Image Denoising Using Curvelet Transform
Using Log Gabor Filter

Vishal Garg, Nisha Raheja

Abstract—In this paper we propose a new method to reduce noise in digital image. Images corrupted by Gaussian Noise is still a classical problem. To reduce the noise or to improve the quality of image we have used two parameters i.e. quantitative and qualitative. For quantity we will compare peak signal to noise ratio (PSNR). Higher the PSNR better the quality of the image. For quality we compare Visual effect of image. Image denoising is basic work for image processing, analysis and computer vision. The Curvelet transform is a higher dimensional generalization of the Wavelet transform designed to represent images at different scales and different angles. In this paper we proposed a Curvelet Transformation based image denoising, which is combined with Gabour filter in place of the low pass filtering in the transform domain. We demonstrated through simulations with images contaminated by white Gaussian noise. Experimental results show that our proposed method gives comparatively higher peak signal to noise ratio (PSNR) value, are much more efficient and also have less visual artifacts compared to other existing methods.

Index Terms—Image Denoising, Discrete Wavelets Curvelet, Log Gabor filter.

I. INTRODUCTION

Noise mainly exists in all digital images to some degree. This noise is oftenly introduced by the camera while taking a picture or during the transmission of image. Image denoising algorithms attempt to remove this noise from the image. Ideally, the resulting image after denoising will not contain any noise or added visual artifacts. Major denoising methods include use of Mean and Median Filters, Gaussian Filtering, Wiener Filtering and Wavelet Thresholding. A large number of more methods have been developed; however, most of those methods make assumptions about the image that can lead to blurring. This paper will also explain these assumptions and one of the measurements used will be the method noise, which is the difference between the image and denoised image [1]. A noisy image can be represented as:

\[ O(i) = t(i) + n(i) \]  

---(1.1)

Where \( O(i) \) is the observed value, \( t(i) \) is the true value and \( n(i) \) is the noise value at the \( i \)th pixel. From the fig. 1, we show that input the original image, introduce the Gaussian Noise in the original image after that the image is decomposed into the wavelets, then applying the Curvelet Transformation with Log Gabor Filter and reconstructing the image to obtain the original image. Output image is the denoised image.

II. FORMING AN IMAGE

Wavelet based image denoising is based on obtaining pixel values of images as proper data sets i.e. a matrix of pixels. Images seen on computer screen have two dimensions in terms of \( x_i \) and \( y_i \) representing \( i \)th position of pixel in the horizontal and vertical axes. A pixel or picture element is known as one of the many tiny dots that make up the representation of a picture in a screen of computer. Each pixel may have different values at each position. With these explanations one can say that image with “256x256” resolution has “256” dots in \( x_i \) and “256” dots in \( y_i \).

III. NOISE IN IMAGE FORMATION

Noise is of many types. One common type of noise is Gaussian noise. This type of noise contains variations in
intensity that is drawn from a Gaussian distribution and is a very good model for many kind of sensor noise. Noise is oftenly introduced by the camera when a picture is taken. Any Image denoising algorithm attempts to remove this noise from the image. Ideally, the resulting denoised image will not contain any noise or added artifacts. The image that is corrupted by different types of noises is a frequently encountered problem in image acquisition and transmission [2]. The noise comes from noisy sensors or channel transmission errors. Several kinds of noises are discussed here. The impulse noise (or the salt and pepper noise) is caused by sharp or sudden disturbances into the image signal; its appearance is randomly scattered white or black (or both) pixels over the whole image. Gaussian noise is an idealized form of white noise, that is caused by random fluctuations in the signal. It is distributed normally as shown in fig. 1.3.

Wavelet theory is based on analyzing signals to their components by using a set of basis functions. One important characteristic of the wavelet basis functions is that they relate to each other by simple scaling and translation. The original wavelet function is used to generate all basis functions. It is designed with respect to desired characteristics of the associated function. For multi-solution transformation, there is also a need for another function which is known as scaling function. It makes the analysis of the function to a finite number of components. WT is a two-parameter expansion of a signal in terms of a Wavelet transformation (WT) uses the concept of DWT and IDWT. DWT (Decomposed Wavelet transformation) can be 1-D or 2-D. WT was developed to overcome the shortcoming of the Short Time Fourier Transform (STFT), which can be used to analyze the non-stationary signals, while STFT always gives a constant resolution at all frequencies, WT uses multi-resolution technique for non-stationary signals by which different frequencies can be analyzed with different resolutions [3].

V. FOURIER TRANSFORM VS. WAVELET TRANSFORM

A. FOURIER TRANSFORM:

The FT is an image representation as a sum of complex exponentials of varying magnitudes, frequencies, and phases. The FT plays an important role in a broad category of image processing applications, that include enhancement, analysis, restoration, and compression. Mathematically, The Fourier analysis process is represented by the Fourier transform:

\[ F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \]  

which is the sum over all time of the signal \( f(t) \) multiplied by a complex exponential. The final results of the transform are the Fourier coefficients \( F(\omega) \), which when multiplied by a sinusoid of frequency \( \omega \) yield the constituent sinusoidal components of the original signal. The process is shown in fig. 1.5.
B. WAVELET TRANSFORM:

Similarly, the Continuous Wavelet Transform (CWT) is defined as the sum over all time of the signal i.e. multiplied by scaled, shifted versions of the results of the CWT are various wavelet coefficients C, which are mainly a function of scale and position.

\[
C(\text{scale, position}) = \int_{-\infty}^{\infty} f(t)\psi(\text{scale, position}, t)dt
\] ----(1.4)

Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal.

VI. IMAGE FUSION BASED ON CURVELET TRANSFORM

The curvelet transform (CT) has evolved as an important tool for the representation of curve shapes in graphical applications. Then, it is extended to the fields of edge detection, image denoising and image fusion etc. (Ali, Dakany, sood and Abd, 2008). When the curvelet transform (CT) is introduced to image fusion, the image after fusion will take not only more characteristics of original images but also more information for fusion is maintained (Sun, li and Yang, 2008). The main aim of curvelet transform is to generate an image of better quality in terms of reduced noise than the original image. Conventional methods have very erratic decision making capabilities as compared to curvelet method. Curvelet Transform is a new multi-scale representation and is most suitable for objects with curves. Curvelet Transformation is an extended technique to reduce image noise and to increase the contrast of structures of interest in image. This method can manage the vagueness and ambiguity in many image reconstruction applications efficiently when compared to other techniques. (Starck, Candes and Donoho, 2002).

VII. LOG GABOR FILTER

Curvelet Gabor filters are mainly recognized as one of the best choices for obtaining frequency information locally. They provide the best simultaneous localization of spatial and frequency information. There are two important characteristics of Gabor Filter, Firstly, Log-Gabor functions, always have no DC component, by definition, that contributes to improve not only the contrast ridges but also edges of images. Secondly, the transfer function of the Log-Gabor function has an extended tail at the high frequency end, that oftenly enables one to obtain a very wide spectral information with localized spatial extent and consequently helps to preserve true ridge structures of images (Lajevardi and Hussain, 2009).

The “Gabor Filter Bank” is a popular technique used to determine a feature domain for representing an image. This technique can be designed by varying the spatial frequency and orientation of a Gabor Filter which mimics a band-pass filter. A Gabor filter can be designed for a bandwidth of only 1 octave maximum with a small DC component into the filter. To overcome this limitation, Field proposes the “Log Gabor Filter”. A Log Gabor Filter has no DC component and can be constructed with any arbitrary bandwidth. Two important characteristics of Log-Gabor filter, First, the Log-Gabor filter function always has zero DC components which not only improve the contrast ridges but also edges of images. Second, the Log Gabor filter function has an extended tail at the high frequency end which allows it to encode images more efficiently than the ordinary Gabor function (Mara and Fookes, 2010). For obtaining the phase information log Gabor wavelet is used for feature extraction. From observation it is clear that the log filters (which use Gaussian transfer functions viewed on a logarithmic scale) can code natural images better than Gabor filters. Statistics of natural images indicate the presence of high-frequency components. Since the ordinary Gabor filters under-represent high frequency components, the log filters is a better choice (Mehrotra, Majhi and Gupta). Log-Gabor filters, consist of a complex-filtering arrangement in p orientations and k scales, whose expression in the log-polar Fourier domain is as shown:

\[
G(p, \theta, p, k) = \exp(-1/2(\rho - \rho_k)/\sigma_\rho)^2 \exp(-1/2(\theta - \theta_{kp})/\sigma_\theta)^2
\] -----(1.5)

in which (p, \theta) both are log-polar coordinates and \sigma_\rho and \sigma_\theta are the angular bandwidth and radial bandwidth respectively (common for all the filters). The pair (\rho_k and \theta_{kp}) corresponds to the frequency center of the filters, where the variables p and k represent the orientation and scale selection, respectively. In addition, the scheme is completed by a Gaussian low-pass filter \( G(p, \theta, p, k) \) (approximation).

VIII. METHODOLOGY USED

The methodology used for Image Denoising is to introduce Log Gabor Filter inside the curvelet transformation so that high pass and low pass filters may be replaced. The technique used will give good PSNR and also the best visual quality. The results of various algorithms will be interpreted on basis of different quality and quantity metrics. Thus, methodology for implementing the these objectives can be summarized as follows :-

Image Denoising is basic work for image processing, analysis and computer vision. This work proposes a Curvelet Transformation based Image Denoising, which is combined with Gabor Filter instead of the low pass filtering in the transform domain. We demonstrated through simulations with images contaminated by white Gaussian noise. And we found that our scheme exhibits better performance in both PSNR (Peak Signal to Noise Ratio) and visual effect.

1). Main Objectives:
1. To study all the methods of the Denoising.
2. Attenuate the color frequencies using Log Gabor Filter.
3. Applying the Curvelet Transform.
4. Compare the image quality using PSNR Tool.

**Work Flow For Image Denoising Using Log Gabor Filter Using Curvelet Transformation**

1. **Input Image**
2. **Add noise**
3. **Subband Decomposition**
4. **Smooth Partitioning**
5. **Renormalization**
6. **Ridgelet Composition**
7. **Smooth Integration**
8. **Renormalization**
9. **Subband Reconstruction**
10. **Obtained Image With Normal Curvelet Transformation**

**A. IMAGE DECOMPOSITION**

1) **Subband decomposition:** The image is filtered into subbands.

\[ f \rightarrow (P_0f, \Delta_1f, \Delta_2f, \ldots) \]  \hspace{1cm} (1)

(a) Divide the image into resolution layers.
(b) Each layer will contain details of different frequencies:
\( P_0 \) – Low-pass filter.
\( \Delta_1, \Delta_2, \ldots \) – Band-pass (high-pass) filters.
(c) The original image can be reconstructed from these sub-bands:
\[ f = P_0(P_0f) + \sum \Delta_s(\Delta_s f) \]  \hspace{1cm} (2)

(d) Low-pass filter \( \Phi_0 \) deals with low frequencies near \( |f| \leq 1 \).
(e) Band-pass (High-pass) filter \( \Psi_{2^j} \) deals with frequencies near domain \( |f| \in [2^{2^j}, 2^{2^j+1}] \).
(f) Recursive construction:
\[ \Psi_{2^j}f(x) = 2^{j}\psi(2^jx). \]  \hspace{1cm} (3)

(g) The sub-band decomposition can be approximated with the well known Wavelet Transform (WT). Using WT, \( f \) is decomposed into \( S_0, D_1, D_2, D_3, \ldots \) etc. \( P_0f \) is partially constructed from \( S_0 \) and \( D_1 \), and may include also \( D_2 \) and \( D_3 \). \( \Delta f \) is constructed from \( D_2 \) and \( D_3 \).

2) **Smooth Partitioning:** The layer will be dissected into small partitions. A grid of dyadic squares can be defined as:
\[ Q_{i,k_1,k_2} = \left[ \frac{k_1}{2^i}, \frac{k_1+1}{2^i} \right] \times \left[ \frac{k_2}{2^i}, \frac{k_2+1}{2^i} \right] \in \mathbb{Q} \]  \hspace{1cm} (4)

\( Q_{i,k_1,k_2} \) – all the dyadic squares of the grid.

(a) Let \( w \) be a smooth windowing function with ‘main’ support of size \( 2^i \times 2^i \). For each square, \( w_Q \) shows a displacement of \( w \) localized near \( Q \). Multiplying \( \Delta f \) with \( w_Q \) \( (\forall Q \in \mathbb{Q}) \) produces a smooth dissection of the function into many ‘squares’:
\[ h_Q = w_Q \cdot \Delta f \]  \hspace{1cm} (5)

The windowing function \( w \) is a non-ve smooth function.

(b) Partition of the energy:
The energy of certain pixel \((x_1, x_2)\) is divided between all sampling windows of the grid:
\[ \sum_{k_1,k_2} w^2(x_1 - k_1, x_2 - k_2) = 1 \]  \hspace{1cm} (6)

3) **Renormalization:** Renormalization is centering each dyadic square to the unit square \([0,1] \times [0,1]\). For each value of \( Q \), the operator \( T_Q \) is defined as:
\[ T_Q f(x_1, x_2) = 2^{j} f\left(2^j x_1 - k_1, 2^j x_2 - k_2\right) \]  \hspace{1cm} (7)

Then each square is renormalized by:
\[ g_Q = T_Q^{-1} h_Q \]  \hspace{1cm} (8)

4) **Ridgelet Analysis:**
(a) Each normalized square is analyzed in the

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Fig. 7 Curvelet transform processing flowchart

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ridgelet system:

\[ \alpha_{(Q,s)} = \langle g_{Q}, \rho_s \rangle \] ---- (9)

(b) The ridge fragment has an aspect ratio of \(2^{-2s} \times 2^{-s}\). After the renormalization, it has a localized frequency in band \([|\vartriangle| = [2^{-2s}, 2^{-s}]\).

(c) A ridge fragment needs only a very few ridgelet coefficients to represent it.

B. IMAGE RECONSTRUCTION:

1) Ridgelet Synthesis:

\[ g_{Q} = \sum_{\lambda} \alpha_{(Q,\lambda)} \cdot \rho_{\lambda} \] ---- (10)

2) Renormalization:

\[ h_{Q} = T_{Q} g_{Q} \] ---- (11)

3) Smooth Integration

\[ \Delta_{s} f = \sum_{\rho_{Q}} w_{\rho_{Q}} \cdot h_{\rho_{Q}} \] ---- (12)

4) Sub-band Reconstruction

\[ f = P_{0} (P_{0} f) + \sum_{s} \Delta_{s} (\Delta_{s} f) \] ---- (13)

IX. PROPOSED ALGORITHM STEPS

**Step I:** Take noisy image.

**Step II:** Applying Curvelet Transform as under:

(a) Sub Band Decomposition
(b) Smooth Partitioning
(c) Renormalization
(d) Ridgelet Analysis

**Step III:** In Sub Band Decomposition

(a) Divide image into resolution layers
(b) Each layer contains details of different frequencies.
(c) These frequencies are attenuates and approximate with the help of Log Gabor Filter.

This Step has following characteristics –

(a).The main particularity of this scheme is the construction of the low-pass and high-pass filters.
(b).Log Gabor Filters basically consist in a Logarithmic Transformation of the Gabor domain which eliminates the annoying DC-component allocated in medium and high-pass filters.
(c).This type of scheme approximates flat frequency response and therefore exact image reconstruction which is obviously beneficial for applications in which inverse transform is demanded, such as texture synthesis, image restoration, image fusion or image compression.

**Step IV:** In Smooth partitioning, we will dissect the layer into small portions as explained in eq 1.4,1.5 and 1.6.

**Step V:** In Renormalization each square is renormalized by using the specified formula.

**Step VI:** Ridgelet Analysis is taking place, which is explained by eq 1.9.

**Step VII:** For Reconstruction inverse of curvelet transform is performed.

**Step VIII:** Output is the final Denoised image.

Flow Chart Of Proposed Algorithm Of Image Denoising

```
Start
Input Original Image
Apply Subband Decomposition
LL     HL
LH     HH
Apply Smooth Partitioning
Renormalization
Ridgelet
Image
Obtained Image With Proposed Transformation
Stop
```

Fig. 8 Diagrammatic depiction of Proposed Algorithm

X. RESULT AND DISCUSSIONS

This section presents the results of combined effect of curvelet transformation and Log gabor Filter on the images to observe the change in PSNR ratio. The noisy images are simulated by adding Gaussian white noise on the original images. The performance of the method is illustrated with both quantitative and qualitative performance measure. The qualitative measure is the visual quality of the resulting image [10]. The peak signal to noise (PSNR) is used as quantitative measure. It is expressed in dB units. The fig. shows the steps applied on the image.
Fig. 9. Decomposed Wavelet Transformation of image “lena”

Fig. 10. Image Denoising with Normal Curvael Transformation of image “lena” with noise variance 0.008052.

Fig. 11. Image Denoising with Curvelet Transformation with Log Gabor Filter of image “lena” with noise variance 0.008052.

Fig. 12. Image Denoising with Normal Curvael Transformation of image “lena” with noise variance 0.015.
Fig 13. Image Denoising with Curvelet Transformation with Log Gabor Filter of image “lena” with noise variance 0.015.

Fig 14. Image Denoising with Normal Curvelet Transformation of image “lena” with noise variance 0.028701.

Fig 15. Image Denoising with Curvelet Transformation with Log Gabor Filter of image “lena” with noise variance 0.028701.

Fig 16. Image Denoising with Normal Curvelet Transformation of image “lena” with noise variance 0.036623.
XI. CONCLUSION

It is known that the reduction of noise is of paramount importance in imaging systems. Indeed, image processing techniques typically adopted in image-based measurements (dealing with image analyzing, image transmission and image processing etc.) are very sensitive to noise. To address this issue, a new technique for images contaminated by white Gaussian noise has been presented. As a result, a very fine control of noise cancellation and detail preservation can be achieved. This paper has shown that satisfactory choices of parameter values can be obtained from a preliminary stimate of the standard deviation of the Gaussian noise. In the proposed approach, this estimate is achieved by preprocessing the noisy image with Wavelet Transform Decomposition and then applying Curvelet Transform With Log Gabor Filter. Results of computer simulations dealing with different test images and noise variances have shown that the new method is very effective in removing Gaussian noise from images and performs better than normal curvelet transform. Table 1 shows the values of PSNR for proposed method that is better than normal curvelet in all cases.

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<th>S.N</th>
<th>Variance</th>
<th>PSNR (For Normal Curvelet)</th>
<th>PSNR (For Curvelet With Gabor)</th>
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<td>64.2457</td>
</tr>
</tbody>
</table>

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