Theory, Kirchhoff’s Laws, Laplace Functions, and Network Functions.

## I. INTRODUCTION

Calculating the parameters such as the driving point impedance, the transfer impedance and the voltage transfer ratio for the two-port ladder networks of the type (Fig. 1) involves different approach and methods depending upon the problem type. The current flowing through the 1,10 port of the network needs to be divided into  $I_1, I_2$  and  $I_3$  [1] [2] within the loops of the network and then writing the KVI and KVL equations. Although such a method does hold good is tricky and the solution is time consuming. Instead if the current is divided only into  $I_1$  and  $I_2$  the general equation to start with becomes simpler and reduces the further simplification and hence this short report.

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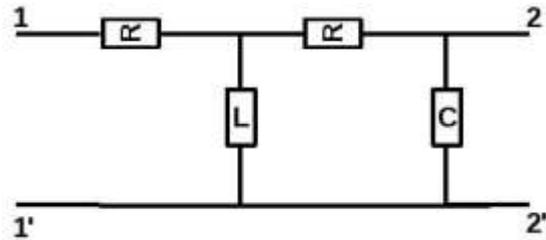


Fig. 1. A General Two-Port Ladder Network

## II. PROBLEM DEFINITION

Consider the typical two-port ladder network of the type 1 as shown in (Fig. 2) for which the transfer functions  $G_{21}(S)$  and  $Z_{21}(S)$  and the driving point impedance  $Z_{11}(S)$  needs to be calculated.

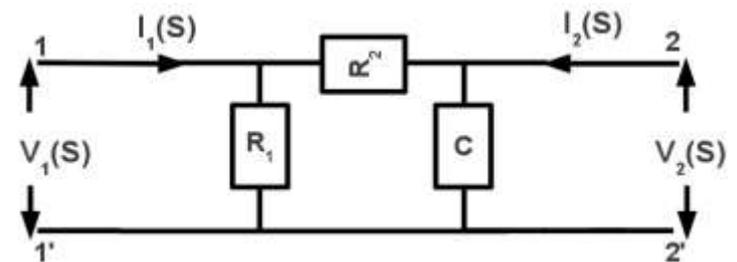


Fig. 2. A typical type 1 two-port ladder network

Determining  $Z_{11}(S)$  is simple. But for determining  $G_{21}(S)$  and  $Z_{21}(S)$ , Kirchhoff’s laws has to be applied. For the network in (Fig. 2), taking into account the current flowing through each loop (Fig. 3), one gets the following three equations ( 1, 2 and3).

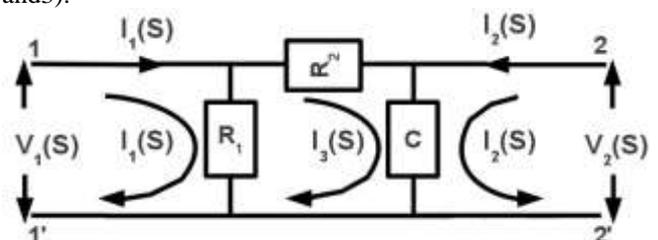


Fig. 3. Current path through a type 1 two-port ladder network

$$V_1(S) = R_1 I_1(S) - R_3 I_3(S) \tag{1}$$

$$R_2 I_1(S) + R_1 [I_3(S) - I_1(S)] + 1/CS [I_3(S) + I_2(S)] \tag{2}$$

$$V_2(S) = [I_2(S) + I_3(S)] * 1/CS \tag{3}$$

Equations ( 1, 2 and 3) are complex and determine  $G_{21}(S)$  requires series of substitutions and simplifications. We report

here a general and simple technique to solve such networks and many more complex networks of the similar type involving more number of components.

### III. METHODOLOGY

Consider the circuit shown in the (Fig. 2). Assuming only the currents  $I_1$  and  $I_2$  we can write the following two equations ( 4 and 5).

$$I_0(S) = [I_1(S) + I_2(S)] \quad (4)$$

$$I_0(S) = V_1(S) * [Y_1(S) + Y_2(S)] \quad (5)$$

Where  $Y_1(S)$  and  $Y_2(S)$  is the admittance and is given by equations ( 7 and 9)

$$Z_1(S) = R_1 \quad (6)$$

$$Y_1 = 1/Z_1(S) \quad (7)$$

$$Z_2(S) = R_2 + 1/CS \quad (8)$$

$$Y_2(S) = 1/Z_2(S) \quad (9)$$

Now write

$$I_2(S) = V_1(S) * Y_2(S) \quad (10)$$

Substituting for  $V_1(S)$  from eqn. (5) [for simplification we do an assumption:  $I_0(S) = I_1(S)$ ], we get

$$I_2(S)/I_1(S) = Y_2(S)/[Y_2(S) + Y_1(S)] \quad (11)$$

It is known that  $G_{21}(S)$  is given by the following eqn. (12).

$$G_{21}(S) = V_2(S)/V_1(S) \quad (12)$$

Substituting the numerator and denominator in the eqn. (12) by equations (13 and 14),

$$V_2(S) = I_2(S)/CS \quad (13)$$

$$V_1(S) = I_1(S) R_1 - I_2(S) R_1 \quad (14)$$

we get  $G_{21}(S)$  as eqn. ( 15)

$$G_{21}(S) = V_2(S)/V_1(S) = I_2(S) * 1/CS [I_1(S) R_1 - I_2(S) R_1] \quad (15)$$

Finally using eqn. ( 11),  $G_{21}(S)$  can be calculated. Also by writing  $Z_{21}(S)$  and  $Z_{11}(S)$  as equations ( 16 and 17) and then using eqn. ( 11), these parameters can be calculated.

$$Z_{21}(S) = V_2(S)/I_1(S) = I_2(S)/[I_1(S)CS] \quad (16)$$

$$Z_{11}(S) = V_1(S)/I_1(S) = [I_1(S) * R_1 - I_2(S) * R_1]/I_1(S) \quad (17)$$

Similarly a typical two-port ladder network of the type 2 as shown in (Fig. 4) can be solved by using the same method as discussed above. Since the type 2 ladder network differs from the type 1 ladder network by an extra component the only modification is with the eqn. ( 14). Due to the presence of the capacitor at the 1,10 port, eqn. ( 14) is modified to be written as eqn. ( 18).

$$V_1(S) = I_1(S) * [R_1 + 1/C_1(S)] - I_2(S) * R_1 \quad (18)$$

All other two-port ladder networks of the similar type can be solved using the above method.

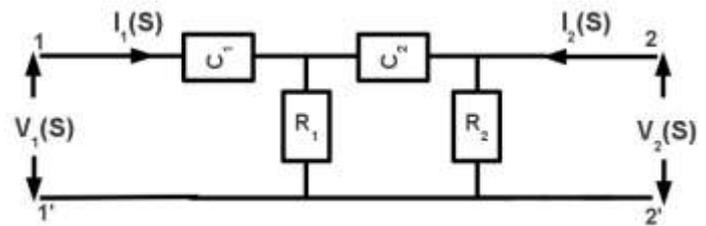


Fig. 4. A typical type 2 two-port ladder network

### IV . OBSERVATIONS:

This method has been successfully tested for almost all kinds of two-port ladder networks with a modification in eqn. ( 14). This general and simple method exactly gives the same value as been calculated by the other methods.

### V. CONCLUSION

Prospect Of The Technique: The proposed method will be useful to solve all kinds of two port ladder networks involving more number of components.

### VI. REFERENCES:

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