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A comparative study of Level II fuzzy sets and Type II fuzzy sets

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Abstract-

This paper reviews and compares theories of level II fuzzy sets and type II fuzzy sets. Two approaches for the usefulness of level II fuzzy sets are reviewed one is in database modeling while other is in GIS. Type II fuzzy sets under set theoretic view seem to be closely related to level II fuzzy sets, but linguistically they deal with different types of variables in database modeling.

Keywords: Type 2 fuzzy sets, Level 2 fuzzy sets.

Section1: Introduction-

Theories of level II fuzzy sets and type II fuzzy sets are generalizations of fuzzy set theory for modeling higher order of uncertainty. Real world problem involve many kinds of imperfections in the data. This imperfect information can be classified in following five categories- Uncertain information, imprecise information, vague information, inconsistent information and incomplete information [4]. Fuzzy sets linguistically deal with above types of information but value of each element being crisp cannot handle higher level of uncertainty [2]. Level II fuzzy sets are mainly used to precise clearly the difference between fuzziness and uncertainty [5]. Details will be discussed in the section 2, while type II fuzzy sets linguistically deal capturing uncertainties with words with capturing uncertainties with words. The fact that "word means different for different people" [13] is handled beautifully by type II fuzzy sets. Details of this will be discussed in section 2.2. There have been many studies on the topic of level II fuzzy sets in database modeling [4,5]. While the studies of type II fuzzy sets in the sense of computing with words are scarce [11,12,13]. The reason behind is that type II fuzzy sets cannot handle different types of uncertainty but can handle the uncertainty handled by fuzzy sets more precisely or with more level of uncertainty [9] while level II fuzzy sets can be used to model different types of uncertainties in the information at a same times [7]. It is very important to realize that our comparisons of two theories are based on very specific interpretations of each theory. Furthermore many issues involved in both theories are not taken into

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consideration. Although conclusions drawn above from such comparisons should be read cautiously, the examination may provide more insights into both theories.

Section 2) Overview of level II fuzzy sets and type II fuzzy sets-

There are many formulations and interpretations of theories of theories of level II fuzzy sets and T2FS [3,14]. Both theories are extensions of fuzzy set theories proposed by L.A.Zadeh [6, 16].

2.1- Type II fuzzy sets-

The notion of type II fuzzy sets provides a convenient tool for representation vague concepts more imprecisely by allowing fuzzy sets as partial memberships.

Let X be nonempty crisp set called as universe of discourse. Then type II fuzzy set [ ] $\tilde{A}$ on X is defined as,

$$\tilde{A} = \int_{x \in X} \left[\int_{u \in J_x^u} f_x(u)/u \right] \Big/ x \quad \text{where}$$

u is dummy variable, J_x^u is foot print of uncertainty and $f_x : J_x^u \to [0,1]$ is a function where J_x^u is subset of $[0,1]$. In short if $F(X)$ denotes set of all fuzzy sets then a type II fuzzy set is a function from $\tilde{A} : X \to F([0,1])$

Set theoretic operations and algebraic operations are defined on type II fuzzy set concentrate more on primary membership [10,11] than secondary membership.

Details of all operations given above are available in [10,15] and hence are not discussed in details here.

2.2- Level II fuzzy sets-

The level II fuzzy set also denoted by L2FS is defined as

$$\breve{\breve{A}} = \int_{T \in S} \mu_{\breve{\breve{A}}}(T) \Big/ \left(\int_{x \in X} \mu_T(x)/x \right)$$

where $\mu_{\breve{\breve{A}}} : F(X) \to [O,1]$ is called outer layer membership, S is collection of all fuzzy subsets of X whose outer layer membership is nonzero while $\mu_T : X \to [O,1]$ is called inner layer membership. Also level II fuzzy set can be treated as function from sets of fuzzy sets $[O,1]$

i.e. $\breve{\breve{A}} : F(X) \to [0,1]$

Set theoretic operations on level II fuzzy sets are given as follows-

If $\breve{\breve{A}}$ and $\breve{\breve{B}}$ are level II fuzzy sets defined on X , given by

$$\breve{\breve{A}} = \int_{T \in S} \mu_{\breve{\breve{A}}}(T) \Big/ \left(\int_{x \in X} \mu_T(x)/x \right)$$

And

$$\breve{\breve{B}} = \int_{T' \in S'} \mu_{\breve{\breve{B}}}(T') \Big/ \left(\int_{x \in X} \mu_{T'}(x)/x \right)$$

Then

$$\breve{\breve{A}} \cup \breve{\breve{B}} = \int_{T'' \in S''} S\left(\mu_{\breve{\breve{A}}}(T), \mu_{\breve{\breve{B}}}(T')\right) \Big/ \left(\int_{x \in X} S\left(\mu_T(x), \mu_{T'}(x)\right)/x \right)$$

Where S denotes s-norm or t-conorm

while $T'' = T \cup T'$

and $S'' = S \cup S'$

$$= \{T \cup T' / T \in S, T' \in S'\}$$

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And in intersection, we replace s-norm by t-norm.

Similarly, for complement we concentrate equally on inner layer membership as well as outer layer membership i.e. complement is given by

$$\breve{\tilde{A}}^{C} = \int_{T \in S} \left(1 - \mu_{\breve{\tilde{A}}}(T)\right) \Big/ \left(\int_{x \in X} \left(1 - \mu_T(x)\right) / x \right)$$

Note that standard complements can be replaced by any functions satisfying complement axioms [8].

Section 3) Comparisons of level II fuzzy sets and type II fuzzy sets –

This section compares theories of level II fuzzy sets and type II fuzzy sets based on the models presented in earlier section.

1) **Number of L2FS and T2FS on a crisp set-**
   The number of level II fuzzy sets on a set are always greater than or equal to number of type II fuzzy sets on the same set . Proof is available in [3] and hence not included as part of paper.
2) **Concentration on memberships in set theoretic and algebraic operations of L2FS and T2FS-**
   As stated above while defining union, intersection on T2FS a emphasis is given on primary membership function. i.e. while defining these operations, extension principle [18] is used ones and the purpose for doing this is its ease to write a computer program. While in defining operations on level II fuzzy set we need extension principle twice for each computation.
3) **Computing with words using T2FS and L2FS-**
   L2FS deals with linguistic variables like “preferably”, “possibly”, “more or less recent” [4,18], while type II fuzzy sets deal with the linguistic variable like “many”, “few”, “fast” (cars), “expensive” (books), [1] linguistic variables of fuzzy sets are same as T2FS but T2FS can handle them more efficiently. One more difference between the theories is that T2FS is developed much more as compared to L2FS. The reason may be stated as follows-Fuzzy set theory had to face many obstacles from traditional mathematical societies. It was accepted with open hearts when its applicability came into the scene and hence common scenario in the development of the theory is develop mathematics according to real world problem use it and keep it. That is, the reason why no proper, purely theoretical developments are found in the field. In next section we will discuss about combination of two types of fuzzy sets L2FS and T2FS.

Section 4) Combinations of L2FS and T2FS-

A very beautiful class of fuzzy sets develops on combination of above two viz. type II level II fuzzy set [7]. It is defined as follows-

Let X be a crisp set then type II level II fuzzy set [7] is a function

$\breve{A} : F(X) \to F([0,1])$ where T is some fuzzy subset X and it maps to some fuzzy set of $[0,1]$.

Mathematically, a type II level II fuzzy set (T2L2FS) can be defined as

$$\breve{A} = \int_{T \in S} \left[\int_{u \in J_T^U} f_T(u)/u \right] \Big/ \int_{x \in X} \mu_T(x)/x$$

where,

1) X is crisp set.
2) T is fuzzy set with outer layer membership as as a fuzzy set (around $\mu_{\breve{A}}(T)$).
3) u is dummy variable.
4) J_T^u is a fuzzy subset of [0,1].
5) $f_T : J_T^u \rightarrow [0,1]$ is a function.
6) S is collection of T which satisfy condition (2).

For example:

In discrete case real number x is preferably more near to 6 than near to 8 can be represented as

$$\breve{A} = \left(\frac{0.7}{0.7} + \frac{1}{8} + \frac{0.6}{0.9}\right) \Big/ \left(\frac{0.7}{5} + \frac{1}{6} + \frac{0.7}{7}\right) + \left(\frac{0.8}{0.2} + \frac{1}{0.3} + \frac{0.7}{0.4}\right) \Big/ \left(\frac{0.7}{7} + \frac{1}{8} + \frac{0.9}{9}\right)$$

Section 5) Conclusion-

In this paper, we have examined the relationships and differences between the theories of level II fuzzy sets and type II fuzzy sets. Although, geometrically they look identical but they are altogether different and deal with different kinds of linguistic variables. And as the proverb goes "A multiplication rules over the division", combining these two theories can handle more degrees of uncertainty and give solution to more perfection for real world problems.

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