

# Optimal Design RRC Pulse Shape Polyphase FIR Decimation Filter for Multi-Standard Wireless Transceivers

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**ABSTRACT:** ----- Pulse Shaping Filters or Root Raised Cosine (RRC) low pass filters emerges as one of the hottest topic in field of Wireless Communication technology. Low pass filters are used for decimation and for interpolation. When decimating, low pass filters are used to reduce the bandwidth of a signal prior to reducing the sampling rate. This is done to minimize aliasing due to the reduction in the sampling rate. When interpolating, low pass filters are used to remove spectral images from the low-rate signal.

In this paper, Area Efficient and Cost Effective Techniques for design of Pulse Shaping Filters have been presented to improve the computational and implementation complexity. Pulse Shaping Filters have been designed and implemented by using Square Raise Cosine Filter and Optimized Multistage polyphase FIR interpolation filter, and polyphase FIR decimation filter.

The results shown that the computational cost of a polyphase Decimation filter is less with comparing the computational cost of a conventional FIR filter, which can be used both at the receiver and the transmitter.

**Keywords:** Bandwidth (BW), Inter Symbol Interference (ISI), Decimation filtering, hardware implementation, Finite impulse response filter (FIR)

## I. INTRODUCTION

Sampling rate reduction is required for efficient transmission, and a sampling rate increase is required for the regeneration of the speech Ronald et al. (1975), The processes of sampling rate reduction (often called decimation) and sampling rate increase (or interpolation).

Sampling rate increase and sampling rate reduction are basically interpolation processes and can be efficiently implemented using finite impulse response (FIR) digital filters Bellanger et al. (1974) found that efficient implementations of low-pass FIR filters could be obtained by a process of first reducing the sampling rate, filtering, and then increasing the sampling rate back to the original frequency.

Interpolation is the process of increasing the sampling frequency of a signal to a higher sampling frequency that differs from the original frequency by an integer value. Interpolation also is known as up-sampling shown in Fig.1.

*Upsampler:*—a basic multirate component that appends  $L-1$  zeros at the end of every sample.

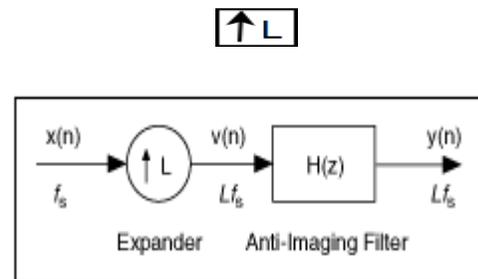


Fig.1 Up-Sampling.

The interpolation filter contains an  $L$ -fold expander followed by a low pass FIR filter  $H(z)$ . The  $L$ -fold expander inserts  $L-1$  zeroes between consecutive samples to the original signal  $x(n)$  and changes the sampling frequency  $f_s$  of the original signal  $x(n)$  to a new sampling frequency  $Lf_s$ . This process introduces images, as shown in the figure below, to the original signal. The interpolation filter then uses the low pass FIR filter  $H(z)$  to remove the images. Therefore, this low pass FIR filter is an anti-imaging filter. The interpolation filter then returns an output signal  $y(n)$  with the new sampling frequency.

The Fig. 2 shows the spectrum of the original signal  $x(n)$  and the spectra from directly interpolating the signal by 2, 3, and  $L$  without using an anti-imaging filter. Notice multiple images emerge in the range from 0 to half of the resulting sampling frequency in parts (b), (c), and (d) of the figure. These images demonstrate the effect of interpolation.

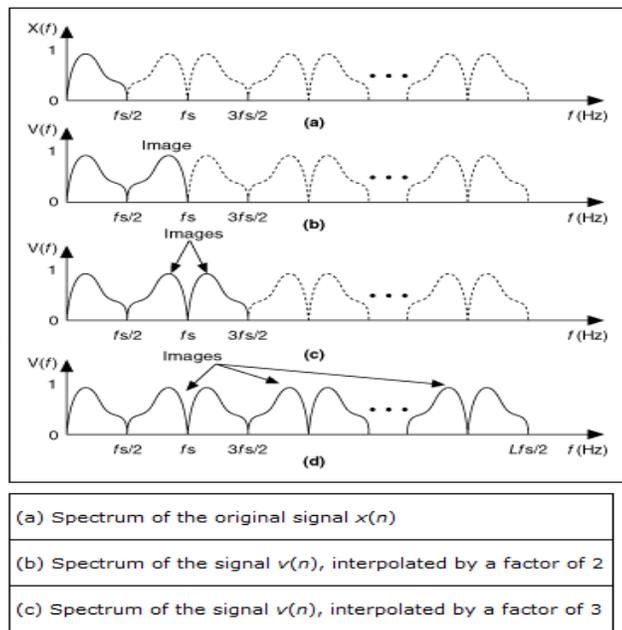


Fig. 2 Spectrum of Up-sampling

The interpolation system uses the lowpass filter  $H(z)$  after the expander to attenuate the frequency components of the signal from  $f_s/2$  to  $Lf_s/2$ . In the time domain, the effect of  $H(z)$  is to replace the inserted zero value samples that the expander introduces with the interpolated values. When replacing the inserted zeroes with interpolated values, the anti-imaging lowpass filter  $H(z)$  might alter the original values.

**Downsampler** – a basic multirate component that picks out every  $M^{\text{th}}$  sample in a data stream and discards the rest of the data.

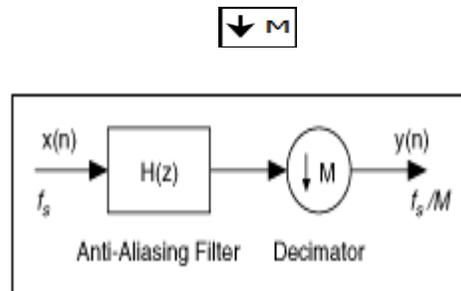


Fig. 3 Down-Sampling.

The fig. 3 shows a typical  $M$ -fold decimation filter, where  $M$  is the integer value by which you want to decrease the sampling frequency. This filter contains a lowpass FIR filter  $H(z)$ . This lowpass FIR filter is an anti-aliasing filter followed by an  $M$ -fold decimator. The decimator passes every  $M^{\text{th}}$  sample and discards the other samples. After this operation, the decimation filter changes the sampling frequency  $f_s$  of the input signal  $x(n)$  to a new sampling frequency  $f_s/M$ . The decimation filter then returns an output signal  $y(n)$  with the new sampling frequency.

To prevent aliasing, this system uses the lowpass filter  $H(z)$  before the  $M$ -fold decimator to suppress the frequency contents above the frequency  $f_s/(2M)$ , which is the Nyquist frequency of the output signal. This system produces the same results as an analog anti-aliasing filter with a cutoff frequency of  $f_s/(2M)$  followed by an analog-to-digital (A/D) converter with a sampling frequency of  $f_s/M$ . Because the system shown in the figure above is in the digital domain,  $H(z)$  is a digital anti-aliasing filter.

Decimation filters that use a polyphase implementation compute only the final expected output samples, not the samples to discard, thus reducing the computational complexity of the filters.

The fig. 4 illustrates the potentially harmful effects of not using an anti-aliasing filter before the decimator. This figure shows the spectrum of the original signal  $x(n)$  and the spectra of the signals resulting from decimating the original signal by 2, 3, and  $M$ . Notice the overlapping spectra in parts (c) and (d) of the figure. The overlapping spectra indicate aliasing due to the decimation operation.

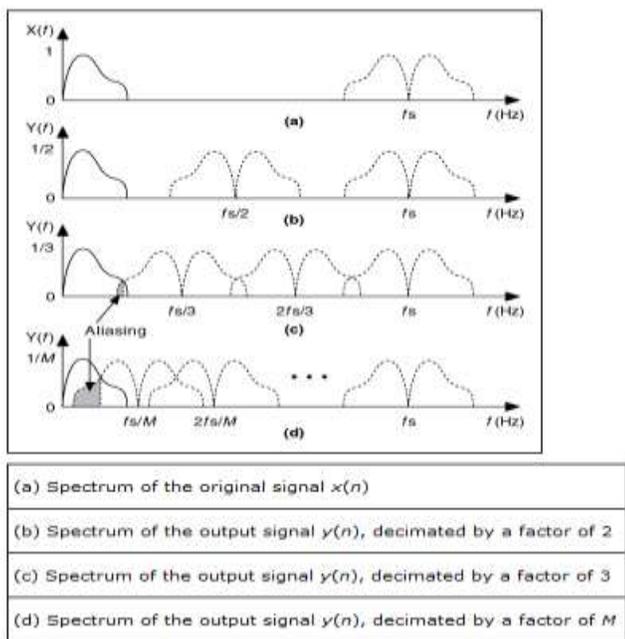


Fig. 4 Spectrum of Down-sampling

**Multirate Filter** – A signal processing system that filters the data and has an output data rate that is different than the input data rate. The ratio of the output data rate to the input data rate is known as the multirate factor.

In decimation and interpolation multirate filters, the normalized transition bandwidth inversely relates to the decimation factor  $M$  and the interpolation factor  $L$ . The order of a decimation or interpolation filter increases as  $M$  or  $L$  increases, and the resulting multirate filter uses more resources to implement. You can use multistage multirate filters to simplify multirate filters that have large sampling frequency conversion factors.

Nyquist filters have the following magnitude response specifications:

TABLE I  
NYQUIST FILTERS MAGNITUDE RESPONSE

Filter Specification	Value Range
Passband edge frequency	$[0, f_s/2M - \epsilon]$
Stopband edge frequency	$[f_s/2M + \epsilon, f_s/2]$

In this Table 1,  $M$  denotes the sampling frequency conversion factor and  $f_s$  denotes the sampling frequency of a Nyquist filter. For an interpolation Nyquist filter,  $f_s$  equals  $L$

times the sampling frequency of the input signal, where  $L$  denotes the interpolation factor. For a decimation Nyquist filter,  $f_s$  equals the sampling frequency of the input signal. You can specify  $\epsilon$  indirectly by using the roll off, which is defined as the following equation:

$$\alpha = \frac{\epsilon}{f_s/M}$$

The following Fig. 5 illustrates the magnitude response of a Nyquist filter.

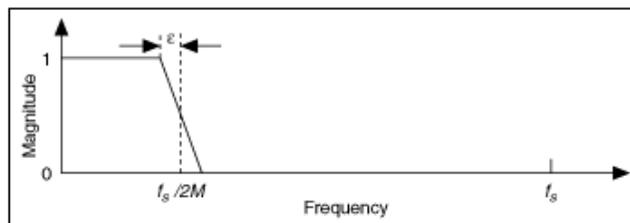


Fig. 5 Nyquist filters magnitude response

## II. STATEMENT OF PROBLEM WITH EQUATIONS

Typically low pass filters are used for decimation and for interpolation. When decimating, low pass filters are used to reduce the bandwidth of a signal prior to reducing the sampling rate. This is done to minimize aliasing due to the reduction in the sampling rate.

When decimating, the bandwidth of a signal is reduced to an appropriate value so that minimal aliasing occurs when reducing the sampling rate. Down sampler is basic sampling rate alteration device used to decrease the sampling rate by an integer factor. An down-sampler with a down-sampling factor  $M$ , where  $M$  is a positive integer, develops an output sequence  $y[n]$  with a sampling rate that is  $(1/M)^{th}$  of that of the input sequence  $x[n]$ . The down sampler is shown in Fig 6.

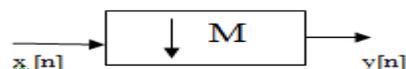


Fig. 6 Down Sample

Down-sampling operation is implemented by keeping every  $M$  sample of  $x[n]$  and removing  $(L-1)$  in between samples to generate  $y[n]$ . The input and output relation of down sampler can be expressed as:

$$y[n] = x(nM) \tag{1}$$

Applying the  $z$ -transform to the input-output relation of a factor-of- $M$  down-sampler, we get

$$Y(z) = \sum_{k=-\infty}^{\infty} x[Mn]z^{-n} \tag{2}$$

The expression has shown the right-hand side of Eq. (2) Cannot be directly expressed in terms of X(z). To get around this problem, a new sequence  $x_{int} [n]$  can be expressed as:

$$x_{int} [n] = \begin{cases} x[n] & n = 0, \pm M, \pm 2M \\ 0 & otherwise \end{cases} \tag{3}$$

$$\begin{aligned} Y(z) &= \sum_{n=-\infty}^{\infty} x[Mn]z^{-n} = \sum_{n=-\infty}^{\infty} x_{int} [Mn]z^{-n} \\ &= \sum_{n=-\infty}^{\infty} x_{int} [k]z^{-k/M} = x_{int} [Mn]z^{-k/M} \end{aligned} \tag{4}$$

Now,  $x_{int} [n]$  can be formally related to  $x[n]$  as follows:

$$x_{int} [n] = c[n].x[n] \tag{5}$$

Where

$$c[n] = \begin{cases} 1, & n = 0, \pm D, \pm 2D \\ 0, & otherwise \end{cases} \tag{6}$$

A convenient representation of  $c[n]$  is given by

$$c[n] = \frac{1}{M} \sum_{k=0}^{M-1} W_M^{kn} \tag{7}$$

Where

$$x_{int} (z) = \frac{1}{M} \sum_{n=-\infty}^{\infty} \left( \sum_{k=0}^{M-1} W_M^{kn} \right) x[n]Z^{-n} \tag{8}$$

Where

$$W_D = e^{-j\frac{2\pi}{M}}$$

Taking the z-transform of Eq. (5) and by making use of Eq. (7), we get

$$x_{int} (z) = \frac{1}{M} \sum_{k=0}^{M-1} \left( \sum_{n=-\infty}^{\infty} x[n] Z^{-n} W_M^{kn} \right)$$

Where

$$x_{int} (z) = \frac{1}{M} \sum_{k=0}^{M-1} X(Z W_M^{-k}) \tag{9}$$

The spectrum of a factor-of-2 down-sampler with an input  $x[n]$  is shown in Fig. 7.

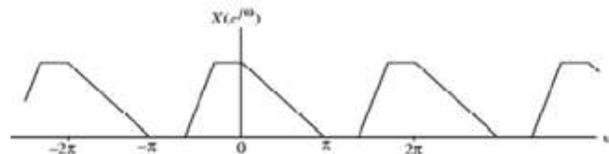


Fig. 7 Spectrum of down sampler

The DTFTs of the output and the input sequences of this down-sampler are then related as

$$Y(e^{j\omega}) = \frac{1}{2} \{ X(e^{j\omega}) + X(-e^{j\omega}) \} \tag{10}$$

The second term in above equation is simply obtained by shifting the first term  $X(e^{j\omega/2})$  to the right by an amount  $2\pi$  as shown in Fig 3. The two terms have an overlap due to which original “shape” of  $X(e^{j\omega/2})$  is lost when  $x[n]$  is down-sampled. This overlap causes the *aliasing* that takes place due to under-sampling. There is no overlap, i.e., no aliasing, only if

$$X(e^{j\omega}) = 0 \text{ for } |\omega| \geq \frac{\pi}{2} \tag{11}$$

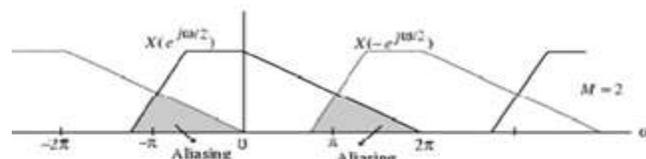


Fig. 8 Aliasing Effect

### III. SOLUTION OF PROBLEM

In general, Aliasing is absent if and only if

$$X(e^{j\omega}) = 0 \text{ for } |\omega| \geq \frac{\pi}{M} \tag{12}$$

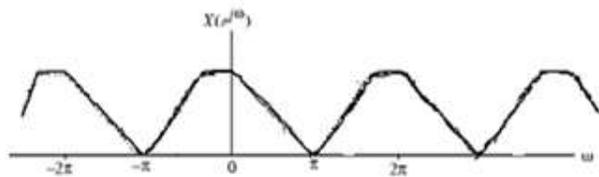


Fig. 9 Aliasing remove by Nyquist design filter

To overcome the effect of aliasing decimation filters are used. The specifications for the low pass decimation filter are given by:

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\omega_c}{D} \\ 0, & \frac{\pi}{M} \leq |\omega| \leq \pi \end{cases} \quad (13)$$

#### IV. RESULTS AND DISCUSSION

A raised cosine filter is typically used to shape and oversample a symbol stream before modulation/transmission. The rolloff factor, determines the width of the transition band. Practical digital communication systems use a roll off factor between 0.1 and 0.5.

The ideal pulse shaping filter has two properties:

- A high stop band attenuation to reduce the inter channel interference as much as possible.
- Minimized intersymbol interferences (ISI) to achieve a bit error rate as low as possible. The first Nyquist criterion states that in order to achieve a ISI-free transmission, the impulse response of the shaping filter should have zero crossings at multiples of the symbol period.

The practical Pulse shaping raised cosine filters are windowed.

- The window length can be controlled in three ways: filter order, filter order in symbol durations, and
- Minimum order to achieve a given stop band attenuation.

Raised cosine filters are used for pulse shaping, where the signal is upsampled. Therefore, we also need to specify the upsampling factor.

In this paper, the given proposed design rolloff factor is 0.5 and upsampling factor is 8 with filter coefficient 4 shown in Fig. 10. The plot compares the digital data and the upsampled, filtered signal.

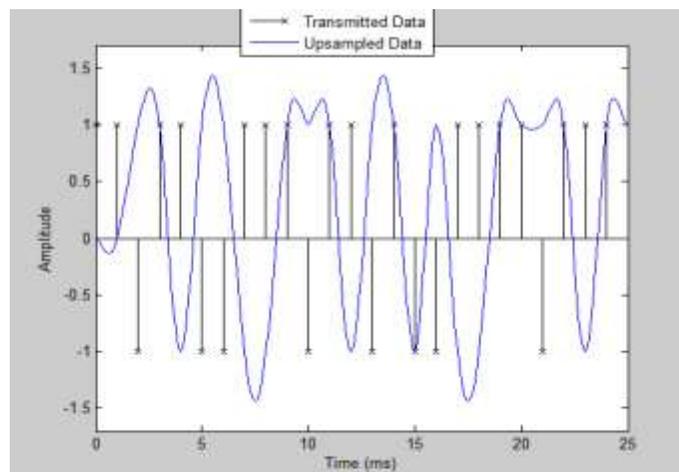


Fig. 10 Pulse shape Transmitted data with upsampled date

Such a filter also has a group delay of  $(16/8) = 2$  two symbol durations shown in Fig.11

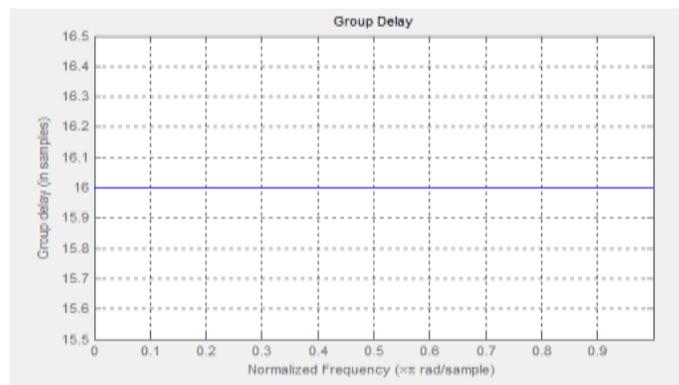


Fig.11 Group Delay

Root raised cosine (RRC) filter introduces a group delay that causes ISI and needs to be compensated by the filter. This group delay can be compensated by delaying the input signal as shown in Fig. 12.

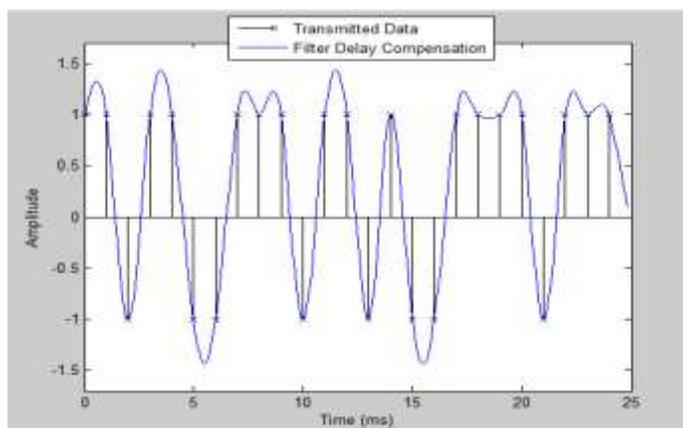


Fig. 12 Removing filter propagation Delay

The 3G standard proposes  $1/T = 3.84 \times 10^6 \text{ s}^{-1}$  and roll off factor = 0.22. In this case, the bandwidth occupied by the modulated raised cosine with these parameters is  $1.22 \times 3.84 = 4.68 \text{ MHz}$  which is less than 5MHz separation between adjacent channels of W-CDMA. Therefore the value of roll off factor used in designing the raised cosine pulse shaping filter has been reduced for 0.5 to 0.22 which results in narrow transition width to further reject the effect of ISI in filtered output as shown in Fig. 13.

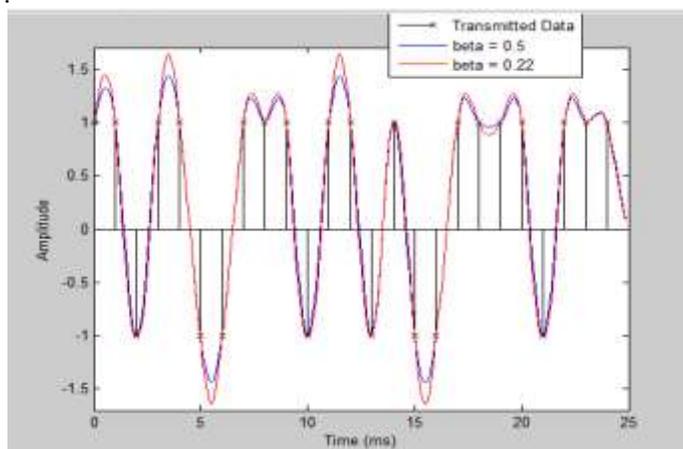


Fig.13 Filtered output with reduced roll off

The use of raised cosine filtering is to split the filtering between transmitter and receiver. Both transmitter and receiver employ square-root raised cosine filters. The combination of transmitter and receiver filters is a raised cosine filter, which results in minimum ISI shown in Fig.14.

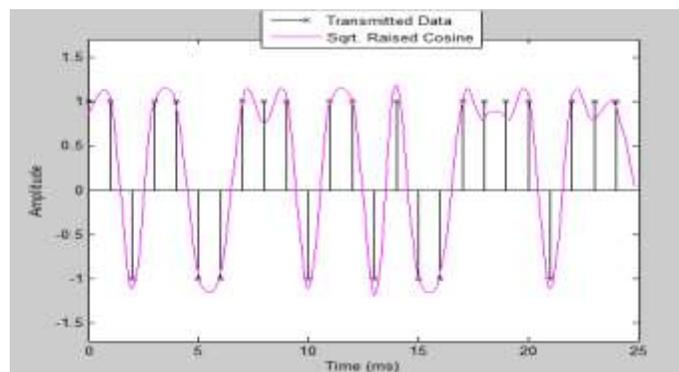


Fig. 14 Square root raised cosine filter design

In theory, the cascade of two square root raised cosine (SqRRC) filters is equivalent to a single normal raised cosine filter (RRC).

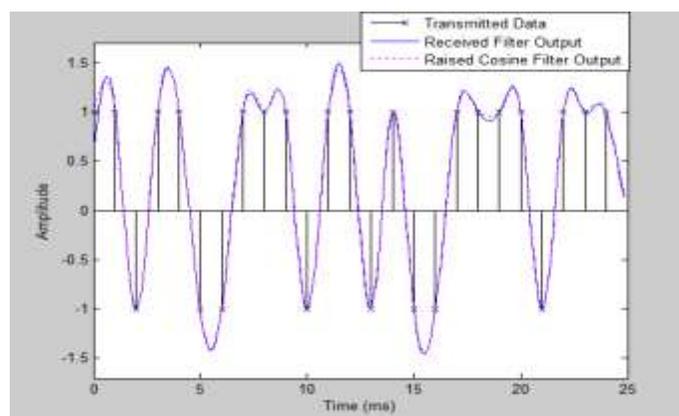


Fig. 15 Conventional FIR transmitted and received Data signal

Raised cosine and square root raised cosine filters are widely used in data transmission systems. Despite their popularity, but they are not optimal in any sense.

But polyphase filters require a very small number of multipliers to implement, they are inherently stable, have low round off noise sensitivity and no limit cycles but other results are exactly the same as the conventional FIR filter shown in Fig.15.

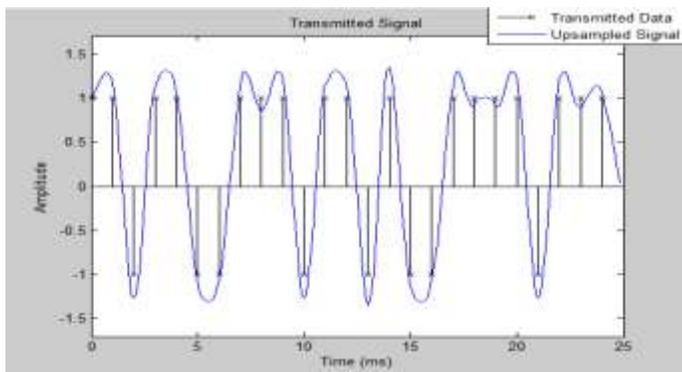


Fig. 16 Design polyphase interpolation filters

Multirate Interpolator (Up-Sampler) are use for Transmit data  
Multirate Decimator (Down-sampler) are use for received data shown in Fig. 16.

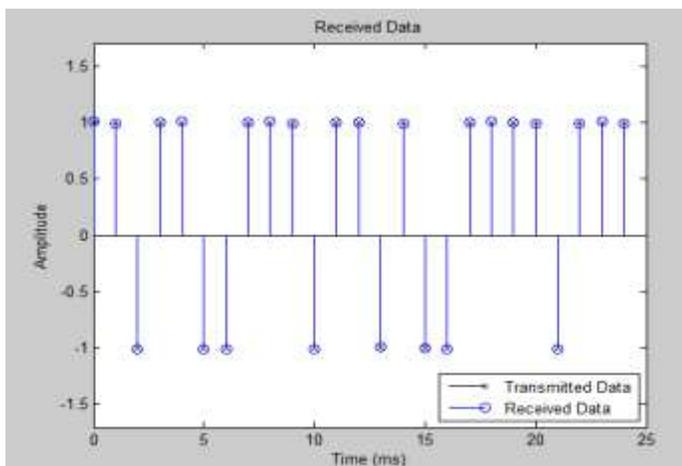


Fig. 17 Transmitted and Received Data

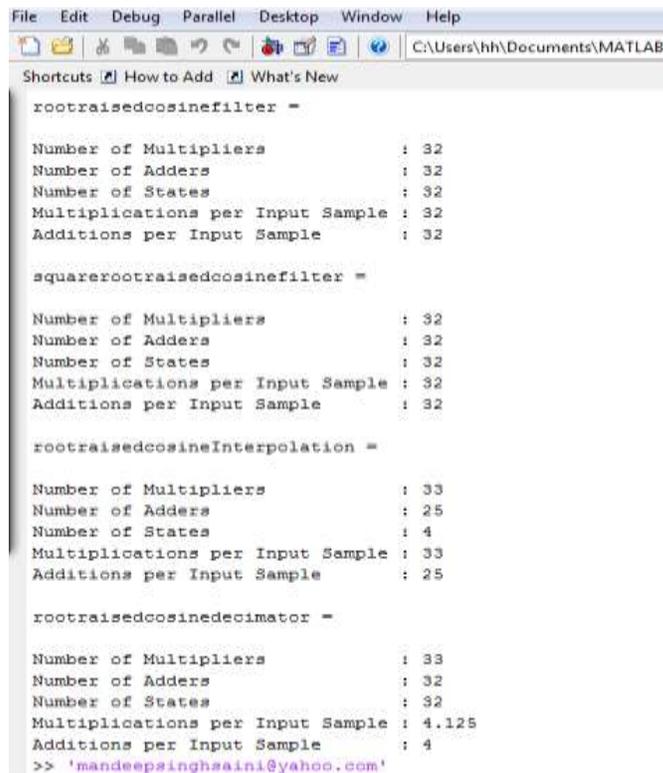


Fig. 18 Cost comparisons of all design

TABLE 2

IMPLEMENTATION COST COMPARISON

Implementation cost comparison					
Technique	Number of Multipliers MUL	Number of Adders ADD	Number of States STATE	Multiplications per Input Sample MULT/SAMPLE	Additions per Input Sample ADD/SAMPLER
Conventional RRC FIR	32	32	32	32	32
Conventional Sqr. RRC FIR	32	32	32	32	32
Multirate Polyphase Interpolator FIR	33	25	04	33	25
Multirate Polyphase Decimator FIR	33	32	32	4.125	4

This Multirate Polyphase method achieves computational costs lower than that of Conventional root raised cosine filter and square root cosine filter since Multirate Polyphase Decimator requires only 4.125 MPIS on average compared to

32 MPIS for the Direct-Form FIR. All design comparing chart are shown in Table 2.

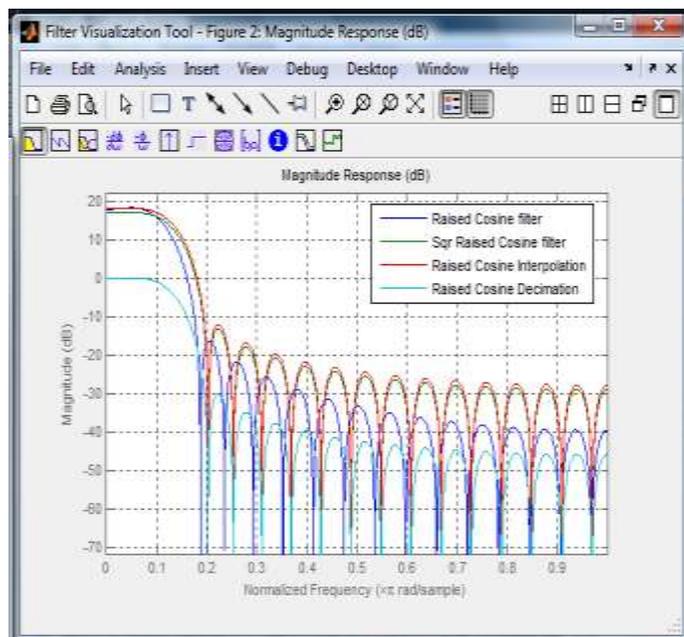


Fig. 19 Comparison pulse shape Decimation design with other design

## V. CONCLUSIONS

Even though the filtering results are exactly the same, the computational cost of a polyphase filter is less. All results shown Table 2, we compare the computational cost of a conventional FIR filter, which can be used both at the receiver and the transmitter, polyphase FIR interpolation filter, and polyphase FIR decimation filter cost is less as compare to other design and pulse shape of filter is best as compare to other design shown in Fig. 19.

So Multistage Polyphase FIR pulse shaping interpolator at the transmitter is associated with a simple downsampler at the receiver.

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