Branch and Bound Technique in Two Stage Flow Shop Scheduling Problem Including idle/waiting Time Operator $O_{i,w}$ For a Job Block Where Jobs Are Performed in The Form of String

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Abstract— **This paper presents an algorithm with the help of branch and bound approach for a flowshop scheduling problems consisting of n jobs to be processed on 2 machines in which idle/waiting Time Operator $O_{i,w}$ for a job block and all the jobs are performed in the form of string .the objective is to getthe optimal sequence of jobs to minimize the total elapsed time.**

Index Terms— Equivalent job, processing time ,elapsed time, branch and bound

INTRODUCTION

The basic study in the field of scheduling was made by Johnson (1954) who developed a polynomial time algorithm to minimize make span in two, three stage flow shop. Convey et al (1963) formulate the integer programming model for scheduling, Ignall E and Scharge (1965) applied Branch and Bound Technique in flow shop problem. Gupta and Dudek (1971) conducted an experimental study of a comprehensive performance measure in the flow shop schedule. Maggu and Das (1977) introduced the equivalent job-block concept in the theory of scheduling. Singh T.P. & Gupta Deepak (2004) made an attempt to study the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria.

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Scheduling is the allocation of resources over time to perform a collection of task. It is an important subject of production and operations management area. Scheduling problems occurred in real world applications generally are flow shop scheduling problems. Each job has the same routing through machines and the sequence of operations is fixed in a flow shop. Branch and bound algorithms are the The basic study of flowshop scheduling was developed by Johnson . Further work was developed by Ignall and Scharge , MacNaughton, Smith and Dudek, Maggu and Das , Yoshida and Hitomi , Singh and Deepak , etc by considering various parameters. The basic concept of equivalent job for a job block was introduced by Maggu and Dass . Heydari `dealt with a flow shop scheduling problem where n jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed and other job block in which order of jobs is arbitrary.
Further we have made an attempt to develop an algorithm in which processing time of each job is given and moreover we are using idle/waiting Time Operator $O_{i,w}$ for a group job for a job block and jobs are performed in the form of string.

2. Theorem 1 Equivalent Job Block Theorem :

Statement: Let n jobs 1, 2, 3, …………n are processed through two machines A & B in order AB with processing time a_i& b_i (i = 1, 2, 3, ……….n) on machine A and B respectively.
If
$(a_p, b_p)\ O_{i,w}\ (a_q, b_q) = (a_\beta, b_\beta)$
then
$a_\beta = a_p + \max(a_q - b_p, 0)$
and
$b_\beta = b_q + \max(b_q - a_q, 0)$
where β is the equivalent job for job blosck (p, q) and p, q ∈ {1, 2, 3, …………..n}.

PROOF:

Starting by the equivalent job block criteria theorem for β = (p, q) given by Maggu & Das (5), we have:

$a_\beta = a_p + a_q$ - min $(b_{p,} a_q)$ …(1)

$b_\beta = b_p + b_q$ min $(b_{p,} a_q)$ …(2)

Now, we prove the above said theorem by a simple logic:

CASE I: When $a_q > b_p$

$a_q > b_p > 0$

max { $a_q > b_p$, 0} = $a_q > b_p$ …(3)

and

$b_p > a_q < 0$

max { $b_p > a_q$, 0} = 0 …(4)

(1) a_β = $a_p + a_q$ – min (b_p, a_q)

= $a_p + a_q - b_p$ as $a_q > b_p$

= a_p + max $\{a_q - b_{p,} 0\}$ using (3)

(2) $b_\beta = b_p + b_q$ – min (b_p, a_q)

= $b_p + b_q - b_p$ as $a_q > b_p$

= $b_q + (b_p - b_p)$

= $b_q + 0$

= b_q + max $(b_p - a_q, 0)$ using (4)

CASE II: When $a_q < b_p$

$a_q - b_p < 0$

max $(a_q - a_q, 0) = b_p - a_q$

and $b_p - a_q > 0$

max $(b_p - a_q, 0) = b_p - a_q$

(1) a_β = $a_p + a_q$ – min (b_p, a_q)

= $a_p + a_q - a_q$ as $a_q > b_p$

= $a_p + a_q - a_q$ as $a_q > b_p$

= $a_p + 0$

= a_p + max $(a_q - b_p, 0)$ using (7)…(9)

(2) b_β = $b_p + b_q$ – min (b_p, a_q)

= $b_p + b_q - a_q$ as $a_q < b_p$

= $b_p + (b_p - a_q)$

= b_p + max $(b_p - a_q, 0)$ using (8)…(10)

CASE III: When $a_q = b_p$

$a_q - b_p = 0$

max $(a_q - b_p, 0) = 0$ …(11)

also

$b_p - a_q = 0$

max $(b_p - a_q, 0) = 0$

(1) $a_\beta = a_p + a_q$ – min (b_p, a_q) …(12)

= $b_p + a_q - a_p$ as $b_q = a_p$

= $a_p + 0$

= a_p + max $(a_q - b_p, 0)$ …(13)

(2) $b_\beta = b_p + b_q$ – min (b_p, a_q) …(12)

= $b_p + b_q - b_p$

= $b_q + (b_p - b_p)$

= $b_q + 0$

= b_q + max $(b_p - a_q, 0)$ using (12) …(14)

by (5), (6), (9), (10), (13) and (14) we conclude:

$a_\beta = a_p + a_q$ – max $(a_q, b_p, 0)$

$b_\beta = b_p$ + max $(b_p, a_q, 0)$

for all possible three cases

The theorem can be generalized for more number of job blocks as stated:

Let n jobs 1, 2, 3, ……….n are processed through two machines A & B in order AB with processing time a_i & b_i (i = 1, 2, 3, ……..n) on machine A & B respectively. If (a_{io}, b_{io}) $O_{i,w}$ (ai_1, bi_1) $O_{i,w}$ (ai_2, bi_2) $O_{i,w}$……………… $O_{i,w}$ $(a_{ip}, b_{ip}) = (a_\beta, b_\beta)$

Then

$$(a_\beta = a_{io} + \sum_{J=1}^{p} \max \{a_{ij} - b_{i(j-1)}\ 0\}$$

and

$$(b_\beta = b_{ip} + \sum_{J=1}^{p} \max \{b_{i(j-1)} - a_{ij}, 0\}$$

where i_0, i_1, i_2, i_3, ……………ip є {1, 2, 3 ………..n} and β is the equivalent job for job block (i_0, i_1, i_2, i_3, ……………ip). The proof can be made using Mathematical induction technique on the lines of Maggu & Das (7).

In the light of above theorem operator $O_{i,w}$ (Idle/Waiting time Operator) is defined as follows

Definition 1:

Let R+ be the set of non negative numbers. Let G = R+ x R+. Then Oi,w is defined as a mapping from G x G → G given by:

Oi,w[(x1, y1), (x2, y2)] = (x1, y1) Oi,w(x2, y2)
= { x1 + max (x2- y1, 0), y2 + max (y1- x2, 0)}
where x1, x2, y1, y2∈ R+

Algorithm

: Step 1 : Determine equivalent jobs for each job blocks using idle/waiting Time Operator as following:
(ap, bp) Oi,w (aq, bq) = (αβ, bβ)

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then
$a_\beta = a_p + \max(a_q - b_p, 0)$
and
$b_\beta = b_q + \max(b_q - a_q, 0)$
where β is the equivalent job for job blosck (p, q) and p, q ∊ {1, 2, 3, ………….n}.

Step 2:
Calculate the lower bounds using the following formula:

$$(i)\ l_1 = t(j_r, 1) + \sum_{i \epsilon j_r^i} G_i + \min_{i \epsilon j_r^i}(H_i)$$

$$(ii)\ l_2 = t(j_r, 2) + \sum_{i \epsilon j_r^i} H_i$$

Step 3:
Calculate $l = \max(l_1, l_2)$

Step 4:
We evaluate l first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 5:
Now explore the vertex with lowest label. Evaluate l for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 6: Prepare in-out table for the optimal sequence obtained in step 3 and get the minimum total elapsed time.

Numerical illustration:

Consider 6 jobs 2 machine flow shop problem whose processing time of the jobs on each machine is given

JOBS	MACHINE A	MACHINE B
	A_i	B_i
1	17	28
2	20	35
3	23	30
4	37	34
5	19	38
6	22	16

As per Step 1: Determine equivalent jobs for each job blocks using idle/waiting Time Operator as following:
$(a_p, b_p)\ O_{i,w}\ (a_q, b_q) = (a_\beta, b_\beta)$
then
$a_\beta = a_p + \max(a_q - b_p, 0)$
and
$b_\beta = b_q + \max(b_q - a_q, 0)$
where β is the equivalent job for job blosck (p, q) and p, q ∊ {1, 2, 3, ………….n}.

JOBS	MACHINE A_I	MACHINE B_I
α	22	34
β	17	34
λ	22	38

Again applying equivalent job block criteria ,we get

JOBS	MACHINE A_I	MACHINE B_I
α	22	34
η	22	50

STEP 2:

$J_1 = (\alpha)$, $J_1^I = (\eta)$

STEP 3:
And $l_1 = 94$ and $l_2 = 84$
LB(1) =max (94,84) = 94
Similarly LB(2) = 84

STEP 4:
So ,required sequence is S = η ,α
Which is S = (1,3,5,6,2,4)

STEP 5 :

In-out table for S1 and the minimum total elapsed time as in tableau-

JOBS	MACHINE A	MACHINE B
	IN -OUT	IN -OUT
1	0-17	17-45
2	17-40	45-75
5	40-59	75-112
6	59-81	112-128
2	81-101	128-163
4	101-138	163-197

SO, the total elapsed time is 197 units

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