ISSN: 2278 – 1323
International Journal of Advanced Research in Computer Engineering & Technology
Volume 1, Issue 4, June 2012

Comparative Study on Performance Analysis of High Resolution Direction of Arrival Estimation Algorithms

Chetan.L.Srinidhi, Dr.S.A.Hariprasad

Abstract*-Array processing involves manipulation of signals induced on various antenna elements. Its capabilities of steering nulls to reduce co channel interferences and pointing independent beams toward various mobiles, as well as its ability to provide estimates of directions of radiating sources, make it attractive to a mobile communications system designer. This paper presents a performance evaluation of robust and high-resolution direction-of-arrival (DOA) estimation algorithms such as MUSIC, ESPRIT and Q-MUSIC used in the design of smart antenna systems. A comparative study is carried out by evaluating the performance of these DOA algorithms for a set of input parameters that include the size of the sensor array, number and angular separation of incident signals from the mobile terminals, as well as noise characteristics of the mobile communication channel. MUSIC, ESPRIT and Q-MUSIC algorithms provide high angular resolution among the other current DOA estimation techniques and hence they are explored in detail. The simulation results shows that Q-MUSIC algorithm is highly accurate, stable and provides high angular resolution even for multidimensional complex data signals compared to ESPRIT and MUSIC.

***Index Terms* - Direction of arrival (DOA), MUSIC, ESPRIT, Quaternion MUSIC.**

1. INTRODUCTION

The application of antenna array processing has been in extensive research in recent years for mobile communication systems to cope up with the problem of limited channel bandwidth and to satisfy an ever growing demand that a large number of mobiles put on the channel.

Manuscript received May, 2012.

Chetan.L.Srinidhi *is with the Department of Electronics and Communication Engineering, Visvesvaraya Technological University Belgaum India, R.V.College of Engineering, Bangalore, India (srinidhipy@gmail.com).*

Dr.S.A.Hariprasad is with the Department of Electronics and Communication Engineering, Visvesvaraya Technological University Belgaum India, R.V.College of Engineering, Bangalore, India (hariprasad@rvce.edu.in).

The application of the array processing requires either the knowledge of a reference signal or the direction of the desired signal source to achieve its desired objectives. There exists a range of schemes to estimate the direction of arrival with conflicting demands of accuracy and processing power.

The technique used for estimating directions of arrival of signals using an antenna array has been study of interest in recent years. Direction of arrival methods has been widely studied in the literature and can be grouped into two categories. In the first category, they are called as classical methods which provides a representation of the sources field (power and angular positions of the sources) by projecting the model vector (directional vector) on the space of observation without considering the determination of the number of sources. However, these conventional methods do not get a good resolution. The second category is known as "parametric" or high-resolution method which requires prior knowledge of the number of uncorrelated sources before estimating their characteristics (angular position, power). The estimation problem is first solved by estimation methods of the number of sources. Then a high-resolution method is applied to estimate the angular position of these sources. These high-resolution methods are known to be more robust than conventional techniques.

The origin of high-resolution methods was first proposed by Prony .This approach has been modified by Pisarenko to estimate the sinusoids [1]. Both these methods are based on linear prediction technique which characterizes the signal model. The modern high-resolution method is based on the concept of subspace, such as MUSIC developed by Schmidt [2], which requires a comprehensive search algorithm to determine the largest peaks for the estimated direction which is a computationally intensive process. MUSIC fails to resolve coherent sources, sources with less number of snapshots and with low SNR conditions. To overcome these drawbacks Root-

MUSIC [3] was proposed which has better probability of resolution than MUSIC and also estimates coherent sources, but not applicable for all array geometry. Further ESPRIT [4] was developed which has identical performance as Root-MUSIC, but is less computationally intensive than the other two algorithms because of the selection of sub arrays. However in all these methods, the multidimensional collection of an EM vector sensor array is organized into a long vector to enable matrix formulation. As a result, the relevant DOA estimators somehow destroy the vector nature of the incident signals carrying multidimensional information in space, time and polarization.

To overcome this drawback in array signal processing, recently quaternion based algorithms were proposed [5]. This quaternion model based algorithms use vector sensors for multidimensional complex data signals, for recovery of vector nature of incident signals and also for polarization estimates of EM waves. As vector sensors become more and more reliable, polarization has been added to estimation process as an essential attribute to characterize sources, in addition to their DOAs. In recent years, MUSIC-like methods for polarized arrays are presented in [6], [7] and ESPRIT techniques [8], [9], [10] which can be used to estimate spatially coherent sources, whereas MUSIC and ESPRIT is only for uncorrelated sources. In this paper we discuss quaternion based MUSIC algorithm for vector sensor array which exploits spatial and polarization diversity for multi-component complex data signals. It also results in accurate estimation of signal subspace on a two component vector sensor array. The quaternion MUSIC provides more robust resolution than the other subspace method even at low SNR conditions and also with less number of samples. Q-MUSIC allows a more accurate estimation of the noise subspace for a two component data, compensating for the loss of performance due to the reduction of data covariance matrix size.

This paper is organized as follows. Section II, describes receiving signal model. Section III, presents description of the MUSIC algorithm along with the simulation results. Section IV, discusses the ESPRIT algorithm. A short description of quaternion's and polarization model along with QUATERNION MUSIC is introduced in Section V. Section VI, presents the Comparison of MUSIC, ESPRIT and Q-MUSIC. Finally Section VII, presents the conclusion of the work.

II. SIGNAL MODEL

In this section, we present the general-rank signal models which frequently appear in array signal processing for communication. The following signal model is applicable for MUSIC and ESPRIT, later the polarization signal model for Q-MUSIC is discussed in Section V. The conventional data model assumes that the signal impinging upon an array of sensors to be narrow-band and emitted from a point source in the far field region. The far field wave is incident at a DOA upon a uniform linear array consisting of M identical antenna elements. Consider K narrowband signals $S_1(t)$, $S_2(t)$, ..., $S_K(t)$ at frequency f_0 arriving with directions q_k (k = 1,.., K) and received by a linear array of M identical elements (M > K) spaced by a distance d, in an additive white Gaussian noise, Fig.1. These signals can be fully correlated as in the case of multiple paths or not correlated as in the case of multiple users.

Depending on the analytical representation, the received signal in baseband can be written as:

$$x(t) = \sum_{k=1}^{K} a(\theta_k) S_k(t) + n(t) \tag{1}$$

Where $x(t) = [x_1(t), x_2(t), \dots, x_m(t)]^T$ is the reception signal or observation vector signal of (M-1) dimension representing the measured complex envelopes of K signals on each antenna.

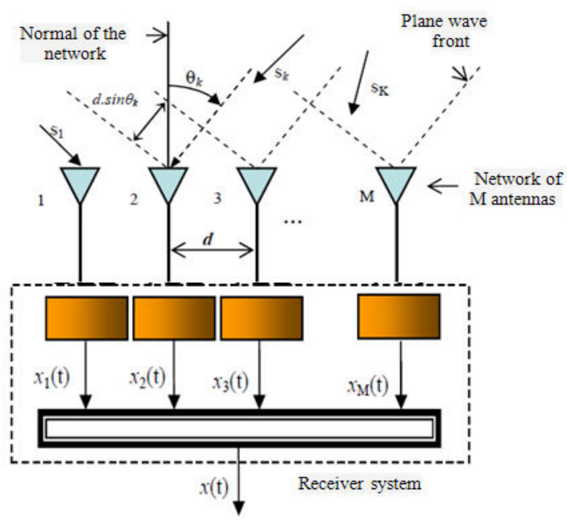

Fig.1 Network of M antennas and receiver system with K incident signals.

Symbol T denotes transpose. $a(\theta_k)$ is the steering or direction vector of the k[th] source defined at the central frequency:

$$a(\theta_k) = \left[e^{-j\varnothing_{1,k}}, e^{-j\varnothing_{2,k}}, \ldots, e^{-j\varnothing_{M,k}} \right]^T \quad (2)$$

$\varnothing_{m,k} = \frac{2\pi}{\lambda}(m-1).d.\sin(\theta_k)$ With m=1... where M is the geometric phase shift introduced at the element m of the network and the k[th] signal depending on the angle of arrival. S_k is the complex envelope of the analytic signal of the k[th] source emitted and is the Gaussian vector of noise on the network. It is noted that noises are stochastic processes assumed to be stationary with zero mean.

Using matrix notation, the matrix of observations can be expressed as:

$$X = AS + \eta \quad (3)$$

Where:

$A = [a(\theta_1 1), a(\theta_1 2), \ldots, a(\theta_1 k)]$

$S = [S_1(t), S_2(t), \ldots, S_K(t)]^T$

$\eta = [\eta_1(t), \eta_2(t), \ldots, \eta_m(t)]^T$

A is a MxK matrix formed by M steering vectors of sources , S is the Kx1 vector of complex envelopes of K sources, and η is the noise vector on the network.

If the number of sources K is less than the number of elements M of the network, then K directional vectors are linearly independent and generate a vector subspace of the observation space of dimension M assuming that signals and noises are stationary and uncorrelated. The correlation matrix or covariance of signals will be given by:

$$R_{xx} = E\{XX^H\} = AR_{ss}A^H + R_{nn} \quad (4)$$

Noisy observation = Signal space + Noise space

Where X^H is the conjugate transpose of X, $R_{nn} = \sigma^2 I$ is the MxM correlation matrix of the noise vector, I is the identity matrix and σ^2 is the noise power identical for each element of network. R_{ss} is the KxK square matrix of covariance of the signal vector given by $R_{ss} = \{SS^H\}$, R_{ss} is a diagonal of full rank. However, it becomes singular when at least two sources are totally correlated.

In practice, the correlation matrix or covariance is estimated by an average over N observations as following:

$$R_{XX} = N^{-1}.X.X^H \quad (5)$$

Where N is the number of samples or observation vectors, and X is the N×K complex envelopes matrix of K measured signals.

From the decomposition of this autocorrelation matrix of observation vectors into orthogonal subspaces, we will compare the performance of the two methods (MUSIC, ESPRIT) of high resolution based on the concept of subspace.

III. MUSIC ALGORITHM

A. Algorithm

MUSIC exploits the technique of eigenvectors decomposition and eigenvalues of the covariance matrix of the antenna array for estimating directions-of-arrival of sources based on the properties of the signal and noise subspaces. For this, the initial hypothesis is that the covariance matrix R is not singular. This assumption physically means that sources are totally uncorrelated between them. MUSIC algorithm assumes that signal and noise subspaces are orthogonal. The signal subspace V_s consists of phase shift vectors between antennas depending on the angle of arrival. All orthogonal vectors to V_s constitute a subspace V_n called noise subspace.

MUSIC algorithm being based on the properties of signal and noise subspaces, vectors derived from V_s generate a signal subspace collinear with steering vectors of sources $a(\theta_k)$ and vectors derived from V_n generate a noise subspace orthogonal to steering vectors of these sources.

If the number of signals impinging on M element array is D, the number of signal eigenvalues and eigenvectors is D and number of noise eigenvalues and eigenvectors is M-D. The array correlation matrix with uncorrelated noise and equal variances is then given by:

$$R_{xx} = A * R_{ss} * A_H + \sigma_n^2 * I \quad (6)$$

Where $A - [a(\theta_1), a(\theta_2), a(\theta_3), \ldots, a(\theta_D)]$ is $M \times D$ array steering matrix.

$$R_{ss} = [[s_1(k), s_2(k), s_3(k), \ldots, s_D(k)]]^T \quad (7)$$

R_{xx} has D eigenvectors associated with signals and M – D eigenvectors associated with the noise. We can then construct the M x (M-D) subspace spanned by the noise eigenvectors such that:

$$V_N = [V_1, V_2, V_3, \ldots, V_{M-D}] \quad (8)$$

The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals θ_1, θ_2, θ_3, θ_D and the MUSIC Pseudo spectrum is given as:

$$P(\theta) = P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) V_n V_n^H a(\theta)} \quad (9)$$

However when signal sources are coherent or noise variances vary the resolution of MUSIC diminishes. To overcome this we must collect several time samples of received signal plus noise, assuming ergodicity and estimate the correlation matrices via time averaging as:

$$R_{xx} = \frac{1}{K}\sum_{k=1}^{K} x(k) * x(k)^H \quad (10)$$

and

$$R_{xx} = A * R_{ss} * A^H + A * R_{sn} + R_{sn} * A^H + R_{nn} \quad (11)$$

We find the 'D' largest peaks of P_{MUSIC} (θ) in eqn(9) to obtain DOA estimates.

B. Simulation Results

DOA estimation using MUSIC algorithm is performed based on a set of input parameters viz. Angle of arrival, inter element spacing, number of elements in an array, SNR range and number of snapshots. The input parameters for simulation of MUSIC algorithm is given in Table 1.

Parameter	Value
Angle of arrival of signals	25° , 80°
Inter element spacing (*d*)	$\frac{\lambda}{2}$
Number of elements in an array	8 ,16
SNR range (dB)	0 to 25
Number of snapshots	200,1000

Table 1: Input parameters for music algorithm

For the set of input parameters, MUSIC algorithm has been tested for the following cases.

Case.1. Varying number of array elements

MUSIC algorithm has been tested in MATLAB for two different sizes of array elements (8 and 16) considering previously defined set of input parameters.

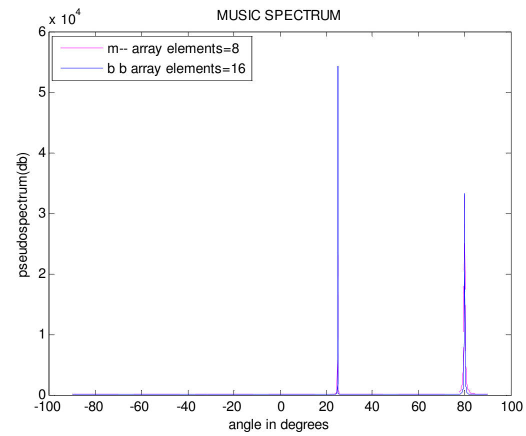

Fig.2: Music spectrum for array elements 8 and 16. (SNR= 25 db and number of snapshots = 200)

From Fig.2 it is observed that with the increase in number of array elements, peak becomes more sharp indicating an increase in ability to resolve two closely spaced targets and hence resulting in an enhanced DOA estimation.

Actual DOA	Estimated DOA	
	Array elements=8	Array elements=16
25°	25°	25°
80°	80°	80°

Table 2: DOA estimation for arrays elements 8 and 16 (SNR= 25 db and number of snapshots = 200)

Table.2 depicts the case where SNR is 25 dB and hence the DOA estimation is equal for both array elements. For a lesser SNR value accurate angular estimation is possible for higher number of array elements.

Case.2. Varying number of SNR values

DOA estimation varies with respect to variations in SNR values. Here MUSIC algorithm has been tested for different values of SNR (0dB and 25dB).

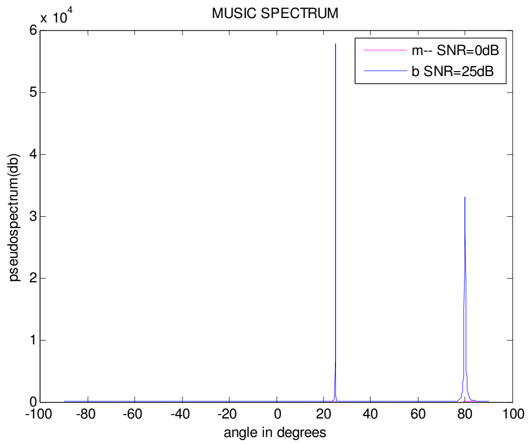

Fig. 3: MUSIC spectrum for SNR of 0dB and 25dB. (Number of array elements = 8 and number of snapshots =200)

From Fig.3 it is observed that as SNR value increases, DOA estimates of MUSIC are more accurate and almost near to estimated DOA. When SNR is very low (close to 0dB) accurate estimation of DOA fails. The actual and estimated DOA's are shown in Table 3.

Actual DOA	Estimated DOA	
	SNR=0dB	SNR=25dB
25°	24.70°	25°
80°	79.41°	80°

Table 3: DOA estimation for SNR 0dB and 25dB (Number of array elements = 8 and number of snapshots =200)

Case.3. Varying number of snapshots

Simulations are carried out for two different snapshots viz. 200 and 1000 keeping other input parameters specified in Table 1 constant.

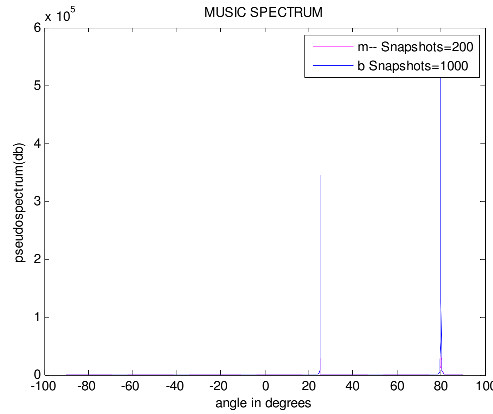

Fig.4: MUSIC spectrum for snapshots 200 and 1000. (SNR= 25 db and number of arrays elements=8)

From the Fig.4 it is clearly observed that accuracy of DOA estimates is directly proportional to the number of snapshots. The estimated DOA for two different snapshots is as shown in Table.4.

Actual DOA	Estimated DOA	
	Snapshots=200	Snapshots=1000
25°	24.82°	25°
80°	79.29°	80°

Table 4: DOA estimation for snapshots 200 and 1000 (SNR= 25 db and number of arrays elements=8)

VI. ESPRIT ALGORITHM

A. Algorithm

ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) was developed by Roy and Kailath in 1989 [4]. It is based on the rotational invariance property of the signal space to make a direct estimation of the DOA and obtain the angles of arrival without the calculation of a pseudo-spectrum on the extent of space, nor even the search for roots of a polynomial.

This method exploits the property of translational invariance of the antenna array by decomposing the main network into two sub-networks of identical antennas which one can be obtained by a translation of the other, Fig.5

ISSN: 2278 – 1323
International Journal of Advanced Research in Computer Engineering & Technology
Volume 1, Issue 4, June 2012

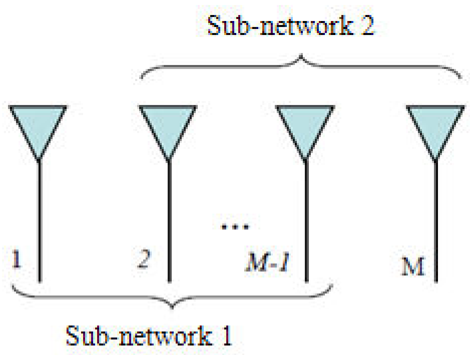

Fig.5 Principle of the ESPRIT algorithm

The main advantage of this method is that it avoids the heavy research of maxima of a pseudo-spectrum or a cost function (therefore a gain calculation) and the simplicity of its implementation. In addition, this technique is less sensitive to noise than MUSIC. It is based on the array elements placed in identical displacement forming matched pairs, with M array elements, resulting in m=M/2 array pairs called "doublets".

Computation of signal subspace for the two sub arrays, P_1 and P_2, results in two vectors V_1 and V_2, such that Range[S]=Range[B]. Also, there should exist a non singular matrix T of D×D such that $V_s = \bar{B}T$, where V_s can be decomposed into V_1 and V_2.

$$V_1 = BT \;,\; V_2 = B\varphi T \quad (16)$$

$$\varphi = diag\left[e^{jkdsin(\theta_1)}, e^{jkdsin(\theta_2)}, \ldots, e^{jkdsin(\theta_D)}\right]$$
(17)

If D×D is diagonal,unitary matrix with phase shifts between doublets for each DOA , there exists a unique rank D matrix FεC such that

$$V_1W_1+V_2W_2=BTW_1+B\varphi TW_2=0 \quad (18)$$

Rearranging eqn(18), we get:

$$BT\psi = B\varphi T \quad (19)$$

With B as full rank and sources are having distinct DOA, then

$$\psi = T^{-1}\varphi T \quad (20)$$

Eqn(20) indicates that if we are able to find out eigenvalues of, which are diagonal elements of φ ,we can estimate DOA as $\varphi = (\alpha_1, \alpha_2, \ldots, \alpha_D)$ where

$$\alpha_1 = e^{jkdsin(\theta_i)} \quad i=0,1,2,\ldots D \quad (21)$$

DOA can be calculated by

$$\theta_i = sin^{-1}\left[\frac{\arg(\alpha 1)}{kd}\right] \quad (22)$$

B. Simulation Results

DOA estimation using ESPRIT algorithm is performed based on the same set of input parameters specified for MUSIC algorithm and are shown in Table 5.

Parameter	Value
Angle of arrival of signals	$25°$, $80°$
Inter element spacing (*d*)	$\frac{\lambda}{2}$
Number of elements in an array	8 ,16
SNR range (dB)	0 to 25
Number of snapshots	200,1000

Table 5: Input parameters for ESPRIT algorithm

DOA estimation using ESPRIT technique has been tested for the following cases.

Case.1. Varying number of array elements

ESPRIT algorithm has been tested for two different sizes of array elements (8 and 16) by considering other parameters as shown in Table 5.

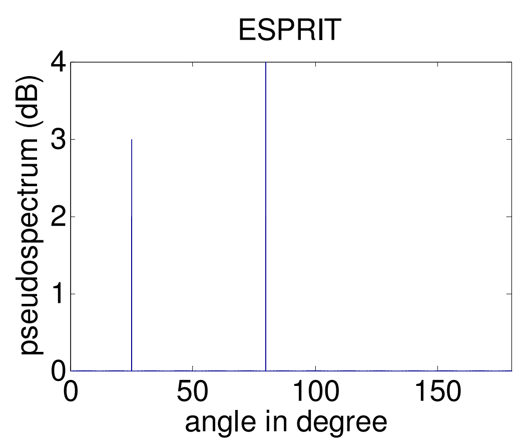

Fig.6: ESPRIT spectrum for array elements 8 and 16. (SNR= 25 db and number of snapshots = 200)

ESPRIT spectrum provides almost similar DOA estimates for different array elements with lesser variance than MUSIC algorithm.

Actual DOA	Estimated DOA	
	Array elements=8	Array elements=16
25°	25.0031°	25.0010°
80°	79.9565°	79.9873°

Table 6: DOA estimation for arrays elements 8 and 16 (SNR= 25 db and number of snapshots = 200)

From Table 6 it is observed that as array size increases from 8 to 16, DOA estimates of signals are almost close to actual DOA which is superior to performance of MUSIC.

Case.2. Varying number of SNR values

ESPRIT technique has been tested for two different SNR values 0dB and 25dB as in the case of MUSIC algorithm.

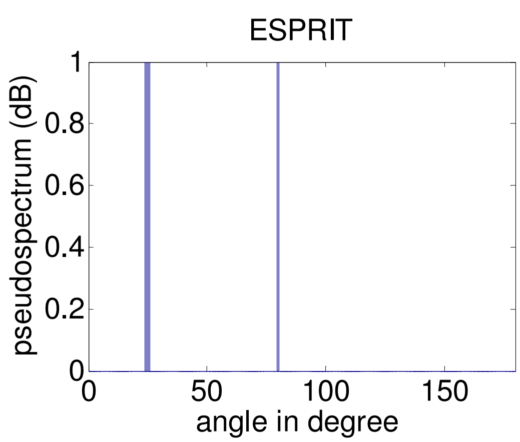

Fig.7: ESPRIT spectrum for SNR of 0dB and 25dB. (Number of array elements = 8 and number of snapshots =200)

Pseudo spectrum obtained using ESPRIT technique for different S.N.R values is shown in Fig.7 and their DOA estimates are shown in Table 7.

Actual DOA	Estimated DOA	
	SNR=0dB	SNR=25dB
25°	24.92°	24.9658°
80°	79.1016°	80.0026°

Table 7: DOA estimation for SNR 0dB and 25dB (Number of array elements = 8 and number of snapshots =200)

From Table 7 it is observed that ESPRIT provides better resolution than MUSIC even at lower SNR.

Case.3. Varying number of snapshots

ESPRIT algorithm has been tested for two different snapshots 200 and 1000 by considering input parameters as shown in Table 5.

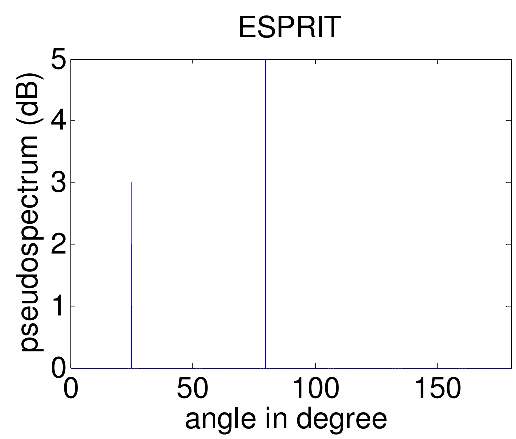

Fig.8: ESPRIT spectrum for two different snapshots 200 and 1000. (SNR= 25 db and number of arrays elements=8)

From Fig.8 it is observed that DOA estimates for ESPRIT technique are almost close to actual DOA even at lesser number of snapshots and hence it outperforms MUSIC algorithm.

Table 8: DOA estimation for snapshots 200 and 1000 (SNR= 25 db and number of arrays elements=8)

Actual DOA	Estimated DOA	
	Snapshots=200	Snapshots=1000
25°	25.0056°	25.0047°
80°	80.1840°	80.0013°

The observation made from Table 8 clearly indicates that ESPRIT provides more precise estimation of DOA than MUSIC.

VI. QUATERNION MUSIC ALGORITHM

A. Quaternions

Quaternions are a four dimensional hyper complex number system. Discovered by Sir R.W. Hamilton in 1843 [11], they are an extension of complex numbers to 4D space. A quaternion q is described by four components (one real and three imaginaries). It can be expressed in its Cartesian form as:

$$q = a + ib + jc + kd, \tag{23}$$

Where

$$i^2 = j^2 = k^2 = ijk = -1,$$

$ij = k \quad ji = -k,$

$ki = j \quad ik = -j,$

$jk = i \quad kj = \quad i.$

Several properties of complex numbers can be extended to quaternions. Some of them are given below:

- The conjugate of q, noted $\bar{q}$, is given by: $\bar{q} = a + ib + jc + kd;$

- A pure quaternion is a quaternion which real part is null:
$q = ib + jc + kd$

- The modulus of a quaternion q is $|q|$ = $\sqrt{q\bar{q}}$= $\sqrt{\bar{q}q}$= = $\sqrt{a^2 + b^2 + c^2 + d^2}$ and its inverse is given by

$$q^{-1} = \frac{\bar{q}}{|q|^2} \tag{24}$$

- A quaternion is said to be null *if* a = b = c = d = 0;

- The set of quaternion's, denoted by $\mathbf{H}$, forms a non-commutative normed division algebra, that means that given two quaternion's q_1 and q_2:

$$q_1 q_2 \neq q_2 q_1 \tag{25}$$

- Conjugation over $\mathbf{H}$ is an anti-involution:

$$\overline{q_1 q_2} = \overline{q_2 q_1} \tag{26}$$

It is that a quaternion is uniquely expressed as: $q = q^1 + jq^2$, where q^1 = $a + ib$ and q^2 = $c - id$ .This is known as the Cayley-Dickson form. It is also possible to express q in an alternate Cayley-Dickson form. Thus, any quaternion q can be written as: $q = q' + iq''$, where q' = $a + jc$ and q'' = $b + jd$. This notation will be used in the polarized signal quaternion model proposed in the next section.

B. Polarization model

Consider a scenario with one polarized source, assumed to be a centred, stationary stochastic process, emitting a wave field in an isotropic, homogenous medium. This wave field is recorded on a two-component noise free sensor, resulting in two highly correlated temporal series $s_1(t)$, $s_2(t)$. In this case, the two signals are linked by phase and amplitude coefficient called polarization parameters (supposed constant in time and frequency). Consider that the two signals in time are transformed in the Fourier domain as $s_\eta(\nu) = FT[s_\eta(t)]$,with $s_\eta(\nu) \in \mathbf{C}^I$ and $\eta = 1,2,$ then the two-component signal recorded on a vector –sensor can be expressed in the frequency domain such as:

$$s(\nu) = s_1(\nu) + i\, s_2(\nu) \tag{28}$$

This signal is pure quaternion valued. Taking the first component $s_1(\nu)$as reference and with the previous assumptions on polarization parameters, $s(\nu)$ can be rewritten as:

$$s(\nu) = (1 + [i\rho]_1 1\, e^{\dagger}(I\varphi_1 1))\ s_1(\nu) \tag{29}$$

Where ρ_1, ρ_2 and φ_1, φ_2 are the amplitude ratios and the phase shifts for the second and third component respectively, with respect to the first one. In the following, the working frequency will be omitted, assuming that we consider narrowband signals or that we work independently at each frequency.

C. Quaternion-MUSIC

MUSIC-like methods are based on the projection of a steering vector on the estimated noise subspace. Here we discuss a new approach based on hypercomplex number system discovered by Hamilton[11],they are an extension of complex numbers to four-dimensional(4-D) space. This new approach improves the signal subspace estimation accuracy and reduces the computational burden. Additionally, the discussed algorithm presents a better resolution power for direction of arrival (DOA) estimation than the long-vector approach, for equivalent statistical performances and also results in a reduction by half of memory requirements for representation of data covariance model.

The Quaternion Music can be summarized as follows [12]:

Step1: Consider a linear, uniform vector-sensor array of N sensors .Assume that the far-field waves from K sources (K-known, K < N) impinge on an antenna as shown in Fig.9. Sources are supposed to be decorrelated, spatially coherent and confined in the array plane. Their polarizations are also stationary in time and space (distance). Noise is spatially white and not polarized.

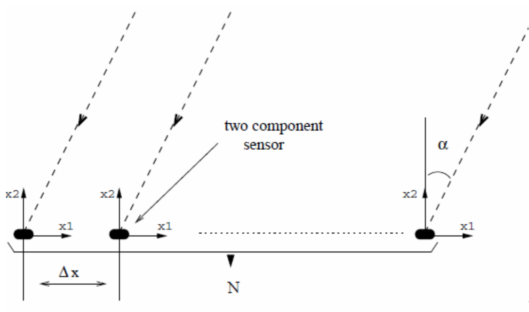

Fig. 9 Acquisition Scheme

Step2: The source direction of arrival (DOA) can then be calculated using the following formula:

$$\theta = 2\pi \mathrm{v} \frac{\Delta x \sin\alpha}{v} \tag{30}$$

Where Δx is the inter-sensors distance, v is the wave propagation velocity and v is the working frequency.

step3: The output of the vector-sensor array is given by a column quaternion vector $x \in \mathbb{H}^N$ equal to the sum of the K sources contributions (added with a noise term):

$$X = \sum_{k=1}^{K} [d_k \beta_{1k} \exp(j\alpha_{1k})] + n \tag{31}$$

Step4: We introduce the equivalent second order representation for a vector-sensor array using quaternion formalism. Quaternion spectral matrix $\Omega = H^{N\mathrm{x}N}$ is defined as:

$$\Omega = \mathbb{E}\{xx^{\triangleleft}\} \tag{32}$$

Step5: Assuming the decorrelation between the noise and the sources and between sources themselves, the expression of the quaternion spectral matrix becomes:

$$\Omega = \Omega_S + \Omega_N \tag{33}$$

Where $\Omega_S = \sum_{k=1}^{K} \sigma_k^2 c_k c_k^{\triangleleft}$ and $\Omega_N = \mathbb{E}\{\mathbf{nn}^{\triangleleft}\}$ is a matrix containing noise second order statistics.

Step6: We associate the first K eigenvalues to the signal part of the observation and the rest of N-K to the noise part. Thus, the projector on the noise subspace is defined as:

$$\prod_N = \sum_{k=K+1}^{N} \mathbf{U}_k \mathbf{U}_k^{\triangleleft} \tag{34}$$

Step7: The quaternion steering-vector $q \in \mathbb{H}^N$ is then generated:

$$q(\theta,\rho,\varphi) = \sqrt{N(1+\rho^2)} \begin{pmatrix} 1 + i\rho e^{j\varphi} \\ e^{-j\theta} + i\rho e^{j(\varphi-\theta)} \\ e^{-j(N-1)\theta} + i\rho e^{j(\varphi-N-1)\theta} \end{pmatrix} \tag{35}$$

Where q is a three-parameter unitary vector, modelling the arrival of a source of DOA and polarization parameters on a 2C sensor array.

Step8: Quaternion-MUSIC estimator (Q-MUSIC) is then computed by projecting the steering-vector $q(\theta,\rho,\varphi)$ on the noise subspace:

$$M_Q(\theta,\rho,\varphi) = \frac{1}{q^{\triangleleft}(\theta,\rho,\varphi) \prod_N q(\theta,\rho,\varphi)} \tag{36}$$

eqn (36) for Q-MUSIC estimator similar to the well-known form of MUSIC algorithm for scalar-sensor array. The functional in (36) has maxima for sets of (θ,ρ,φ) corresponding to sources present in the signal.

D. Simulation Results

DOA estimation using Q-MUSIC algorithm has been tested for one polarized source impinging on a two component (2C) - sensor array for input parameters shown in Table 9.

Parameter	Value
Angle of arrival of signals	0.44rad ≈ 25°
Inter element spacing (d)	$\frac{\lambda}{2}$
Number of elements in an array	8 ,16
SNR range (dB)	0 to 25
Number of snapshots	200,1000
constant modulus ratio (ρ)	3
constant phase shift (φ)	0.27rad

Table 9: Input parameters for Q-MUSIC algorithm

Q-Music algorithm has been tested for the following cases.

Case.1. Varying number of array elements

For two different array elements 8 and 16 Q-MUSIC algorithm has been tested with input parameters shown in Table 9.

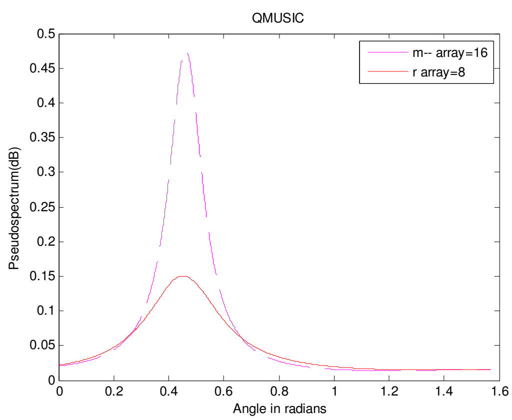

Fig.10: Q-MUSIC spectrum for two different array of sensor 8 and 16. (SNR= 25 db and number of snapshots = 200)

From the Fig.10 it is observed that as array elements increases from 8 to 16 the 3dB detection lobe width for Q-MUSIC becomes narrower compared to MUSIC and ESPRIT and hence provides accurate DOA and polarized estimates for multidimensional signals.

Actual DOA	Estimated DOA	
	Sensors=8	Sensors=16
25°	24.97°	25°

Table 10: DOA estimation for array of sensor 8 and 16 (SNR= 25 db, number of snapshots = 200, ρ =3 and $\varphi = 0.27rad$)

Case.2. Varying number of SNR values

Q-MUSIC algorithm simulation has been carried out for two different SNR values 0dB and 25dB for input parameters shown in Table 9.

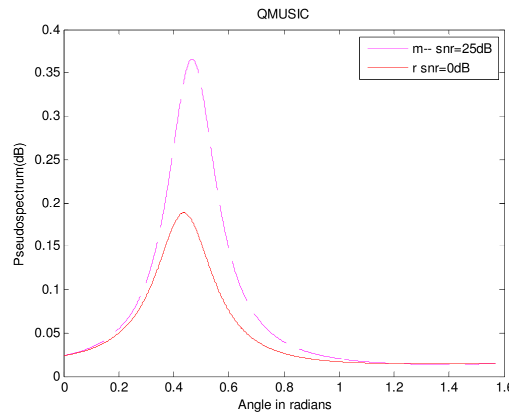

Fig.11: QMUSIC spectrum for S.N.R values 0dB and 25dB. (Number of array elements = 8 and number of snapshots =200)

From Fig.11 it is observed that even when SNR is 0dB, Q-MUSIC still provides better estimation of DOA of signals when compared to MUSIC. The estimates are provided in Table 11.

Actual DOA	Estimated DOA	
	SNR=0dB	SNR=25dB
25°	25.0675°	25°

Table 11: DOA estimation for two different SNR 0dB and 25dB (array of sensor=8, number of snapshots = 200, ρ =3 and $\varphi = 0.27rad$)

Case.3. Varying number of snapshots

Q-MUSIC algorithm has been tested for two different snapshots 200 and 1000 by considering input parameters shown in Table 9.

ISSN: 2278 – 1323
International Journal of Advanced Research in Computer Engineering & Technology
Volume 1, Issue 4, June 2012

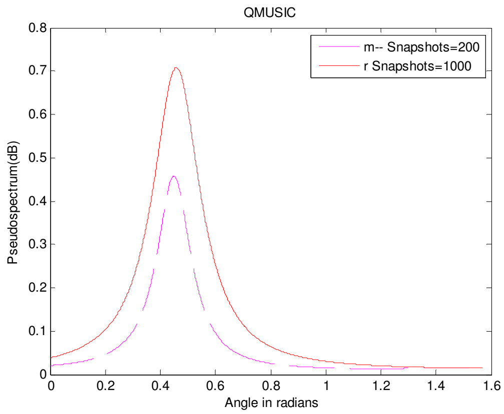

Fig.9: QMUSIC spectrum for snapshots 200 and 1000. (SNR= 25 db and number of arrays elements=8)

The estimation of DOA using Q-MUSIC algorithm is directly proportional to the number of snapshots similar to MUSIC. But from Fig.9, it is observed that Q-MUSIC provides better estimation of signal arrival even at lower number of samples than MUSIC and the estimated DOA is equal to actual DOA as shown in Table 12.

Actual DOA	Estimated DOA	
	Snapshots=200	Snapshots=1000
25°	25.0092°	25.0023°

Table 12: DOA estimation for snapshots 200 and 1000 (SNR= 25 db, number of sensors = 8, ρ =3 and $\varphi = 0.27rad$)

VII. COMPARISON OF MUSIC, ESPRIT AND Q-MUSIC ALGORITHMS

Section IV, V and VI discussed the various algorithms used in smart antenna systems for robust DOA estimation. The performance analysis of MUSIC, ESPRIT and Q-MUSIC algorithm based on high resolution direction of arrival estimation is compared in this section for the following cases.

Case.1. Varying number of array elements

For array elements 8and 16 all the three algorithms have been simulated and results are tabulated in Table 13.

Array elements=8			
Algorithm	Actual DOA	Estimated DOA	% Error
MUSIC	25°	25°	0°
ESPRIT	25°	25.00314°	0.00314°
Q-MUSIC	25°	24.97°	0.03°
Array elements=16			
Algorithm	Actual DOA	Estimated DOA	% Error
MUSIC	25°	25°	0°
ESPRIT	25°	25.0010°	0.001°
Q-MUSIC	25°	25°	0°

Table 13: DOA estimation for array elements 8 and 16 (SNR= 25 db and number of snapshots = 200)

From Table 13, it is clear that as the number of array elements increase, accurate DOA estimation can be achieved. Amongst the three algorithms discussed above, Q-MUSIC has a lesser % error and the estimated DOA is almost equal to the actual DOA.

Case.2. Varying number of SNR values

For SNR= 0 dB and 25dB, all the three discussed algorithms have been simulated and results are tabulated in Table 14.

SNR=0dB			
Algorithm	Actual DOA	Estimated DOA	% Error
MUSIC	25°	24.70°	0.3°
ESPRIT	25°	24.92°	0.08°
Q-MUSIC	25°	25.0675°	0.0675°
SNR=25dB			
Algorithm	Actual DOA	Estimated DOA	% Error
MUSIC	25°	25°	0°
ESPRIT	25°	24.9658°	0.0342°
Q-MUSIC	25°	25°	0°

Table 14: DOA estimation for SNR 0db and 25dB. (Number of array elements=8 and number of snapshots = 200)

In general, performance of DOA estimation degrades as signal to noise ratio decreases. But Q-MUSIC accurately estimates DOA even at a very lower gain of 0dB.

Case.3. Varying number of snapshots

The comparison table for performance of three discussed algorithms is shown in Table 15 for two different snapshots 200 and 1000.

Snapshots=200			
Algorithm	Actual DOA	Estimated DOA	% Error
MUSIC	25°	24.82°	0.3°
ESPRIT	25°	25.0056°	0.08°
Q-MUSIC	25°	25.0092°	0.0675°
Snapshots=1000			
Algorithm	Actual DOA	Estimated DOA	% Error
MUSIC	25°	25°	0°
ESPRIT	25°	25.0047°	0.0342°
Q-MUSIC	25°	25.0023°	0°

Table 15: DOA estimation for snapshots 200 and 1000 (SNR= 25 db and number of array elements=8)

From Table 15, it is observed that Q-Music provides better detection capability for lower number of samples compared to the other two algorithms. Hence Q-MUSIC algorithm is best suited in multipath environment whereas the received signal is difficult to estimate accurately.

VIII. CONCLUSION

In this paper, we discussed three high resolution direction finding algorithms based on their performance for different number of array elements, snapshots and different SNR's. From the simulated results, it is observed that MUSIC and ESPRIT provides an accurate estimation of DOA with improved resolution power than the other direction finding techniques. But in the case of diversely polarized antenna arrays where the SNR values and number of snapshots are less, these two algorithms failed to achieve precise estimation of DOA of signals. To overcome these drawbacks we have discussed a new technique for vector sensor array processing using Quaternion's for diversely polarized array. This Quaternion based algorithm (Q-MUSIC) of EM vector sensors can achieve better performance than scalar–sensor arrays, while occupying less space and cost for hardware implementation. However, the processing requirements are somewhat higher for the diversely polarized array. Further we have shown that Q-MUSIC provides higher direction finding accuracy and higher resolving power even with lower SNR and snapshots than scalar techniques (MUSIC, ESPRIT). Thus Q-MUSIC can be quite successfully used in radar's to improve clutter rejection and discriminate between different types of target. It also provides more efficient spectrum utilization for mobile communication systems.

REFERENCES

[1] V. F. Pisarenko, "*The retrieval of harmonics from a covariance function,*" Geophysical J. of the Royal Astron. Soc., vol. 33, pp. 347–366, 1973.

[2] R. O. Schmidt, "*Multiple emitter location and signal parameter estimation,*" IEEE Trans. Antennas Propagation, vol. 34, no. 3, pp. 276–280, Mar. 1986.

[3] A. J. Barabell, "*Improving the resolution performance of eigenstructure based direction-finding algorithms,*" Proc. of ICASSP, Boston, MA, USA, pp. 336–339, 1983.

[4] Richard Roy and Thomas Kailath, "*ESPRIT-Estimation of signal parameters via rotational invariance techniques,*" IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 37, N° 7, pp. 984-995, July 1989.

[5] H. D. Schutte and J. Wenzel, "Hypercomplex numbers in digital signal processing," *Proc. IEEE Int. Symp. Circ. Syst.*, pp. 1557–1560, 1990.

[6] K. T. Wong and M. D. Zoltowski, "Self- initiating MUSIC direction finding and polarization estimation in spatio polarizational beamspace," *IEEE Trans. Antennas Propagation*, vol. 48, pp. 1235–1245, Sept. 2000.

[7] K. T. Wong and M. D. Zoltowski, "Diversely polarized root-MUSIC for azimuth- elevation angle of arrival estimation," *Digest of the 1996 IEEE Antennas Propagation Soc. Int. Symp., Baltimore*, pp. 1352–1355, July 1996

[8] M. D. Zoltowski and K. T. Wong, "ESPRIT-based 2D direction finding with a sparse array of electromagnetic vector-sensors," *IEEE Trans. Signal Processing,* vol. 48, pp. 2195–2204, Aug. 2000.

[9] K. T. Wong and M. D. Zoltowski, "Uni-vector-sensor ESPRIT for multi-source azimuth, elevation and polarization estimation," *IEEE Trans. Antennas Propagation*, vol. 45, pp. 1467–1474, Oct. 1997.

[10] J. Li and J. R. Compton, "Angle and polarization estimation using ESPRIT with a polarization sensitive array," *IEEE Trans. Antennas Propagation*, vol. 39, pp. 1376–1383, Sept. 1991.

[11] W.R.Hamilton, " On quaternions," in *Proc. Royal Irish Academy*,1843

[12] Miron S., Bihan N.L. and Mars J. "Quaternion-MUSIC for Vector-Sensor Array processing," *IEEE Trans. Signal Processing*, 2006, 54(4):1218-1229.

[13] N. Le Bihan and J.Mars," Singular value decomposition of matrices of quaternions: A new tool for vector-sensor signal processing." *Signal Process,* vol.84, n0 7, pp 1177-1199, 2004.

[14] CUI Wei, "A Novel Quaternion DOA estimation algorithm based on magnetic loop antenna".

[15] J.P. Ward, *Quaternion's and Cayley Numbers, Algebra and applications,*Kluwer Academic, 1997

[16] F. Zhang, "Quaternions and matrices of quaternions," *Linear algebra and its applications,* vol. 251, pp. 21–57, 1997.

[17] Kantor and A. Solodovnikov, *Hypercomplex numbers, an elementary introduction to algebras. Springer-V erlag, 1989.*

[18] Kantor and A. Solodovnikov, *Hypercomplex numbers, an elementary introduction to algebras. Springer-V erlag, 1989.*

[19] L.C. Godra, "Application of antenna arrays to mobile communications-PartI: Performance improvement, feasibility and system considerations," *Proc. IEEE,* vol. 85, pp. 1031-1060, Jul, 1997.

[20] L.C. Godra,"Application of antenna arrays to mobile communications_Part II: Beam – forming and direction-of-arrival considerations," *Proc. IEEE*,vol. 85,pp.1195-1245, Aug 1997.

BIOGRAPHIES

Chetan.L.Srinidhi graduated in Electronics and Communication Engineering from Nitte Meenakshi Institute of Technology, Bangalore India. He is Currentlty pursuing his M.Tech degree in Communication syatems, at R.V.College of Engineering, Bangalore, India. His research interests are in the fields of digital Signal Processing, Antennas and Digital Communication.

Dr.S.A.Hariprasad obtained his PhD from Avinashilingam University for Women, Coimbatore in the area of digital controller's .He is having teaching experience of 22 years and five years of research experience. He has published nearing 35 papers in international/national journals and conferences. He has also published a text book on advanced Microprocessor and reviewed books on Microwave engineering and won best teacher award (twice) from RSST and appreciation award from ISTE. His research areas of interest are embedded systems and RF systems.