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Intuitionistic Fuzzy Rough Relation in Some Medical Applications

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***Abstract*— The concept of intuitionistic fuzzy rough set is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. In this paper we propose intuitionistic fuzzy rough relations through the well known Sanchez's approach for medical diagnosis using max-min-max composition and exhibit the technique with a hypothetical case study.**

***Index Terms*— IF rough sets, IF rough relations, Max-min-max composition, Min-max-min composition.**

I. Introduction & Preliminaries

Medical diagnosis process vary in the degree to which they attempt to deal with different complicating aspects of diagnosis such as relative importance of symptoms, varied symptom pattern and the relation between diseases themselves. Based on decision theory, in the past many mathematical models such as crisp sets, fuzzy sets [7], intuitionistic fuzzy [IF] sets [3] were developed to deal with complicating aspects of diagnosis. But, many such models are unable to include important aspects of the expert decisions. Therefore, an effort has been made to process imprecision, vagueness, and uncertainty in data by Pawlak [8] with the introduction of rough set theory. There have been many researches in combining IF sets with rough sets. IF rough sets are generalizations of fuzzy rough sets, which incorporates the beneficial properties of both rough set and IF set techniques.

De et al. [1] have studied Sanchez's[4] method of medical diagnosis using IF set. Also, Saikia et al.[2] have extended De, Biswas and Roy's method in using IF soft set theory. Our proposed method is an attempt to improve the results in[1] using IF rough sets to formulate a pair of medical knowledge. In this article, we propose a method to study Sanchez's approach of medical diagnosis through max-min-max and min-max-min composition of IF rough relations.

Some of the relevant preliminaries required for this paper are presented in this section. IF rough sets are generalizations of fuzzy rough sets that give more information about the uncertain, or boundary region. They follow the definitions for partitioning of the universe into equivalence classes as in traditional rough sets, but instead of having a simple boundary region, there are basically two boundaries formed from the membership and non-membership functions.

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Definition 1.1 Let U be a universal set. An IF set A in U is an object having the form $A=\{x,\mu_A(x),\nu_A(x):x\in U\}$ where $\mu_A:U\to[0,1]$, $\nu_A:U\to[0,1]$ respectively the membership function and the non-membership function and $\mu_A(x)$ denotes membership degree and $\nu_A(x)$ the non-membership degree of the element $x\in U$ to the set A such that $0\le\mu_A(x)+\nu_A(x)\le 1$. The amount $\pi_A(x)=1-(\mu_A(x)+\nu_A(x))$ is called the hesitation part, which may cater either membership value or non-membership value or both.

Definition 1.2 Let U be a non-empty universe of discourse and R an equivalent relation on U, which is called an indistinguishable relation, $U/R=\{X_1,X_2,....,X_n\}$ is all the equivalent class derived from R. $W=(U,R)$ are called an approximation space. $\forall X\subseteq U$, suppose $X_L=\{x\in U\,|\,[x]\subseteq X\}$ and $X_U=\{x\in U\,|\,[x]\cap X\ne\phi\}$, a set pairs (X_L,X_U) are called a rough set in W, and denoted as $X=(X_L,X_U)$; X_L and X_U are the lower approximation and the upper approximation of X on W respectively.

Definition 1.3 Let S be the set of the whole rough sets, $X=(X_L,X_U)\in S$, then an IF rough set $A=(A_L,A_U)$ in X can be described by a pair mapping $\mu_{A_L}:A_L\to[0,1]$, $\nu_{A_L}:A_L\to[0,1]$ and $\mu_{A_U}:A_U\to[0,1]$, $\nu_{A_U}:A_U\to[0,1]$ with the property that $\mu_{A_L}(x)\le\mu_{A_U}(x)$ and $\nu_{A_L}(x)\ge\nu_{A_U}(x)$. Then, an IF rough set A in X could be denoted by $A=\{\langle x,\mu_{A_L}(x),\mu_{A_U}(x),\nu_{A_L}(x),\nu_{A_U}(x)\rangle\,|\,\forall x\in X\}$.

II. Methodology and Algorithm

In this section we introduce some concepts for IF rough relations.

Definition 2.1 Let $X=(X_L,X_U)$ and $Y=(Y_L,Y_U)$ be two rough sets, IF rough relation $R=(R_L,R_U)$ from X to Y is an IF rough set in $X\times Y$ characterized by the two IF relations $\mu_{R_L}:X_L\to Y_L$, $\nu_{R_L}:X_L\to Y_L$ and $\mu_{R_U}:X_U\to Y_U$,

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$\nu_{R_U} : X_U \to Y_U$ with $\mu_{R_L} \subseteq \mu_{R_U}$ and $\nu_{R_L} \supseteq \nu_{R_U}$. An IF rough relation from X to Y will be denoted by IF rough $R\,(X \to Y)$.

Definition 2.2 If $A = (A_L, A_U)$ is an IF rough set, $R = (R_L, R_U)$ is an IF rough relation from X to Y, the max-min-max composition of R with A is an IF rough set B of Y denoted by $B = R \circ A$, and is defined by :

$\mu_{(R\circ A)_U}(y) = \vee_{x\in X_U} [\mu_{A_U}(x) \wedge \mu_{R_U}(x,y)]$ for any $y \in Y_U$

$\nu_{(R\circ A)_U}(y) = \wedge_{x\in X_U} [\nu_{A_U}(x) \vee \nu_{R_U}(x,y)]$ for any $y \in Y_U$

and $\mu_{(R\circ A)_L}(y) = \wedge_{x\in X_L} [\mu_{A_L}(x) \vee \nu_{R_L}(x,y)]$ for any $y \in Y_L$

$\nu_{(R\circ A)_L}(y) = \vee_{x\in X_L} [\nu_{A_L}(x) \wedge \mu_{R_L}(x,y)]$ for any $y \in Y_L$

Definition 2.3 Let $R(X \to Y)$ and $Q(Y \to Z)$ be two IF rough relations. The max-min-max and min-max-min compositions $R \circ Q$ is the IF rough relation from X to Z, defined by

$\mu_{(R\circ Q)_U}(x,z) = \vee_{y\in Y_U} [\mu_{R_U}(x,y) \wedge \mu_{Q_U}(y,z)]$ $\forall x \in X_U, z \in Z_U$

$\nu_{(R\circ Q)_U}(x,z) = \wedge_{y\in Y_U} [\nu_{R_U}(x,y) \vee \nu_{Q_U}(y,z)]$ $\forall x \in X_U, z \in Z_U$ and

$\mu_{(R\circ Q)_L}(x,z) = \wedge_{y\in Y_L} [\mu_{R_L}(x,y) \vee \nu_{Q_L}(y,z)]$ $\forall x \in X_L, z \in Z_L$

$\nu_{(R\circ Q)_L}(x,z) = \vee_{y\in Y_L} [\nu_{R_L}(x,y) \wedge \mu_{Q_L}(y,z)]$ $\forall x \in X_L, z \in Z_L$

Suppose S is a set of symptoms of disease, D is a set of diseases and P is a set of patients. By applying IF rough set technology, a technique through Sanchez's method to diagnose which patient is suffering from what diseases. For this, we construct an IF rough subset A in S and an IF rough relation Q from S to D. Then the max-min-max and min-max-min composition B of the IF rough set A with the IF rough relation $Q[S \to D]$ denoted by $B = A \circ Q$ signifies the state of the patient in term of diagnosis as an IF rough set B of D with the membership and non-membership functions given in the following way

$\mu_{B_U}(d) = \vee_{s\in S_U} [\mu_{A_U}(s) \wedge \mu_{Q_U}(s,d)]$, for any $d \in D_U$

$\nu_{B_U}(d) = \wedge_{s\in S_U} [\nu_{A_U}(s) \vee \nu_{Q_U}(s,d)]$, for any $d \in D_U$

and $\mu_{B_L}(d) = \wedge_{s\in S_L} [\mu_{A_L}(s) \vee \nu_{Q_L}(s,d)]$, for any $d \in D_L$

$\nu_{B_L}(d) = \vee_{s\in S_L} [\nu_{A_L}(s) \wedge \mu_{Q_L}(s,d)]$, for any $d \in D_L$

If the state of a given patient P is described in terms of the IF rough set A of S, then P is assumed to be assigned to diagnosis in terms of IF rough set B of D, through an IF rough relation $Q[S \to D]$. This concept can be extended to a .finite number of patients. Let there are m patients, $p_i, i = 1,2......m$, in a hospital. Thus $p_i \in P$. Let $Q[S \to D]$ be an IF rough relation on $S \times D$ and construct an IF rough relation $R[P \to S]$ from the set of patients P to the set of symptoms S. Clearly, the composition T of IF rough relations Q and $R(T = Q \circ R)$ describes the state of patients p_i in terms of the diagnosis as an IF rough relation from P to D given by the membership and non-membership functions

$\mu_{T_U}(p_i,d) = \vee_{s\in S_U} [\mu_{R_U}(p_i,s) \wedge \mu_{Q_U}(s,d)]$,

$\nu_{T_U}(p_i,d) = \wedge_{s\in S_U} [\nu_{R_U}(p_i,s) \vee \nu_{Q_U}(s,d)]$, and

$\mu_{T_L}(p_i,d) = \wedge_{s\in S_L} [\mu_{R_L}(p_i,s) \vee \nu_{Q_L}(s,d)]$,

$\nu_{T_L}(p_i,d) = \vee_{s\in S_L} [\nu_{R_L}(p_i,s) \wedge \mu_{Q_L}(s,d)]$ $\forall p_i \in P$ and $\forall d \in D$.

For a given Q and R, the relation $T = Q \circ R$ can be computed. The selection index S_{T_U} and S_{T_L} for the final decision can be computed in this way:

$S_{T_U} = \mu_{T_U}(p_i,d) - \nu_{T_U}(p_i,d) \times \pi_{T_U}(p_i,d)$ and

$S_{T_L} = \mu_{T_L}(p_i,d) - \nu_{T_L}(p_i,d) \times \pi_{T_L}(p_i,d)$

assigns a single value of diagnosis to the patients. It emphasizes high values of the membership function $\mu(p_i,d)$ (selection) and reduces low values of the non-membership function $\nu(p_i,d)$ (non-selection). If almost equal values for different diagnosis in T are obtained consider the case for which hesitation is least. In case, the doctor is not satisfied with the results then T is modified. An algorithm for medical diagnosis using compositions of IF rough relations are presented.

Algorithm

We present an algorithm for medical diagnosis using max-min-max and min-max-min composition of the IF rough relation-

Step I: Input the number of objects and attributes in lower approximation and upper approximation to obtain patient-symptom matrix R_U and R_L.

Step II: Input the number of objects and attributes in lower approximation and upper approximation to obtain symptom-disease matrix Q_U and Q_L.

Step III: Perform the composition of IF rough relations to get the patient-diagnosis matrix

$$T_U = Q_U \circ R_U \text{ and } T_L = Q_L \circ R_L.$$

Step IV: Compute the selection index S_{T_U} and S_{T_L} for final decision.

Step V: Find $S_{T_U} = \max(p_i,d)$ and $S_{T_L} = \max(p_i,d)$

Then we conclude that the patient p_i is suffering from the disease d.

Step VI: If S_{T_U} and S_{T_L} has more than one value then go to step IV and repeat the process by reassessing the symptoms for the patients.

III. Case Study

Suppose there are six patient's p_1, p_2, p_3, p_4, p_5 and p_6 in a hospital with symptoms temperature, headache, cough, chest pain, stomach pain and vomiting. Let the possible diseases relating to the above symptoms be viral fever, malaria, dengue fever, typhoid, jaundice, chest problem, pneumonia and stomach problem. We consider $S_1 = (U_1, R_1)$ and $S_2 = (U_2, R_2)$ be two approximation spaces, where U_1 = {temperature, headache, vomiting, cough, chest pain, stomach pain} be the universal set of symptoms and U_2 = {viral fever, malaria, dengue fever, typhoid, Jaundice, chest problem, stomach problem, pneumonia} be the universal set of possible diseases of patients.

Let X be a subset of U_1 as: {temperature, cough, stomach pain, vomiting} and R_1 be an equivalence relation on $X \subseteq U_1$ with the elementary sets [{temperature}, {headache}, {vomiting}, {cough, chest pain}, {stomach pain}]. By applying rough set theory, the lower and upper approximation of X can be defined as follows:

X_L = {temperature, stomach pain, vomiting} and

X_U = {temperature, cough, chest pain, stomach pain, vomiting}.

Now if Y be a subset of U_2 as {viral fever, malaria, typhoid, chest problem, stomach problem} and R_2 be an equivalence relation on $Y \subseteq U_2$ with the elementary sets [{viral fever}, {malaria}, {dengue fever}, {typhoid, Jaundice}, {stomach problem}, {chest problem, pneumonia}]. Again applying rough set theory, the lower and upper approximation of Y is defined as follows:

Y_L = {viral fever, stomach problem, malaria} and

Y_U = {viral fever, typhoid, chest problem, stomach problem, jaundice, pneumonia, malaria}.

Suppose the patient P_1 whose symptoms were recorded by routine case-taking practice. A pair of values was attached to each symptom: first value showing the strength of association (membership value) of that symptom with the patient and the other showing non-association (non-membership value) as perceived by the practitioner. The symptoms, practitioner decided to include, are like this: Suppose patient P_1 had been suffering from stomach-pain (0.4,0.4) . Here the first value is membership value and the other one is non-membership value.

The IF rough relations R_L and Q_L are given as in (hypothetical) Table I and Table II.

Table I: IF rough relation R_L (P $\rightarrow$S)

R_L	Temperature	Vomiting	Stomach-pain
P1	(.3,.5)	(.2,.6)	(.4,.4)
P2	(.2,.6)	(.0,.8)	(.4,.4)
P3	(.1,.6)	(.6,.2)	(.3,.5)
P4	(.2,.7)	(.3,.6)	(.1,.7)
P5	(.1,.8)	(.7,.2)	(.3,.6)
P6	(.5,.3)	(.3,.4)	(.1,.6)

Table II: IF rough relation Q_L (S $\rightarrow$D)

Q_L	Viral fever	Stomach problem	Malaria
Temperature	(.3,.5)	(.5,.4)	(.3,.6)
Vomiting	(.2,.7)	(.3,.6)	(.7,.2)
Stomach-pain	(.4,.5)	(.9,0)	(.4,.5)

The state of patient in terms of diagnosis is defined by the composition $T_L = Q_L \circ R_L$ using min-max-min composition in Table III and S_{T_L} is calculated in Table IV.

Table III: Composition relations $T_L = Q_L \circ R_L$

T_L	Viral fever	Stomach problem	Malaria
P1	(.4,.4)	(.5,.3)	(.4,.4)
P2	(.4,.4)	(.6,.2)	(.4,.4)
P3	(.2,.6)	(.3,.6)	(.5,.3)
P4	(.6,.3)	(.4,.3)	(.4,.2)
P5	(.2,.7)	(.3,.6)	(.6,.3)
P6	(.3,.5)	(.4,.4)	(.3,.5)

Table IV: Selection index S_{T_L}

S_{T_L}	Viral fever	Stomach problem	Malaria
P1	.38	.44	.38
P2	.38	.56	.38
P3	.08	.24	.44
P4	.57	.31	.32
P5	.13	.24	.57
P6	.20	.38	.20

The IF rough relations R_U and Q_U are given as in (hypothetical) Table V and Table VI.

Table V: IF rough relation R_U (P $\rightarrow$S)

R_U	Temperature	Cough	Chest pain	Stomach pain	Vomiting
P1	(.3,.5)	(.7,.2)	(.6,.3)	(.4,.4)	(.2,.6)
P2	(.2,.6)	(.3,.4)	(.2,.7)	(.4,.4)	(0,.8)
P3	(.1,.6)	(.3,.3)	(.4,.5)	(.3,.5)	(.6,.2)
P4	(.2,.7)	(.2,.7)	(.1,.8)	(.1,.7)	(.3,.6)
P5	(.1,.8)	(.4,.5)	(.5,.4)	(.3,.6)	(.7,.2)
P6	(.5,.3)	(.3,.4)	(.5,.4)	(.1,.6)	(.3,.4)

Table VI: IF rough relation Q_U (S → D)

Q_U	Viral fever	Typhoid	Chest problem	Stomach problem	Jaundice	Pneumonia	Malaria
Temperature	(.3,.5)	(.6,.3)	(.1,.8)	(.5,.4)	(.8,.1)	(.8,.1)	(.3,.6)
Cough	(.7,.1)	(.5,.4)	(.4,.4)	(.3,.5)	(.5,.4)	(.4,.5)	(.3,.4)
Chest pain	(.5,.4)	(.3,.5)	(.3,.6)	(.7,.1)	(.2,.5)	(.4,.5)	(.3,.3)
Stomach pain	(.4,.5)	(.5,.4)	(0,.8)	(.9,0)	(.8,.1)	(.2,.7)	(.4,.5)
Vomiting	(.2,.7)	(.3,.6)	(.1,.7)	(.3,.6)	(.4,.1)	(.2,.7)	(.7,.2)

Similarly the state of patient in terms of diagnosis is defined by the composition $T_U = Q_U \circ R_U$ using max-min-max composition in Table VII and S_{T_U} calculated in Table VIII.

Table VII: Composition relations $T_U = Q_U \circ R_U$

T_U	Viral fever	Typhoid	Chest problem	Stomach problem	Jaundice	Pneumonia	Malaria
p1	(.7,.2)	(.5,.4)	(.4,.4)	(.6,0)	(.5,.4)	(.4,.5)	(.4,.3)
p2	(.4,.4)	(.4,.4)	(.3,.4)	(.4,.4)	(.4,.4)	(.3,.5)	(.4,.4)
p3	(.4,.3)	(.3,.4)	(.3,.4)	(.4,.5)	(.4,.2)	(.4,.5)	(.6,.2)
P4	(.2,.7)	(.3,.6)	(.2,.7)	(.3,.6)	(.3,.6)	(.2,.7)	(.3,.6)
P5	(.5,.4)	(.4,.5)	(.4,.5)	(.5,.4)	(.4,.2)	(.4,.5)	(.7,.2)
P6	(.5,.4)	(.5,.3)	(.3,.4)	(.5,.4)	(.5,.3)	(.5,.3)	(.3,.4)

Table VIII: Selection index S_{T_U}

S_{T_U}	Viral fever	Typhoid	Chest problem	Stomach problem	Jaundice	Pneumonia	Malaria
P1	0.68	0.46	0.32	0.60	0.46	0.35	0.31
P2	0.32	0.32	0.18	0.32	0.32	0.20	0.32
P3	0.31	0.18	0.18	0.35	0.36	0.35	0.56
P4	0.13	0.24	0.13	0.24	0.24	0.13	0.24
P5	0.46	0.35	0.35	0.46	0.32	0.35	0.68
P6	0.46	0.44	0.18	0.46	0.44	0.44	0.18

IV. Result

Using the algorithm for medical diagnosis, the disease for which the membership value is maximum gives the final decision. If almost equal values for different diagnosis in composition are obtained, the case for which non-membership is minimum and hesitation is least is considered. The output matched well with the doctor's diagnosis.

Table IX:

Patients	*P1,P2*	*P3,P5*	*P4*	*P6*
S_{T_L}	Stomach problem	Malaria	Viral fever	Stomach problem
S_{T_U}	Viral fever with stomach problem	Malaria	Viral fever	Stomach problem

From the above Table IX it is clear that, if the doctor agrees then patient P_1 and P_2 are suffering from viral fever with stomach problem, patient's P_3 and P_5 from malaria, P_4 from viral fever where as P_6 from Stomach problem.

V. Conclusion

In this paper we have applied the notion of IF rough relation with Sanchez's method of medical diagnosis. A case study has been taken to exhibit the simplicity of the technique. Intuitionistic fuzzy set, as a generalization of fuzzy set developed by Atanassov, can process imprecision and vague data by using degree of membership and non-membership. The non-membership function have more important role here in comparison to the membership function because there is always a fair chance of the existence of some indeterministic part while evaluating the relations. Mean-time rough set can efficiently deal with uncertain events by employing the lower and upper approximation. So it is significant to combine the intuitionistic fuzzy set with rough set and apply them to decision making in medical diagnosis.

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