

Simulative analysis for Image Denoising using wavelet thresholding techniques

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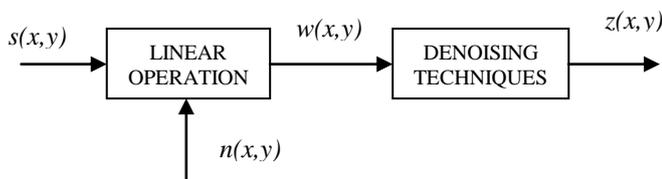
Abstract— This paper deals with the threshold estimation techniques in the wavelet transform domain filtering for image denoising. Wavelet transform based image denoising techniques are of greater interest due to its excellent localization property. The aim of this paper is to present an improved threshold in comparison of universal hard threshold which gives a better result by preserving the original image details. As a numerous papers have been published to restore an image from noisy distortions but selecting an appropriate threshold plays a major role in getting the desired image. Our improved threshold is a forward step towards this approach.

Index Terms— Image Denoising, Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), Universal threshold, Wavelet thresholding.

I. INTRODUCTION

Images are often corrupted with noise during acquisition, transmission, and retrieval from storage media. Many dots can be spotted in a photograph taken with a digital camera under low lighting conditions. A noise is also introduced in the transmission medium due to a noisy channel, errors during the measurement process and during quantization of the data for digital storage. Each element in the imaging chain such as lenses, film, digitizer, etc. contributes to the degradation [1,2,3].

In case of image denoising methods, the characteristics of the degrading system and the noises are assumed to be known beforehand. The image $s(x,y)$ is blurred by a linear operation and noise $n(x,y)$ is added to form the degraded image $w(x,y)$. This is convolved with the restoration procedure $g(x,y)$ to produce the restored image $z(x,y)$.



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Fig.1 Denoising Concept

The “Linear operation” shown in Fig.1 is the addition or multiplication of the noise $n(x,y)$ to the signal $s(x,y)$. Once the corrupted image $w(x,y)$ is obtained, it is subjected to the denoising technique to get the denoise image $z(x,y)$. The point of focus in this paper is on comparing and contrasting “denoising techniques”.

II. WAVELET IMAGE DENOISING TECHNIQUES

A. Wavelet Transform

The Wavelet Transform [4,5] (WT) of image produces a non-redundant image representation, which provides better spatial and spectral localization of image formation. Recently, Wavelet Transform has attracted more and more interest in image de-noising. The WT can be interpreted as image decomposition in a set of independent, spatially oriented frequency channels. The image signal is passed through two complementary filters and emerges as two image signals, approximation and details. This is called decomposition. The components can be assembled back into the original signal without loss of information. This process is called reconstruction. The mathematical manipulation, which implies decomposition and reconstruction, is called wavelet transform and inverse wavelet transform.

All wavelet transform denoising algorithms involve the following three steps in general.

- Forward Wavelet Transform: Wavelet coefficients are obtained by applying the Wavelet transforms.
- Estimation: Clean coefficients are estimated from the noisy ones. In this step all high frequency sub bands are threshold as per thresholding scheme.
- Inverse Wavelet Transform: A denoised Image is obtained by applying the inverse Wavelet transforms.

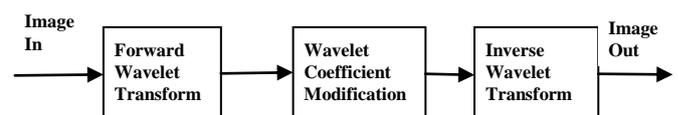


Fig.2 Denoising using Wavelet Transform Filtering

B. Decomposition

An image can be decomposed into a sequence of different spatial resolution image using WT. In case of an image, an N level decomposition can be performed resulting in $3N+1$ different frequency bands (sub bands) namely, LL, LH, HL

and HH as shown in Fig.3

LL ₃	HL ₃	HL ₂	HL ₁
LH ₃	HH ₃		
LH ₂		HH ₂	
LH ₁			HH ₁

Fig.3 Decomposition of Image

The sub-bands HH_k, HL_k, LH_k are called the details coefficients, where $k = 1, 2, \dots, j$; k is the decomposition level and j denotes the largest or coarsest scale in decomposition and LL_k is the approximation coefficient which is low resolution component. The next level of wavelet transform is applied to the low frequency sub band image LL only. The Gaussian noise will nearly be averaged out in low frequency wavelet coefficients. Therefore, only the wavelet coefficients in the high frequency levels need to be threshold. As a final step in the denoising algorithm, the inverse discrete wavelet transform is applied to build back the modified image from its coefficients.

C. THRESHOLDING METHODS

Thresholding methods use a threshold and determine the clean wavelet coefficients based on this threshold.

a. Hard Thresholding Method

If the absolute value of a coefficient is less than a threshold, then it is assumed to be 0, otherwise it is unchanged. Mathematically it is

$$f_h(x) = \begin{cases} x, & \text{if } |x| \geq \lambda \\ 0, & \text{otherwise} \end{cases}$$

The hard-thresholding function chooses all wavelet coefficients that are greater than the given threshold λ and sets the others to zero. The threshold λ is chosen according to the signal energy and the noise variance (σ^2).

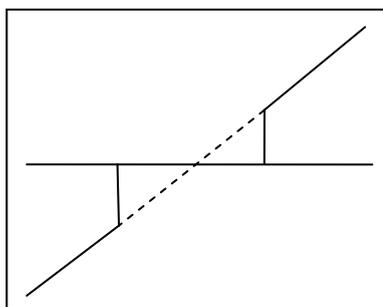


Fig.4(a) Hard Thresholding

b. Soft Thresholding Method

The soft-thresholding function has a somewhat different rule from the hard-thresholding function. It shrinks the wavelet coefficients by λ towards zero,

Hard thresholding is discontinuous. To overcome this, Donoho introduced the Soft Thresholding method. If the absolute value of a coefficient is less than a threshold λ , then is assumed to be 0, otherwise its value is shrunk by λ . Mathematically it is

$$f(x) = \begin{cases} x - \lambda, & \text{if } x \geq \lambda \\ 0, & \text{if } |x| < \lambda \\ x + \lambda, & \text{if } x \leq -\lambda \end{cases}$$

The soft-thresholding rule is chosen over hard-thresholding, for the soft-thresholding method yields more visually pleasant images over hard thresholding.

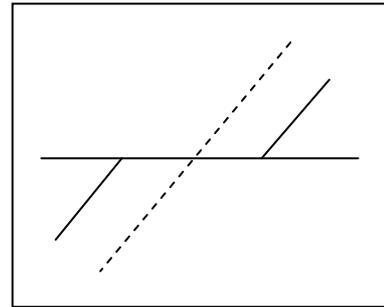


Fig.4(b) Soft Thresholding

c. Universal Threshold

Universal threshold is formulated as,

$$T = \sigma \sqrt{2 \log(M)} \tag{1}$$

where σ^2 is the noise variance and M is the number of pixels. This asymptotically yields a MSE estimate as M tends to infinity. As M increases, we get bigger and bigger threshold, which tends to over smoothen the image.

d. Improved Threshold

In this new threshold, we multiply a term X in the Universal Threshold and it is formulated as,

$$T = \sigma \sqrt{2X \log(M)} \tag{2}$$

where X is the median of absolute value of the image matrix.

e. Bayes Shrink

This method is based on the Bayesian mathematical framework. The wavelet coefficients of a natural image are modeled by a Generalized Gaussian Distribution (GGD). This is used to calculate the threshold using a Bayesian framework. S. Grace Chang suggests an approximation and simple formula for the threshold:

$$T = \frac{\hat{\sigma}_n^2}{\hat{\sigma}_x} \tag{3}$$

If $\hat{\sigma}_x$ is non-zero, otherwise it is set to some predetermined maximum value.

$$\hat{\sigma}_x = \sqrt{\max(\hat{\sigma}_y^2 - \hat{\sigma}^2, 0)} \tag{4}$$

$\hat{\sigma}_y$ is calculated as,

$$\hat{\sigma}_y^2 = \frac{1}{n^2} \sum_{i,j=1}^n Y_{ij}^2; \quad Y_{ij} \in \text{subband}(HH_1) \tag{5}$$

The noise variance $\hat{\sigma}_n$ is estimated from the HH band as,

$$\hat{\sigma}_n = \frac{\text{Median}(|Y_{ij}|)}{0.6745} \quad (6)$$

f. Neigh Shrink

Let $d(i,j)$ denote the wavelet coefficients of interest and $B(i,j)$ is a neighborhood window around $d(i,j)$. Also let $S2 = \sum d(i,j)$ over the window $B(i,j)$. Then the wavelet coefficient to be thresholded is shrunk according to the formulae,

$$d(i,j) = d(i,j) * B(i,j) \quad (7)$$

where the shrinkage factor can be defined as $B(i,j) = (1 - T2 / S2(i,j))_+$, and the sign $+$ at the end of the formulae means to keep the positive value while set it to zero when it is negative.

III. LITERATURE SURVEY FOR COMPARATIVE STUDY

Literature on topic image denoising Techniques is much old and frequent papers also published. Image denoising was first studied by Nasser Nahi in early 1970s. In later 1970s, this problem was attacked by computer vision pioneers A. Rosenfeld in their paper "Iterative enhancement of noisy images". In 1980, JS Lee published an important paper titled "Digital image enhancement and noise filtering by use of local statistics". The invention of wavelet transforms in late 1980s has led to dramatic progress in image denoising which originated in Simoncelli and Adelson's 1996 paper "Noise removal via Bayesian wavelet coring". Since then, numerous wavelet-based image denoising algorithms have appeared. Some references are as-

(i) "Image Denoising Using Discrete Wavelet Transform" by Dr. S. Arumuga Perumal et al. (Jan 2008) [6] shown a modified neigh shrink factor $B(i,j) = (1 - (3/4) * T2 / S2(i,j))_+$ instead of $B(i,j) = (1 - T2 / S2(i,j))_+$. Experimental results also show that modified Neighshrink gives better result (marginally improved PSNR) than Neighshrink, Weiner filter and Visushrink.

(ii) "An Improved Threshold Estimation Technique For Image Denoising Using Wavelet Thresholding Techniques" by Rohtash Dhiman et al. (Oct 2011) [7] shown for salt & pepper noise, in global thresholding function, threshold value λ is replaced by $\lambda_{proposed} = \lambda + 2\beta$; Where, λ = global threshold value and it is given by $\lambda = \sigma\sqrt{2\ln(N)}$ and β = penalized threshold value. The proposed threshold mentioned in this paper shows better performance (PSNR = 47.2) over other techniques.

(iii) "Wavelet Transform Based Image Denoise Using Threshold Approaches" by Akhilesh Bijalwan et al. (Jun 2012) [8] shown a simple and sub band semi soft threshold proposed method to address the issue of image recovery from its noisy counterpart. It is based on the discrete wavelet transform and Gaussian distribution modeling of sub band coefficients. The image denoise algorithm uses semi thresholding to provide smoothness and better image details preservation. The wavelet semi soft thresholding denoise algorithm produce overall better PSNR and MSE result compared with other traditional denoise approaches.

(iv) "Image Denoising Using Wavelet Threshold Methods" by Namrata Dewangan et al. (Jun 2012) [9] shown that Bivariate shrinkage function is used to reduce Gaussian, Salt & Pepper and speckle noise at different resolution levels. These results obtained have shown significant noise reduction then standard denoising methods such as Sure Shrink, Bayes Shrink, Neigh Shrink and Block Shrink.

IV. EVALUATION

In order to quantify the performance of the various denoising algorithms used in this paper, a test image is taken and some known noise is added to it. This would then be given as input to the denoising algorithm, which produces an image close to the original test image. The performance of each algorithm is compared by computing MSE and PSNR [10,11], besides the visual interpretation.

MSE and PSNR are the two parameters used in this paper for comparison of denoising techniques. In statistics, the mean squared error of an estimator is the difference between an estimator and the true value of the quantity being estimated i.e. difference between the original image and the denoised image.

Let original image is $f(x,y)$ of size $m \times n$ and $\hat{f}(x,y)$ is estimated image after restoration then MSE can be defined as-

$$MSE = \frac{1}{m \times n} \sum_{x=1}^m \sum_{y=1}^n [f(x,y) - \hat{f}(x,y)]^2 \quad (8)$$

PSNR is the peak signal to noise ratio, here signal is the original Image and noise is error introduced by restoration. PSNR is the most commonly used parameter to measure the quality of reconstruction image with respect to the original image. A higher PSNR would normally indicate that the reconstruction is of higher quality. PSNR is usually expressed in terms of the logarithmic decibel scale (dB).

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right) \quad (9)$$

Using (8), we get

$$PSNR = 10 \log_{10} \left(\frac{255^2}{\frac{1}{m \times n} \sum_{x=1}^m \sum_{y=1}^n [f(x,y) - \hat{f}(x,y)]^2} \right) \quad (10)$$

Here $f(x,y)$ is the original image of size $m \times n$ and $\hat{f}(x,y)$ is the reconstructed image after restoration. In the expression of PSNR, the value of the numerator is 255^2 ; it is the square of the maximum possible pixel value of the image $f(x,y)$. When $f(x,y)$ is the 8-bit gray scale image, the maximum possible pixel value of the Image $f(x,y)$ is 255.

V. SIMULATED RESULTS AND DISCUSSION

We have taken two test images i.e., Lena (512 x 512) and House (256 x 256) as input image for simulation of results.

The noise added in test image is Additive White Gaussian Noise (AWGN) with noise variance (σ) ranges from 10 to 60.

Now the noisy image is denoised by Universal Hard Threshold and Improved Threshold using Daubechies (db1) wavelets. The simulated results are tabulated as follows:

Table-1(a) MSE under different sigma in AWGN

IMAGE	Universal Hard Threshold	Improved Threshold (db1)
LENA (512 x 512)	MSE	MSE
$\sigma = 10$	55.9452	70.4979
$\sigma = 20$	146.2687	145.7300
$\sigma = 30$	283.5315	271.1169
$\sigma = 40$	471.2244	446.6586
$\sigma = 50$	711.3749	672.3551
$\sigma = 60$	1.0054 e+003	948.2063
HOUSE (256 x 256)	MSE	MSE
$\sigma = 10$	53.1726	82.8734
$\sigma = 20$	154.9345	159.5313
$\sigma = 30$	303.7978	286.8340
$\sigma = 40$	503.8861	465.0578
$\sigma = 50$	758.9837	694.2026
$\sigma = 60$	1.0694 e+003	974.2686

Table-1(b) Performance under different sigma of AWGN

IMAGE	Universal Hard Threshold (db1)	Improved Threshold (db1)
LENA (512 x 512)	PSNR	PSNR
$\sigma = 10$	30.6532	29.6490
$\sigma = 20$	26.4793	26.4953
$\sigma = 30$	23.6048	23.7992
$\sigma = 40$	21.3985	21.6310
$\sigma = 50$	19.6098	19.8548
$\sigma = 60$	18.1073	18.3618
HOUSE (256 x 256)	PSNR	PSNR
$\sigma = 10$	30.8739	28.9467
$\sigma = 20$	26.2293	26.1023
$\sigma = 30$	23.3050	23.5545
$\sigma = 40$	21.1075	21.4557
$\sigma = 50$	19.3285	19.7159
$\sigma = 60$	17.8396	18.2440

The graphs for sigma (noise variance) vs MSE of both hard and improved threshold are as follows:

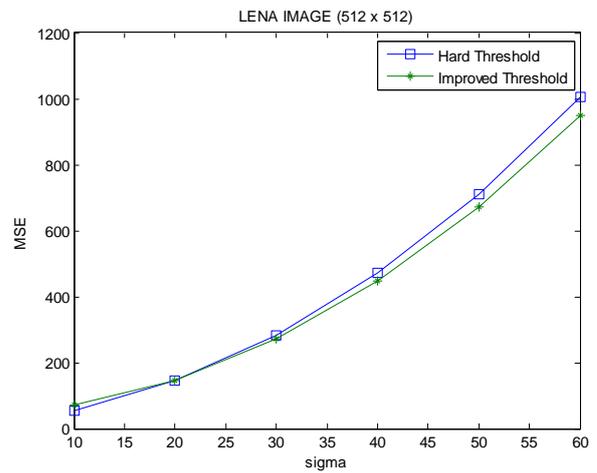


Fig.5(a) Sigma Vs MSE for Lena image (512 x 512)

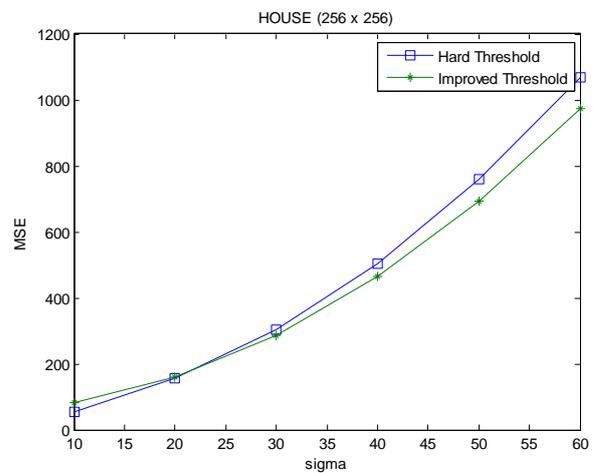


Fig.5(b) Sigma Vs MSE for House image (256 x 256)

The graphs for sigma (noise variance) vs PSNR of both hard and improved threshold are as follows:

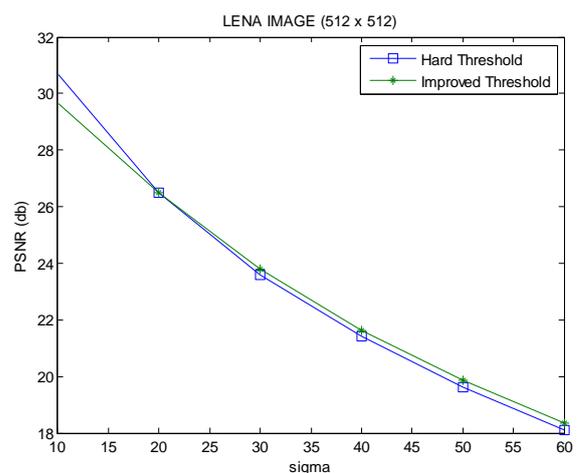


Fig.5(c) Sigma Vs PSNR for Lena image (512 x 512)

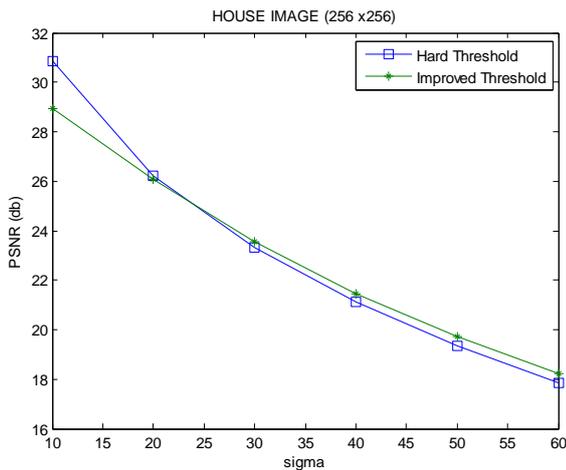


Fig.5(d) Sigma Vs PSNR for House image (256 x 256)

The visual interpretations of simulated results are as follows:



Fig.6(a) Test Lena image (512 x 512)

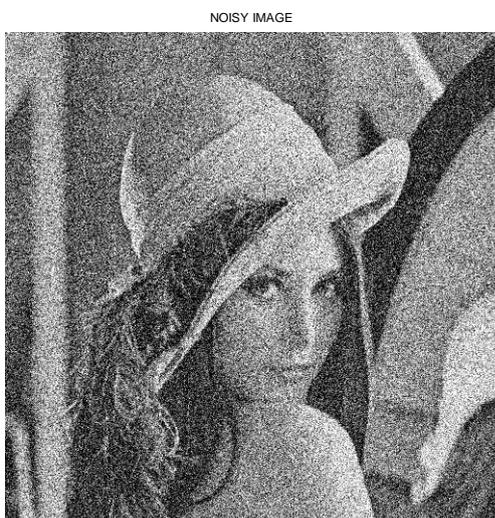


Fig.6(b) Noisy image with sigma = 60 in AWGN

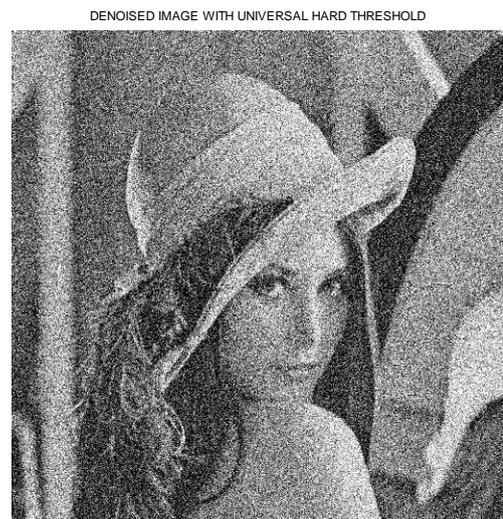


Fig.6(c) Denoised image with Hard Threshold

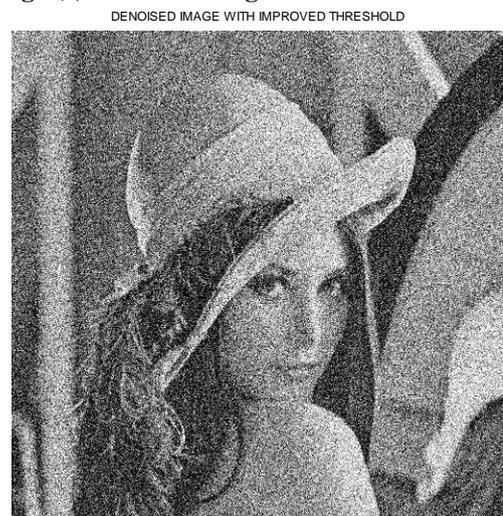


Fig.6(d) Denoised image with Improved Threshold

VI. CONCLUSION

The Improved Threshold for image denoising gives better performance than the Universal hard Threshold. Although the improvement is marginally but it is noticeable that as the noise variance (sigma) value increases beyond a specific value, the performance of the Improved Threshold for image denoising gets better significantly in respect of Universal Hard Threshold. Universal Hard threshold gives better performance than Bayes Shrink in image denoising. Neigh Shrink performance is better than the Universal Hard threshold but Neigh Shrink is slower (*more elapsed time*) than the Universal Hard Threshold.

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