

Survey of Image Denoising Methods using Dual-Tree Complex DWT and Double-Density Complex DWT

Mr. R. K. Sarawale¹, Dr. Mrs. S.R. Chougule²

Abstract— Image denoising is a method of removal of noise while preserving as much as possible important information. Basically there are two classes of the image denoising, namely spatial filtering methods and transform filtering methods. A wavelet transform domain method gives a superior performance in image denoising applications. But due to attractive merits of the extensions of the DWT, the research was shifted towards the extensions of the DWT. The first three sections of this paper explain about the basic introduction to the subject, about Wavelet Transform and its types, and about present theories of the topic. Remaining sections explain about DWT extensions and methodologies of the DWT extensions. The wavelet based denoising algorithms use DWT (Discrete Wavelet Transform) in the decomposition stage. But they suffer from lack of directionality and shift variance. To overcome this, CWT (Complex Wavelet Transform) can be used in two modes namely, Dual-Tree Complex DWT and Double-Density Complex DWT. In this paper, these two modes are compared.

Index Terms— DWT, Dual-Tree Complex DWT, Double-Density Complex DWT, Image denoising.

I. INTRODUCTION

Generally in images, noise suppression is a particularly delicate task. In this task, the main focusing parts are noise reduction and the preservation of actual image features. Real world signals usually contain departures from the ideal signal. Such departures are referred to as noise. An image is always affected by noise in its capture, acquisition and processing. Generally, noise reduction is an important part of image processing systems.

A good image denoising model is one which will remove noise while preserving edges and contours. The wavelet transform is a simple and elegant tool that can be used for many digital signal and image processing applications. The wavelet transform provides a multiresolution representation using a set of analyzing functions that are dilations and translations of a few functions (wavelets). It overcomes some of the limitations of the Fourier transform with its ability to represent a function simultaneously in the frequency and time domains using a single prototype function (or wavelet) and its scales and shifts [21].

The wavelet transform comes in several forms. The critically-sampled form of the wavelet transforms provides the most compact representation; however, it has several limitations.

It lacks the shift-invariance property, and in multiple dimensions it does a poor job of distinguishing orientations, which is important in image processing. For these reasons, it turns out that for some applications, improvements can be obtained by using an expansive wavelet transform in place of a critically-sampled one [32].

II. WAVELET TRANSFORM

The Fourier Transform (FT) is only able to retrieve the global frequency content of a signal, the time information is lost.

This is overcome by the short time Fourier transform (STFT) which calculates the Fourier transform (FT) of a windowed part of the signal and shifts the window over the signal. The STFT gives the time-frequency content of a signal with a constant frequency and time resolution due to the fixed window length. This is often not the most desired resolution. For low frequencies often a good frequency resolution is required over a good time resolution. For high frequencies, the time resolution is more important.

A multi-resolution analysis becomes possible by using wavelet analysis. The types of wavelet transform are given below.

A. Continuous Wavelet Transform

It is calculated analogous to the Fourier transform (FT), by the convolution between the signal and analysis function. However the trigonometric analysis functions are replaced by a wavelet function. The continuous wavelet transform retrieves the time-frequency content information with an improved resolution compared to the STFT.

B. Discrete Wavelet Transform

It uses filter banks to perform the wavelet analysis. The DWT decomposes the signal into wavelet coefficients from which the original signal can be reconstructed again. The wavelet coefficients represent the signal in various frequency bands. The coefficients can be processed in several ways, giving the DWT attractive properties over linear filtering.

The dilation and translation factors are elements of the real line. For a particular dilation a and translation b , the wavelet coefficient $W_f(a, b)$ for a signal f can be calculated as,

$$W_f(a, b) = \langle f, \psi_{a,b} \rangle = \int f(x) \psi_{a,b}(x) dx \quad (1)$$

where, $f(x)$ is the original signal and $\psi_{a,b}(x)$ is the wavelet function.

Wavelet coefficients represent the information contained in a signal at the corresponding dilation and translation. The original signal can be reconstructed by applying the inverse transform:

$$f(x) = \frac{1}{C_w} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(a, b) \psi_{a,b}(x) db \frac{da}{a^2} \quad (2)$$

where, C_w is the normalization factor of the mother wavelet and $\psi_{a,b}(x)$ is the wavelet function [24].

The term discrete wavelet transform (DWT) is a general term, encompassing several different methods. It must be noted that the signal itself is continuous; discrete refers to discrete sets of dilation and translation factors and discrete sampling of the signal. For simplicity, it will be assumed that the dilation and translation factors are chosen so as to have dyadic sampling, but the concepts can be extended to other choices of factors. At a given scale J , a finite number of translations are used in applying multi resolution analysis to obtain a finite number of scaling and wavelet coefficients [6]. The signal can be represented in terms of these coefficients as,

$$f(x) = \sum_k C_{jk} \phi_{jk}(x) + \sum_{j=1}^J \sum_k d_{jk} \psi_{jk}(x) \quad (3)$$

where C_{jk} are the scaling coefficients, d_{jk} are the wavelet coefficients, $\phi_{jk}(x)$ is the scaling function and $\psi_{a,b}(x)$ is the wavelet function.

The first term in Equation (3) gives the low-resolution approximation of the signal while the second term gives the detailed information at resolutions from the original down to the current resolution J [24].

C. Complex Wavelet transform

Orthogonal wavelet decompositions, based on separable, multirate filtering systems have been widely used in image and signal processing, largely for data compression. Kingsbury introduced a very elegant computational structure, the dual-tree complex wavelet transform [17], which displays near-shift invariant properties. Other constructions can be found such as in [26]-[28]. As pointed out by Kingsbury [17], one of the problems of Mallat-type algorithms is the lack of shift invariance in such decompositions. A manifestation of this is that coefficient power may dramatically redistribute itself throughout subbands when the input signal is shifted in time or in space.

Complex wavelets have not been used widely in image processing due to the difficulty in designing complex filters which satisfy a perfect reconstruction property. To overcome this, Kingsbury proposed a dual-tree implementation of the CWT (DT CWT) [19], which uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately.

The DWT suffers from the following two problems.

- 1] Lack of shift invariance - this results from the down sampling operation at each level. When the input signal is shifted slightly, the amplitude of the wavelet coefficients varies so much.
- 2] Lack of directional selectivity - as the DWT filters are real and separable the DWT cannot distinguish between the opposing diagonal directions.

These problems hinder the use of wavelets in other areas of image processing. The first problem can be avoided if the filter outputs from each level are not down sampled but this increases the computational costs significantly and the resulting undecimated wavelet transform still cannot distinguish between opposing diagonals since the transform is still separable.

To distinguish opposing diagonals with separable filters the filter frequency responses are required to be asymmetric for positive and negative frequencies. A good way to achieve this is to use complex wavelet filters which can be made to suppress negative frequency components. The Complex DWT has improved shift-invariance and directional selectivity than the separable DWT.

D. The DWT extensions

Though standard DWT is a powerful tool for analysis and processing of many real-world signals and images, it suffers from three major disadvantages,

- (1) Lack of phase information
- (2) Poor directionality
- (3) Shift- sensitivity

Hence to overcome above drawbacks, it becomes necessary to go with the extensions of DWT namely Dual-Tree and Double-Density DWTs.

III. PRESENT THEORIES

There has been a lot of research work dedicated towards image denoising. Fourier Transform (FT) with its fast algorithms (FFT) is an important tool for analysis and processing of many natural signals. FT has certain limitations to characterize many natural signals, which are non-stationary (e.g. speech). Though a time varying, overlapping window based FT namely STFT (Short Time FT) is well known for speech processing applications, a new time-scale based Wavelet Transform (WT) is a powerful mathematical tool for non-stationary signals [34].

With standard Discrete Wavelet Transforms (DWTs), signal has a same data size in transform domain and therefore it is a non-redundant transform. Standard DWT can be implemented through a simple filterbank structure of recursive FIR filters. A very important property; Multiresolution Analysis (MRA) allows DWT to view and process different signals at various resolution levels.

A wavelet is a small wave which has its energy concentrated in time. It has an oscillating wave like characteristic but also has the ability to allow simultaneous time and frequency analysis and it is a suitable tool for transient, non-stationary or time-varying phenomena [33].

The Discrete Wavelet Transforms (DWT), obtained by iterating a perfect reconstruction (PR) filter bank (FB) on its low-pass output, decomposes a discrete-time signal according to octave-band frequency decomposition [1]. The new version of DWT, known as double-density DWT has the following important additional properties.

- (1) It employs one scaling function and two distinct wavelets which are designed to be offset from one another by one half.
- (2) The double density DWT is over complete by a factor of two [2].

Many different noise removal techniques have been applied to images, but the wavelet transform has been viewed by many as the preferred technique for noise

removal. Rather than a complete transformation into the frequency domain, as in DCT (Discrete Cosine Transform) or FFT (Fast Fourier Transform), the wavelet transform produces coefficient values which represent both time and frequency information. The hybrid spatial-frequency representation of the wavelet coefficients allows for analysis based on both spatial position and spatial frequency content. The hybrid analysis of the wavelet transform is excellent in facilitating image denoising algorithms [33].

Reference [4] illustrates the classification of the image denoising methods. The image denoising methods can be categorized according to estimation approach of wavelet coefficient and wavelet type. Table I shows classification according to wavelet coefficient. The Thresholding and Shrinkage methods give noticeable results. The related works and approaches are tabulated in below Table I.

Table I Classification according to wavelet coefficient

Category	Approach	Related works
Thresholding	Universal thresholding	[13, 15, 12]
	Stein's Unbiased Risk Estimation (SURE)	[14]
	Cross validation	[25]
	BayesShrink	[7]
Shrinkage	MMSE	[22]
	Bivariate shrinkage using level dependency	[33]
	Neighbour dependency	[5, 8]
	Adaptive Bayesian wavelet shrinkage (ABWS)	[9]
	Markov Random Field	[23]
	Hidden Markov Tree	[10]
Other	Gaussian scale mixture	[27]

The different types of wavelet and related works are tabulated in Table II.

Table II Classification according to wavelet type

Wavelet Type	Related works	
Orthogonal separable wavelet	Most of works	
Translation-invariant wavelet	[11, 3, 8, 22]	
Multiwavelet	[16, 3, 8]	
Complex wavelet	[10, 18, 33]	
Others	Curvelet	[35]
	Steerable pyramid	[27]
	Brushlet	[20]

IV. BASICS TO THE DWT EXTENSIONS

A filter bank plays an important role in image denoising applications. It consists of two banks namely, analysis filter bank and synthesis filter bank. A schematic representation of a filter bank is shown in Fig. 1 below. In Fig. 1, $x[n]$ is the input to the Analysis Filter Bank. The $V_0[n]$ is the output of the Analysis Filter bank. The output of the Synthesis Filter Bank is $y[n]$.

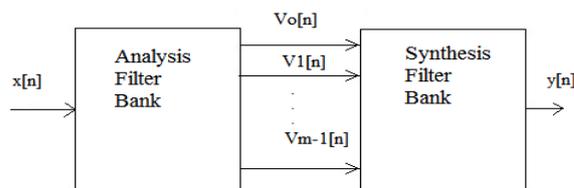


Fig. 1 A Filter Bank

A. One dimensional filter bank

The one dimensional filter bank is constructed with analysis and synthesis filter bank which is shown in Fig. 2 below. The analysis filter bank decomposes the input signal $x(n)$ into two sub band signals, $c(n)$ and $d(n)$. The signal $c(n)$ represents the low frequency part of $x(n)$, while the signal $d(n)$ represents the high frequency part of $x(n)$.

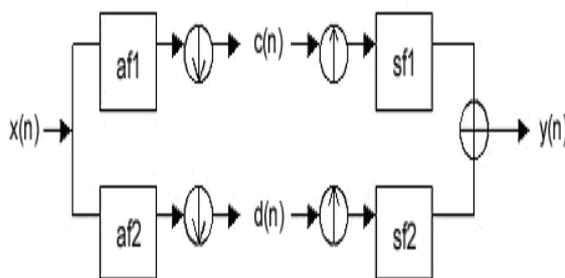


Fig. 2 One dimensional filter bank

It uses filter banks to perform the wavelet analysis. The DWT decomposes the signal into wavelet coefficients from which the original signal can be reconstructed again. The wavelet coefficients represent the signal in various frequency bands. The coefficients can be processed in several ways, giving the DWT attractive properties over linear filtering.

B. Basic differences between the two DWT extensions

The basic differences between the dual tree DWT and double density DWT are given below.

- 1] For the dual-tree DWT there are fewer degrees of freedom for design, while for the double-density DWT there are more degrees of freedom for design.
- 2] The dual-tree and double-density DWTs are implemented with totally different filter bank structures.
- 3] The dual-tree DWT can be interpreted as a complex-valued wavelet transform which is useful for signal modeling and denoising (the double-density DWT cannot be interpreted as such).
- 4] The dual-tree DWT can be used to implement two-dimensional transforms with directional wavelets, which is highly desirable for image processing [31].

By introducing this concept in discrete wavelet transform (DWT) we can achieve dual-tree DWT system. Also combining the double-density DWT and dual-tree DWT we can achieve the double-density dual-tree DWT system.

V. METHODOLOGIES OF THE DWT EXTENSIONS

Complex wavelet transforms (CWT) use complex-valued filtering (analytic filter) that decomposes the real/complex signals into real and imaginary parts in transform domain. The real and imaginary coefficients are used to compute amplitude and phase information.

A. Dual-Tree Complex DWT

Kingsbury found that the dual-tree DWT is nearly shift-invariant when the lowpass filters of one DWT interpolate midway between the lowpass filters of the second DWT [29].

Kingsbury's complex dual-tree DWT is based on (approximate) Hilbert pairs of wavelets [30].

The complex dual-tree can be implemented as critically-sampled separable two dimensional Discrete Wavelet Transforms (DWTs) operating in parallel. Fig. 3 shows that implementation of Dual Tree Discrete Wavelet Transform.

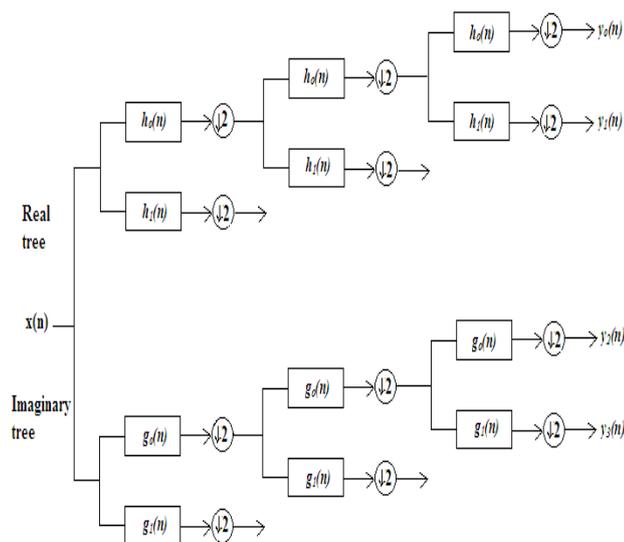


Fig. 3 Implementation of Dual-Tree Discrete Wavelet Transform

The filter banks $h_i(n)$ and $g_i(n)$ (where $i = 0, 1, 2$) are designed in a specific way so the subband signals of the

upper DWT can be interpreted as the real part of a complex wavelet transform, and the subband signals of the lower DWT can be interpreted as the imaginary part.

The implementation steps of dual-tree complex DWT are given below.

- 1] Take the natural image as an input and denote it as $x(n)$.
- 2] Add noise $r(n)$ into the original image $x(n)$. Denote noisy image as $c(n)$ and it is given as $c(n) = x(n) + r(n)$.
- 3] Compute forward dual-tree complex DWT on noisy image $c(n)$. Use analysis filter bank.
- 4] Compute Bivariate Shrinkage Function.
- 5] Compute inverse dual-tree complex DWT. Use synthesis filter bank.
- 6] Reconstruct the original image.

B. Double-Density Complex DWT

In two dimensions, this transform outperforms the standard DWT in terms of enhancement; however, there is need of improvement because not all of the wavelets are directional. That is, although the double-density DWT utilizes more wavelets, some lack a dominant spatial orientation, which prevents them from being able to isolate those directions [21].

A solution to this problem is provided by the double-density complex DWT, which combines the characteristics of the double-density DWT and the dual-tree DWT. The double-density complex DWT is based on two scaling functions and four distinct wavelets, each of which is specifically designed such that the two wavelets of the first pair are offset from one other by one half, and the other pair of wavelets form an approximate Hilbert transform pair. By ensuring these two properties, the double-density complex DWT possesses improved directional selectivity and can be used to implement complex and directional wavelet transforms in multiple dimensions. We construct the filter bank structures for both the double-density DWT and the double-density complex DWT using finite impulse response (FIR) perfect reconstruction filter banks. These filter banks are then applied recursively to the low pass subband, using the analysis filters for the forward transform and the synthesis filters for the inverse transform. By doing this, it is then possible to evaluate each transform's performance in several applications including signal and image enhancement [21].

This system can be implemented by applying two dimensional double-density DWTs in parallel to the same input with distinct filter sets for the rows and columns. Fig. 4 shows that implementation of Double-Density Complex DWT.

There are two separate filter banks denoted by $h_i(n)$ and $g_i(n)$ where $i = 0, 1, 2$. The filter banks $h_i(n)$ and $g_i(n)$ are designed in such a way that the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and the subband signals of the lower DWT can be interpreted as the imaginary part.

The implementation steps of double-density complex DWT are given below.

- 1] Take the natural image as an input and denote it as $x(n)$.
- 2] Add noise $r(n)$ into the original image $x(n)$. Denote noisy image as $c(n)$ and it is given as $c(n) = x(n) + r(n)$.
- 3] Compute forward double-density complex DWT on noisy image $c(n)$. Use analysis filter bank.
- 4] Compute Bivariate Shrinkage Function.

- 5] Compute inverse double-density complex DWT. Use synthesis filter bank.
- 6] Reconstruct the original image.

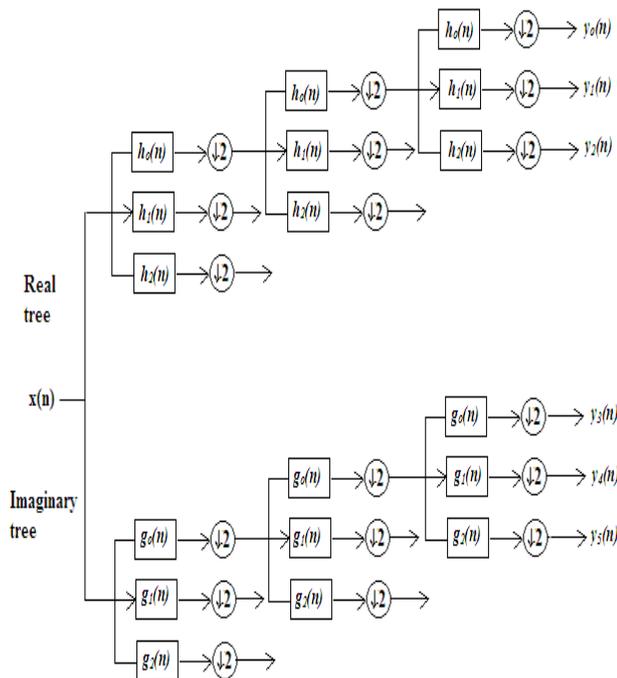


Fig. 4 Basic design of the Double-Density Complex DWT

The performance of Complex Dual Tree DWT and Double Density Complex DWT image denoising methods can be compared by comparing PSNR (Peak-Signal-to-Noise Ratio) value of each system.

VI. CONCLUSION

With the recent inventions and newly developed techniques, the Wavelet Transform plays a vital role in image processing applications. In this paper, the concept highlighted is wavelet based enhancement of gray scale digital images which is corrupted by additive Gaussian noise.

Here, the techniques used are the extensions of the wavelet transform namely, dual-tree complex DWT and double-density complex DWT. These techniques can output noticeable results as compared to the conventional one.

The future work can be extended by applying above mentioned techniques to three dimensional signals.

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Mr. R. K. Sarawale¹ received B.E. and pursuing M.E. in Electronics and Telecommunication in Bharati Vidyapeeth's College of Engg., Kolhapur.



Dr. Mrs. S. R. Chougule² received Ph.D. in Electronics Engg. Currently she is working as a Principal in Bharati Vidyapeeth's College of Engg., Kolhapur.

