

# Branch and Bound Technique To Two Stage Flow Shop Scheduling Problem In Which Setup Time is Separated From Processing Time And Both Are Associated With Probabilities

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**Abstract—** This paper presents an algorithm with the help of branch and bound approach for a flow shop scheduling problems consisting of  $n$  jobs to be processed on 2 machines in which setup time is separated from processing time and both are associated with probabilities.

**Index Terms—** Processing time, Elapsed time, Branch and Bound, Setup time

## I. INTRODUCTION

Scheduling problems are common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, programs to be run in a sequence at a computer center etc. Such problems exist whenever there is an alternative choice in which a number of jobs can be done. Now-a-days, the decision makers for the manufacturing plant have interest to find a way to successfully manage resources in order to produce products in the most efficient way. They need to design a production schedule to minimize the flow time of a product. The optimal solution for the problem is to find the optimal or near optimal sequence of jobs on each machine in order to minimize the total elapsed time. Johnson (1954) first of all gave a method to minimise the makespan for  $n$ -job, two-machine scheduling problems. The scheduling problem practically depends upon the important factors namely, Transportation time, break down effect, Relative importance of a job over another job etc. These concepts were separately studied by Ignall and Scharge (1965), Maggu and Dass (1981), Temiz and Erol (2004), Yoshida and Hitomi (1979),

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Lomnicki (1965), Palmer (1965), Bestwick and Hastings (1976), Nawaz et al. (1983), Sarin and Lefoka (1993), Koulamas (1998), Dannenbring (1977), etc. Further we are using branch and bound technique to minimize the total elapsed time in which setup time is separated from processing time.

## II. PRACTICAL SITUATION

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern. The break down of the machines (due to delay in material, changes in release and tails date, tool unavailability, failure of electric current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) have significant role in the production concern

## III. NOTATIONS

$S$  : Sequence of jobs 1, 2, 3, ...,  $n$

$S_k$  : Sequence obtained by applying Johnson's procedure,  $k = 1, 2, 3, \dots, r$ .

$M_j$  : Machine  $j = 1, 2,$

$a_{ij}$  : Processing time of  $i^{th}$  job on machine  $M_j$

$p_{ij}$  : Probability associated to the Processing time  $a_{ij}$

$A_{ij}$  : Expected processing time.

$S_{ij}$  : Set up time of  $i^{th}$  job on machine  $M_j$ .

$q_{ij}$  : Probability associated to the set up time  $A_{ij}$

$S_{ij}$  : Expected set up time of  $i^{th}$  job on machine  $M_j$

$t_{ij}(S_k)$  : Completion time of  $i^{th}$  job of sequence  $S_k$  on machine  $M_j$

IV. PROBLEM FORMULATION

Let n jobs 1,2,...,n be processed on three machines  $M_1$ ,  $M_2$  and  $M_3$  in a way such that no passing is allowed. Let  $a_{ij}$  be the processing time of  $i^{th}$  job on  $j^{th}$  machine with probabilities  $p_{ij}$  and  $s_{ij}$  be the set up time of  $i^{th}$  job on  $j^{th}$  machine with probabilities  $q_{ij}$ . Let  $A_{ij}$  be the expected processing time and  $S_{ij}$  be the expected set up time of  $i^{th}$  job on  $j^{th}$  machine.

Also consider the following structural relation.

either  $\text{Min}(A_{i1} - S_{i2}) \geq \text{max}(A_{i2} - S_{i1})$   
 or  $\text{Max}(A_{i3} - S_{i2}) \geq \text{max}(A_{i2} - S_{i3})$  or both.

THE MATHEMATICAL MODEL OF THE PROBLEM IN MATRIX FORM CAN BE STATED AS:

Job s	Machine $M_1$				Machine $M_2$				Machine $M_3$			
	$a_{i1}$	$p_{i1}$	$s_{i1}$	$q_{i1}$	$a_{i2}$	$p_{i2}$	$s_{i2}$	$q_{i2}$	$a_{i3}$	$p_{i3}$	$s_{i3}$	$q_{i3}$
1	$A_{11}$	$p_{11}$	$s_{11}$	$q_{11}$	$a_{21}$	$p_{21}$	$s_{21}$	$q_{21}$	$a_{31}$	$p_{31}$	$s_{31}$	$q_{31}$
2	$A_{12}$	$p_{12}$	$s_{12}$	$q_{12}$	$a_{22}$	$p_{22}$	$s_{22}$	$q_{22}$	$a_{32}$	$p_{32}$	$s_{32}$	$q_{32}$
3	$A_{13}$	$p_{13}$	$s_{13}$	$q_{13}$	$a_{23}$	$p_{23}$	$s_{23}$	$q_{23}$	$a_{33}$	$p_{33}$	$s_{33}$	$q_{33}$
-	-	-	-	-	-	-	-	-	-	-	-	-
n	$A_{1n}$	$p_{1n}$	$s_{1n}$	$q_{1n}$	$a_{n2}$	$p_{n2}$	$s_{n2}$	$q_{n2}$	$a_{n3}$	$p_{n3}$	$s_{n3}$	$q_{n3}$

Step 2:

Calculate the lower bounds using the following formula

$$(i) l_1 = t(j_r, 1) + \sum_{i \in j_r^i} G_i + \min_{i \in j_r^i} (H_i)$$

$$(ii) l_2 = t(j_r, 2) + \sum_{i \in j_r^i} H_i$$

Step 3:

Calculate  $l = \text{max}(l_1, l_2)$

Step 4:

We evaluate  $l$  first for the n classes of permutations, i.e. for these starting with 1, 2, 3, ..., n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 5:

Now explore the vertex with lowest label. Evaluate  $l$  for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 6: Prepare in-out table for the optimal sequence obtained in step 3 and get the minimum total elapsed time.

V. NUMERICAL ILLUSTRATION :

let 4 jobs are to be processed on two machines in which processing time and setting times are given with their respective probabilities

Job s	Machine $M_1$				Machine $M_2$			
	$A_i$	$p_i$	$s_i^a$	$r_i$	$B_i$	$Q_i$	$S_i^b$	$s_i$
1	4	.2	2	.2	6	.1	3	.2
2	6	.5	3	.3	4	.2	2	.2

3	5	.1	1	.3	3	.3	3	.2
4	3	.2	2	.2	5	.4	2	.4

Our objective the total elapsed time.

Solution:

Step 1. define expected time

JOB S	MACHIN E A	MACHIN E B
	$A_i$	$B_i$
1	.5	.2
2	1.4	-.1
3	-.1	.8
4	-.2	.6

Step 2:  $l_1 = t(j_r, 1) + \sum_{i \in j_r^i} G_i + \min_{i \in j_r^i} (H_i)$   
 $l_2 = t(j_r, 2) + \sum_{i \in j_r^i} H_i$

Step 3 and Step 4:

We have  $LB(1) = 2, LB(2) = 2.9, LB(3) = 1.4, LB(4) = 1.5$

step 5:  $LB(3,1) = 1.5, LB(3,2) = 2, LB(3,3) = 1.5,$

$LB(321) = 3.2, LB(324) = 2.4$

therefore, sequence is S : (3,2,4,1)

Step 6:

JOB S	MACHIN E A	MACHIN E B
	IN -OUT	IN -OUT
3	0-0.5	0.5-1.1
2	0.9-1.8	1.8-2.6
4	2.7-3.3	3.3-4.3
1	3.7-7.7	7.7-8.3

So .total elapsed time is 8.3

REFERENCES

[1] Brah, S.A. and Loo, L.L.,(1999), "Heuristics for Scheduling in a Flow Shop with Multiple Processors," *European Journal of Operation Research*, Vol. 113, No.1, pp.113-122.

[2] Brown, A.P.G. and Lomnicki, Z.A. (1966), "Some applications of the branch and bound algorithm to the machine scheduling problem", *Operational Research Quarterly*, Vol. 17, pp.173-182.

[3] Chander Shekharan, K, Rajendra, Deepak Chanderi (1992), "An efficient heuristic approach to the scheduling of jobs in a flow shop", *European Journal of Operation Research* Vol. 61, pp. 318-325.

[4] Futatsuishi, Y., Watanabe, I., and Nakanishi, T. ( 2002), "A Study of the Multi-Stage Flowshop Scheduling Problem with Alternative Operation Assignments," *Mathematics and Computers in Simulation*, Vol. 59, No. 3, pp.73-79.

[5] Ignall, E. and Schrage, L. (1965), "Application of the branch-and-bound technique to some flowshop scheduling problem", *Operations Research*, Vol. 13, pp.400-412.

[6] Johnson, S.M. (1954), "Optimal Two and Three Stage Prouction Schedules with Setup Times Include," *Naval Research Logistics Quarterly*, Vol. 1 No. 1, pp. 61-68.

[7] Kim, J.S., Kang, S.H., and Lee, S.M.(1997), " Transfer Batch Scheduling for a Two-Stage Flowshop with Identical Parallel Machines at Each Stage," *Omega*, Vol. 25, No. 1, pp. 547-555

[8] Lomnicki, Z.A. (1965), "A branch-and-bound algorithm for the exact solution of the threemachine scheduling problem", *Operational Research Quarterly*, Vol. 16, pp.89-100.

- [9] Maggu & Das (1981), "On  $n \times 2$  sequencing problem with transportation time of jobs", *Pure and Applied Mathematika Sciences*, 12-16.
- [10] Yang, D.L. and Chern, M.S. (2000), "Two-Machine Flowshop Group Scheduling Problem," *Computers and Operations Research*, Vol.27, No.10, pp.975-985.
- [11] Yoshida, T. and Hitomi, K., (1979), "Optimal Two-Stage Production Scheduling with Setup Times Separated," *AIE Transactions*, Vol. 11, No. 1, pp. 261-273.
- [12] Maggu, P.L. and Das, G.(1980), "On idle/waiting time operator  $O_{i,w}$  having applications to solution of some sequencing/scheduling problems", *PAMS*, Vol. XI, No., pp 1-2.



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