

GENETIC OPERATORS IN SOLVING TRAVELLING SALESMAN PROBLEM

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Abstract— In this paper we will see that genetic algorithm applied to a optimization problem uses genetic operator to solve the problem. In this paper introduction of modified form of genetic operator is done and then we tried to solve the problem. We have taken TSP problem and applied the operator to TSP problem. We have also tried to understand the actual meaning of optimization both in theoretical and mathematical sense. This paper presents the strategy which is used to find the nearly optimized solution to these type of problems. It is the order crossover operator (OX) which was proposed by Davis, which builds up the offspring by choosing a subsequence of one parent and preserving the relative order of chromosomes of the other parent.

Keywords: Genetic Algorithm, Optimisation, Traveling salesman problem, Order Crossover, PMX

I. INTRODUCTION

One of the most fundamental principle in world is to look for optimal solution. This principle applies everywhere ranging from physics where atoms try to form bonds to minimize the energy of their electrons to water where it acquires crystal structure during freezing to have energy optimal structure. The same principle goes with biology given by Darwin which states “Survival of the fittest” is only possible. As long as human kind exist we will always go on for perfection. We want to have maximum happiness with least effort. In our economy, we want to have maximum profit, maximum sales with least cost. So optimization is a concept which extends into our daily life.

As the optimization has a significance in our daily lives and anything which has importance has a mathematical background dealing with it and so is the case with it too. Optimisation is defined as finding

out the best possible x^* elements from a set X according to a set of criteria $F = \{f_1, f_2, f_3, f_4, \dots\}$. These criteria is expressed in a form of mathematical function which is known as Objective function.

Objective Function: An objective function [3][7] is written mathematically as $f : X \rightarrow Y$ with Y which is a subset of R is a mathematical function which is subject to optimization. The domain X of F is called the problem space and codomain and of F must be a subset of real numbers

Global Optimisation actually consist of all the techniques that can be used to find the best element x^* from X and also which satisfies the criteria defined by f .

As we are discussing about optimality and there is lot of mathematics involved in it, we must understand in terms of mathematics only that what is optimality. First we will discuss optimality in terms of Single Objective function and then we will explore the multiobjective function.

SINGLE OBJECTIVE FUNCTION: Whenever we talk about Single objective function [1,7], we actually talk about optimizing a problem as per the single criteria defined. Now that single criteria can be either “Maximum” or “Minimum” depending on our problem. For example

Example 1: Let us take the problem of recruiting the staff in a organization.

Optimal Solution: The optimal solution to this problem will be the recruitment of the staff should be in such a way that maximizes the profit of organization

Example 2: Let us take the problem of assigning jobs to a manufacturing firm in a organization

Optimal Solution: The optimal solution to this problem will be to assign jobs in such a way that minimizes the time taken for completion

But in global optimization problems it is convention to define optimization as minimization, and if the

constraint is maximization then we go for minimization of its negation(-f) ,so that the maximization problem is converted into minimization problem.We need to discuss some more terms to make our concept of optimality more clearer

Local Maxima [7]: A local maxima is defined as $x_i \in X$ of a fn $f \rightarrow R$ an input element with $f(x_i) \geq f(x)$ for all x neighbouring x_i

Local Minima [7]: A local minima is defined as $x_i \in X$ of a fn $f \rightarrow R$ an input element with $f(x_i) \leq f(x)$ for all x neighbouring x_i

Global Maxima [7]: A global maximum $x_i \in X$ of a function $f \rightarrow R$ is an input element with $f(x_i) \geq f(x)$ for all $x_i \in X$

Global Minima [7]: A global minimum $x_i \in X$ of a function $f \rightarrow R$ is an input element with $f(x_i) \leq f(x)$ for all $x_i \in X$

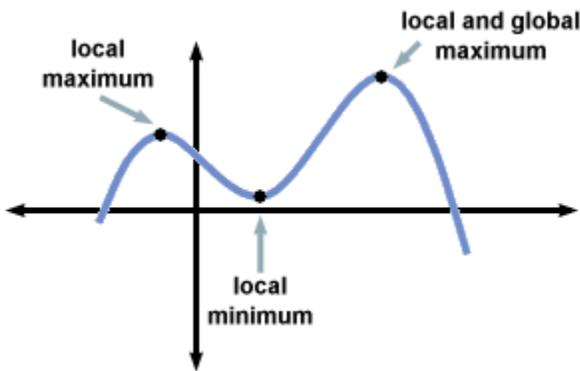


Figure 1

MULTIOBJECTIVE FUNCTION: Optimization techniques are not meant just for finding the maxima or minima of a function,the constraint can be multiple So whenever we will talk about multiple objective function[1][2][3][7],we actually talk about optimizing a problem satisfying multiple criterias defined.Now the multiple criteria can be many..For example:

Example1:Let us take the problem of improving the performance of factory.The multiple criterias which need to be fulfilled are:

1. Minimize the time between the incoming order and shipment of the product

2. Maximize the profit
3. Maximize product quality
4. Minimize the impact of production on environment

The mathematical foundation of MultiObjective function was laid down by Vilfredo Pareto.Pareto optimality become an important notion in Economy,Social Sciences,game theory and Engineering.The notion of optimality is strongly based on the concept of domination.

DOMINATION: The domination[7] is defined as the process where one element is better than other.For example: Let us say we have two elements x_1 & x_2 .We say that x_1 dominates(preferred to) x_2 if x_1 is better than x_2 in one objective function and not worse in all other objective functions.

In solving these optimization problems,we will be using genetic algorithm.A genetic algorithm is a subclass of evolutionary algorithm, which is first proposed for single objective function.But now multiobjective function problems can also be solved by it.The lifecycle of genetic algorithm is as under:

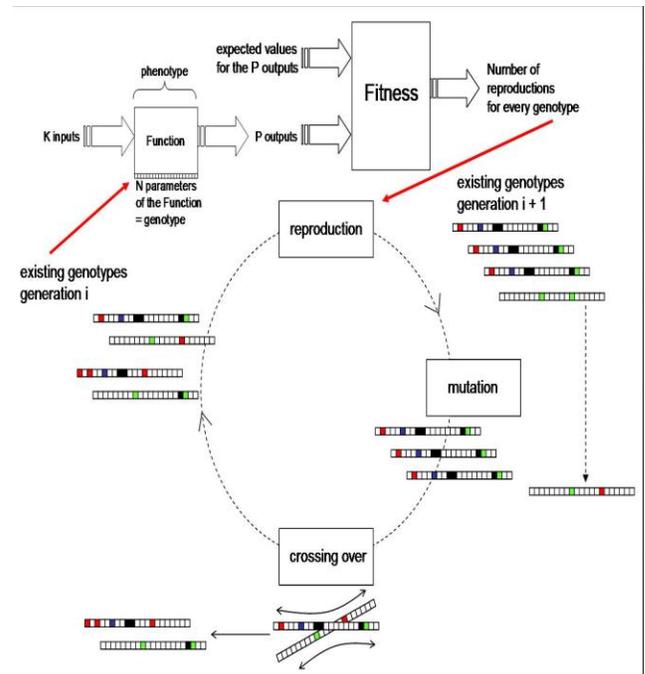


Figure 2

Since they are applied first to single objective function the objective function is known as fitness function.

The above lifecycle can be explained as: Genetic algorithms are iterative loops of optimisation : a fitness function measures the adaptation of a solution to the problem needs. Every solution is represented by a set of numbers that we will call a set of "parameters". These parameters are called "genes".

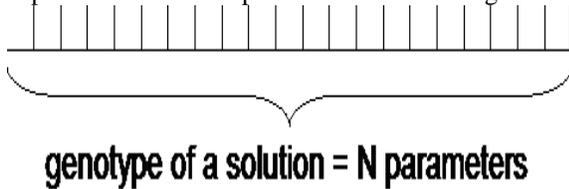


Figure 3

There exist a "function" that uses the parameters. The "function" is called the phenotype, and the problem consist in finding phenotypes [4]adapted to the problem.

Phenotypes [3][4]are filtered by the fitness function that reproduces the most adapted phenotypes. The more the adpatation the more the reproduction. Phenotypes are modified through 2 ways :

- **Mutation** : it consists in moving one parameter through a random modification,

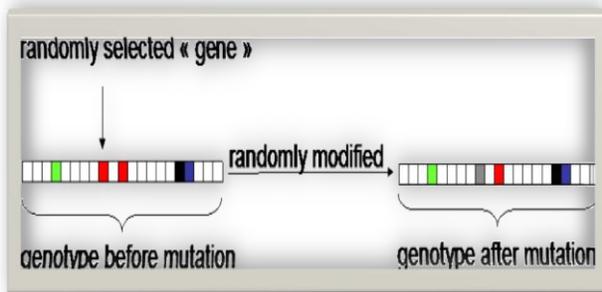


Figure 5

-Crossover: Crossover is a genetic operator used to have variations in the programming of a chromosome or chromosomes. It is analogous to reproduction and biological crossover, upon which genetic algorithms are based[8] Cross over is a process of having taken more than one parents and producing a child solution from them

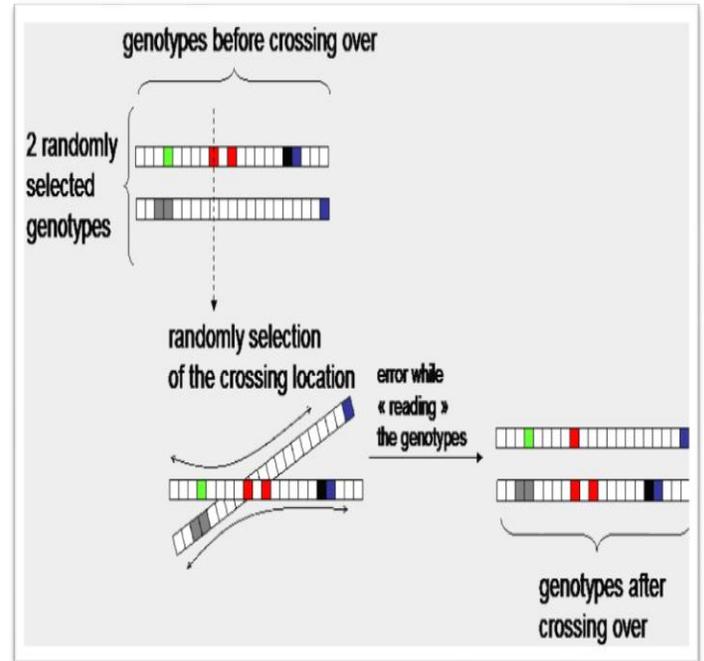


Figure 4

The genetic algorithm make use of genetic operators to solve optimization problem. A **genetic operator** is an operator used in genetic algorithms to maintain genetic diversity, known as Mutation (genetic algorithm) and to combine existing solutions into others, Crossover (genetic algorithm). The main difference between them is that the mutation operators operate on one chromosome, that is, they are unary, while the crossover operators are binary operators.

Genetic variation is a necessity for the process of evolution. Genetic operators used in genetic algorithms are analogous to those in the natural world: survival of the fittest. The different type of genetic operators are as under:

Many crossover techniques exist for organisms which use different data structures to store themselves.

One-point crossover

A single crossover point[8] on both parents' organism strings is selected. All data beyond that point in either organism string is swapped between the two parent

organisms. The resulting organisms are the children:

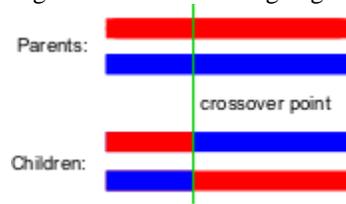


Figure 6

Two-point crossover

Two-point crossover[8] as the name suggest has incorporated two points to be selected on the parent organism strings. Everything between the two points is swapped between the parent organisms, giving rise to two child organisms:

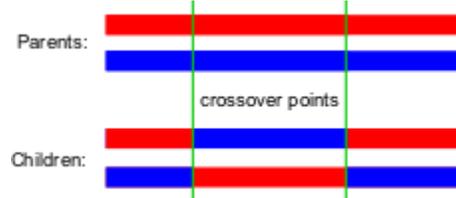


Figure 7

"Cut and splice"[8]

Another crossover variant, the "cut and splice"[8] approach, give rise to change in the length of the children strings. The reason for this difference is that each parent string has a separate choice of crossover point.

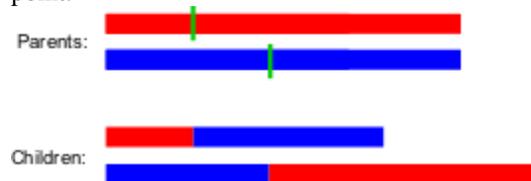


Figure 8

II. PROBLEM DESCRIPTION

The Traveling Salesman Problem (TSP) is a classic combinatorial optimization problem. The problem is stated as : Find the shortest possible tour through N

vertices so that each vertex is visited exactly once. This problem is known to be NP-hard[1], and cannot be solved exactly in polynomial time. Many algorithms have been developed in the field of operations research (OR) to solve this problem. . In the sections we briefly introduce the operation Research problem-solving approaches to the TSP.

III. Exact Algorithm

The algorithms is used to find the shortest possible tour covering all the vertices and finding out the tour and that too of minimum length. The genetic algorithm when applied tries to find many solutions and consider only the optimal one,so they are little more expensive as compared to OR approach. Here d_{ij} is the distance between vertices i and j and the x_{ij} 's are the decision variables: x_{ij} is set to 1 when arc (i,j) is included in the tour, and 0 otherwise. $(x_{ij}) X$ denotes the set of subtour-breaking constraints

$$\text{Min } \sum_{i,j} d_{ij} x_{ij}$$

Subject to:

$$\sum_j x_{ij} = 1, i=1,\dots,N$$

$$\sum_i x_{ij} = 1, j=1,\dots,N$$

$$(x_{ij}) X$$

$$x_{ij} = 0 \text{ or } 1,$$

But as we are seing here the feasible solution is restricted to those consisting of a single tour. Even the formulation of sub tour-breaking constraints can be done in many different ways Without the sub tour breaking constraints, the TSP reduces to an assignment problem (AP), and a solution like the one shown in would then be feasible. Branch and bound algorithms which are mostly found in algorithmic theory are commonly used to find the optimal solution to the TSP[2],

IV. METHOD USED

A. Order Crossover (OX) Davis (85), Oliver et al.

The way the problem is represented here is in the simple two dimension matrix form and here it is

termed as Path matrix which is considered and drawn under topic Figures. The crossover operator which we are going to discuss is different from the Davis’s crossover as it allows two cut points to be randomly chosen on the parent chromosomes. In order to create an offspring, the string between two cut points in the first parent is first copied to the offspring.

and the remaining blanks are filled by considering the position of the chromosomes in the second parent, starting after the second cut point .The above said will get clearer by one of the example shown below.Here the substring 564 in parent 1 is first copied to the offspring (step 1). Then, the remaining positions are filled one by one after the second cut point, by considering the corresponding sequence of cities in parent 2, namely 57814236 (step 2). Hence, city 5 is first considered to occupy position 6, but it is discarded because it is already included in the offspring. City 7 is the next city to be considered, and it is inserted at position 6. Then, city 8 is inserted at position 7, city 1 is inserted at position 8, city 4 is discarded, city 2 is inserted at position 1, city 3 is inserted at position 2, and city 6 is discarded.

parent 1 : 1 2 | 5 6 4 | 3 8 7

parent 2 : 1 4 | 2 3 6 | 5 7 8

offspring

(step 1) : - - 5 6 4 - - -

(step 2) : 2 3 5 6 4 7 8 1

Figure 9.

Clearly, Order CrossOver tries to preserve the relative ordering of the cities in parent 2

V. Modified Better order crossover

In order to improve the efficiency of order crossover operator ,a change is added to it which suggest to detect the minimum edge and hence a minimum edge is detected from the second chromosome and by selecting the first node of this edge as first crossover point and by selecting 2nd crossover point after first by randomly choosing

parent 1 : 1 2 5 6 4 3 8 7

parent 2 : 1 4 2 3 6 5 7 8

Step 1 detect shortest edge from second parent

e.g 3,6 from second parent

Step 2 select second crossover point randomly after first for e.g 5

crossover points are before 3 and after 5 in second chromosome

Step 3 apply order crossover

offspring

Step 3.1 : - - - 3 6 5 - -

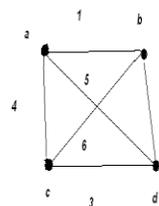
Step 3.2 : 1 2 5 3 6 5 8 7

Step3.3: 1 2 4 3 6 5 8 7

Fig Modified order crossover

VI. RESULTS

A. Tables, Figures and Equations



	a	b	c	d
a	0	1	4	5
b	1	0	6	2
c	4	6	0	3
d	5	2	3	0

Fig 10- Path matrix

parent 1 : 1 2 5 6 4 3 8 7

parent 2 : 1 4 2 3 6 5 7 8

Step 1 find minimum edge from second parent using adjacency matrix

for e.g 3,6

then crossover point is shown in fig

1 4 2 |3 6 5 7 8

Step 2 decide second crossover point randomly after first

for e.g

1 4 2 |3 6 5 |7 8

Step 3 apply crossover

(step 1) 1 2 5 3 6 5 8 7

(step 2) : 1 2 4 3 6 5 8 7

Figure 2- Knowledge augmented PMX

Table 1- Result of OX and My OX

Sample no.	ox, No. of iteration	Shortest path	Myox, No of iteration	Shortest Path
1	1	86	1	87
2	1	348	3	339
3	1	1727	1	2225
4	1	605	2	610
5	2	2432	2	2388

B. ABBREVIATIONS AND ACRONYMS

TSP- Travelling Salesman Problem

GA- Genetic Algorithm

OX- Order Crossover

AP- Assignment Problem

OR- Operation Reserach

PMX- Partially mapped Crossover

C. EQUATIONS

$$F(x) = g(F(x)) \quad (1)$$

Where *f* is an objective function , *g* transforms the value of the objective function to a non-negative number and *F* is relative fitness.

The most fit individuals and the fitness of the others is determined by the following rules:

- $INC = 2.0 \times (MAX - 1.0) / n$
- $LOW = INC / 2.0 \quad (2)$
- $MIN = 2.0 - MAX$

The fitness of individuals in the population may be calculated directly as,

$$f(xi) = 2 - MAX + 2 (MAX - 1) xi - 1 \quad n- 1 \quad (3)$$

Probability of each chromosomes selection is given by: N

$$Ps(i) = f(i) / f(j) \quad J = 1 \quad (3)$$

Ps(i) and *f(i)* are the probability of selection and fitness value

VII. CONCLUSION

Having seen the results of experiments which compares the proposed method with the conventional approach which also suggests that the number of children generated by the traditional crossover operators is limited because of calculation costs factor involved. So, with the help of traditional methods generation of better individuals with limited number of children take place. So a proposal for a new crossover operator take place. In this method first, the children generated by the first parents are evaluated for their fitness. Then, some number of top

children with quite a good fitness rate set beforehand are selected for the position of the next parents. TSP is optimization problem which is used to find minimum path for salesperson. The Actual use of tsp is routing in network. The finding of minimum path will help to reduce the overall time of the Salesman and thus help to improve the overall performance. The work proposed here intends to test the performance of different Crossover used in GA and compare the performance for each of them and compare to others. This paper presents an investigation on different crossover techniques used in GA. Since there are various other methods also which are traditionally used to obtain the optimum distance for TSP. This work aims at establishing the superiority of Genetic Algorithms in achieving optimizing solutions for TSP. One of the objectives of this research work is to find a way to come to the solution fast. But still as can be found from the experimental results the conclusion can be drawn that different methods might outperform the others in different situations.

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