

Performance Study of Interweave Spectrum Sharing Method in Cognitive Radio

Pinki Yadav¹, Partha Pratim Bhattacharya²

Abstract—

Spectrum scarcity is one of the most critical recent problems in the field of wireless communication. One of the promising approaches to resolve this issue is the use of Cognitive Radio technique. CR technique is performed by Secondary User (SU) to utilize the spectrums more efficiently. SU transmission mode can be broadly divided into two modes: interference avoidance mode (white space) and interference management mode (black and gray space). In this paper, interference avoidance mode is considered. The interference avoidance mode is often referred to as interweave access. In interweave access method, SU finds the spectrum hole by sensing the radio frequency. Here we have used continuous time Markov chain model to calculate QoS parameter of the SU for fixed payload length and exponential distribution payload length for a single channel of PU and SU. Numerical results show that the blocking probability and force termination probability are greater in fixed payload length than in exponential distribution payload length.

Index Terms— wireless communication; interweave access; cognitive radio; blocking probability; force termination probability; CTMC model.

I. INTRODUCTION

The demand for spectrum access is growing rapidly in wireless communication. Therefore, an efficient spectrum management and spectrum access techniques are needed. However, studies on spectrum usage have revealed that most of the allotted spectrum is not used efficiently due to the static frequency allocation methods. With the evolution of cognitive radio, spectrum access techniques shift from static spectrum allocation to dynamic allocation. CR enables the secondary user (SU) to build transmission links in vacant PU channels such that there is no/minimum interference to PUs. CR has the following functionalities: spectrum sensing; spectrum access; spectrum allocation and management among different SUs. SU transmission modes can be broadly categorized into two modes: interference avoidance (white space) mode and the Interference management (black and grey space) mode [1]. The interference avoidance mode is often termed as interweave access. In interweave access, the SU finds spectrum holes (white space) by sensing the radio frequency spectrum. These spectrum holes are used by the SU for its

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transmission. This scheme is often referred to as opportunistic spectrum access (OSA) [1]. On the other hand the interference management mode in a CR can be categorized as overlay and underlay. The underlay approach assumes that a PU transmits all the time and imposes severe restrictions on the transmission power of a SU. The overlay approach relies on spectrum holes detection by SU in the network and does not impose severe restrictions on their transmission power [2].

In this paper, the quality of service of secondary users is analyzed in an interweave access scenario. A continuous time Markov model is used to calculate the forced termination probability and blocking probability. Here Simple closed form expressions for the forced termination probability, blocking probability are derived for both fixed and exponential payload lengths [3].

II. SYSTEM MODEL

A widely acceptable spectrum pooling model [4] is adopted, in which a vacant wideband PU channel is divided into multiple narrowband sub channels. These narrowband sub channels are used by SU groups for their opportunistic transmissions. Fig.1 shows the system model, in which there is K available PU channels $k \in \{1, 2, 3 \dots K\}$. Each of the PU channels has a fixed bandwidth W_p , which is subdivided into N sub channels of bandwidth, $W_s = W/N_p$. We consider spectrum sharing scenarios: a single SU group is interweaving the PU channels, denoted by S_1 .

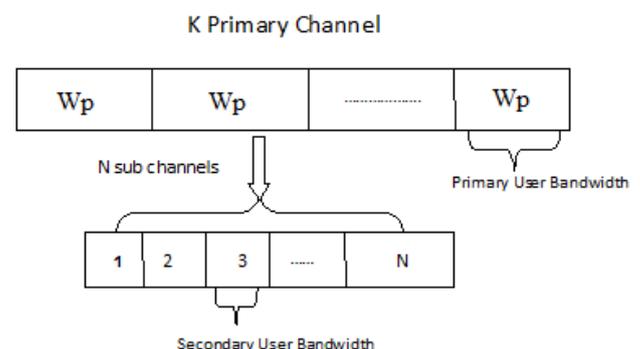


Fig. 1 Channel arrangements of PU and SU Channels

- 1) The service duration of the PU connections are assumed to be exponentially distributed with a mean $1/\mu_p$. The arrival of new connections from the PU group follows a Poisson process with a mean rate λ_p .
- 2) PU network is an M/M/m/m loss network. In this network channel occupancy only depends on the mean service rate of PU group.

3) In a single PU channel and single SU channel group two types of SU connections are considered: fixed header with fixed payload and fixed header with exponentially distributed payload.

The fixed header duration is denoted by $1/\mu_h$, the fixed payload length denoted by $1/\mu_{sf}$, and the mean exponential payload length is denoted by $1/\mu_{se}$. The Poisson arrival rate of a single SU group is denoted by λ_s .

III. PERFORMANCE ANALYSIS

CTMC is used the spectrum sharing scenario between the PU and SU groups. CTMC has been extensively used in conventional cellular systems to model the cell residence time, call dropping probability and handover probability [5]. In these scenarios, the birth-death process (often referred to as a loss system) is one of the commonly used CTMC models. In a CTMC model, the number of connections from the user groups are represented by states (written as n tuples), such as (x_1, x_2) , where x_1 and x_2 may represent the number of PU and SU connections. The transition of one state to another is based on the Markov property [6] which assumes memory less arrivals and departures. Under stationary assumptions, the rate of transition from and to a state is equal. This fundamental fact is used to calculate the state probabilities under the constraint that the sum of all state probabilities is equal to 1.

A. Fixed Payload Length

When the payload is fixed, the total connection length can be expressed by the sum of the overhead and payload length, i.e. [3]

$$\frac{1}{\mu_{fp}} = \frac{1}{\mu_h} + \frac{1}{\mu_{sf}} \tag{1}$$

(1)

When the fixed length SU connection on the channel, the system is not purely Markovian and also it cannot represent by a simple birth-death process. In this approach, the connection with fixed length is replaced by a continuous group of Y exponentially distributed connection which are independent and identically distributed (i.i.d.). The total length of this Y i.i.d. connection is Erlang distributed and individual connection has an average length of $1/Y\mu_{fp}$. It is well established that as $Y \rightarrow \infty$, then the total length is the same as the fixed length connections $1/\mu_{fp}$.

Since the fixed length connection is replaced by a group of Y i.i.d connection, a Y node CTMC is used the number of the arrivals of the connection in the group, and i.e. the y_{th} node models the arrival of the y_{th} connection in the group. These states are referred to as the phases of a fixed SU connection [6]. Complete CTMC model is shown in Fig.2, where the state is $(i; j; y)$, where, i and k ($\in \{1,0\}$) represents the presence (1) and absence (0) of a SU and PU connection on the channel, $1 \leq y \leq Y$ represents the y_{th} phases of a SU connection when $i=1$ shown in fig. The total mean departure rate from each phase of the SU connection is $(Y\mu_{fp} + \lambda_p)$, where $Y\mu_{fp}$ is the completion rate of each i.i.d connection and λ_p , is the termination rate due to the arrival of a PU

connection. A set of balance equations for the CTMC in figure is given as

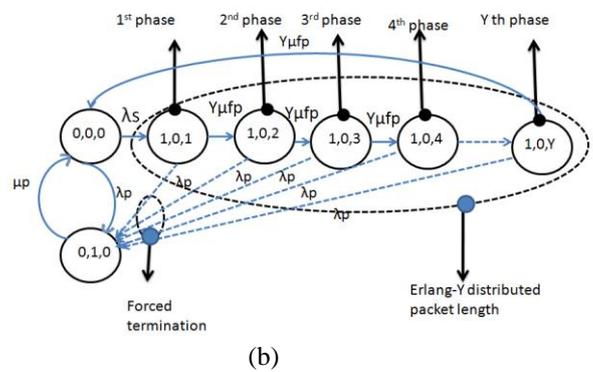
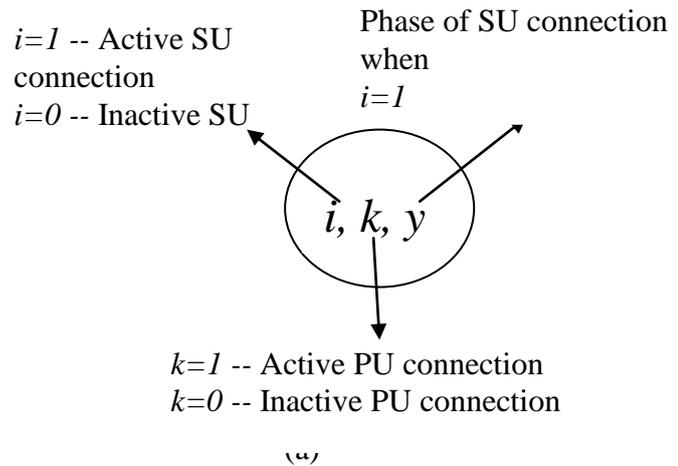


Fig. 2. CTMC model of SU connections with fixed payload

$$\left\{ \begin{aligned} (\lambda_s + \lambda_p) P(0,0,0) &= \lambda_p P(0,1,0) + Y\mu_{sf} P(1,0,Y); \\ Y\mu_{fp} P(1,0,1) &= \lambda_s P(0,0,0); \\ (Y\mu_{fp} + \lambda_p) P(1,0,2) &= Y\mu_{fp} P(1,0,1); \\ &\vdots \\ (Y\mu_{fp} + \lambda_p) P(1,0,Y) &= Y\mu_{fp} P(1,0,Y-1) \end{aligned} \right. \tag{2}$$

Such that

$$P(0,0,0) + P(0,1,0) + \sum_{y=1}^Y P(1,0,y) = 1 \tag{3}$$

Where, P (i; k; y) represents the probability of the state (i; k; y). By simple algebraic manipulation of equations in (2), the probability of the y_{th} phase P (1; 0; y) in terms of P (0, 0, and 0) can be written as [3]

$$P(1,0,y) = \left(\frac{Y\mu_{fp}}{Y\mu_{fp} + \lambda_p} \right)^{y-1} \frac{\lambda_s}{(Y\mu_{fp} + \lambda_p)} P(0,0,0), \quad y \geq 1 \tag{4}$$

Using P (i; k; y), the expression for the force termination probability, blocking probability and throughput are derived.

1) Blocking probability: In Fig. 2(b), an incoming SU connection is blocked when the channel is occupied by a PU or SU connection. The blocking probability $P_B (h;f) (Y)$ in this scenario can be expressed as [3]

$$P_B (h;f) (Y) = 1 - P (0, 0, 0) \tag{5}$$

Since the arrival of a PU connection is independent of a SU connection, the Probability of a PU connection on the channel P (0, 1, and 0) can be written as

$$P (0,1,0) = \frac{\lambda_p}{\lambda_p + \mu_p} \tag{6}$$

Using (2), (4) and (6), the blocking probability of a SU connection with Erlang-Y distributed connection length has the following mathematical expression

$$P_B (h;f) (Y) = 1 - \frac{\mu_p \lambda_p}{(\lambda_p + \mu_p) \left(\lambda_s + \lambda_p - \left(\frac{Y \mu_f p}{Y \mu_f p + \lambda_p} \right)^Y \lambda_s \right)} \tag{7}$$

Let $P_B (h;f) = \lim_{Y \rightarrow \infty} P_B (h;f) (Y)$ the blocking probability of SU connections with fixed payload length becomes

$$P_B (h;f) = 1 - \frac{\mu_p \lambda_p}{(\lambda_p + \mu_p) \left(\lambda_s + \lambda_p - e^{-\frac{\lambda_p}{\mu_f p} \lambda_s} \right)} \tag{8}$$

2) Force termination probability: Forced termination probability is defined as the probability that an ongoing SU connection is terminated prematurely. Based on the fact that the sum of forced and unforced termination rates equals to the incoming connection rate, the forced termination probability can be written as [3]

$$P_F (h;f) (Y) = \frac{\text{SU forced termination rate}}{\text{SU connection rate}} \tag{9}$$

Using (9), the forced termination probability of SU with Erlang-Y distributed connection length can be written as

$$P_F (h;f) (Y) = \frac{\sum_{y=1}^Y P (1,0,y) \lambda_p}{(1 - P_B (h;f) (Y)) \lambda_s} \tag{10}$$

Using (2) and (5), $P_F (h;f)$ can be expressed as

$$P_F (h;f) (Y) = \frac{\lambda_p}{Y \mu_f p} \sum_{y=1}^Y \left(\frac{Y \mu_f p}{Y \mu_f p + \lambda_p} \right)^Y \tag{11}$$

– Note that the geometric series in the above equation is convergent (common ratio $\frac{Y \mu_f p}{Y \mu_f p + \lambda_p} < 1$); hence (11) can be simplified to

$$P_F (h;f) (Y) = \frac{Y \mu_f p}{Y \mu_f p + \lambda_p} - \left(\frac{Y \mu_f p}{Y \mu_f p + \lambda_p} \right)^Y \tag{12}$$

Let $P_F (h;f) = \lim_{Y \rightarrow \infty} P_F (h;f) (Y)$ be the force termination probability of SU connection with fixed payload. The $P_F (h;f)$ can be written as

$$P_F (h;f) = 1 - e^{-\left(\frac{\lambda_p}{\mu_f p} \right)} \tag{13}$$

B. Exponential Payload Length

In the following, the forced termination probability, blocking probability of the SU connection with a fixed header and exponentially distributed payload length are calculated.

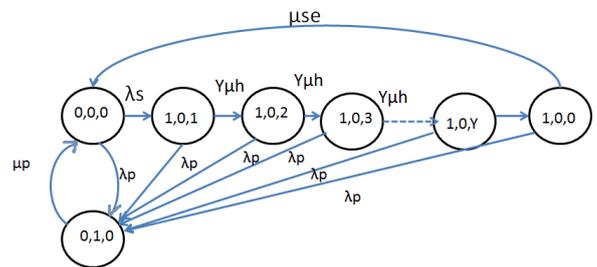


Fig. 3 CTMC model of SU connections with fixed header and exponentially distributed payload.

Fig. 3 shows that the CTMC of the system. In this figure the states (1, 0, y), $y = \{1 \dots Y\}$ represent the phases of fixed header portion, whereas the state (1, 0, 0) represents the exponentially distributed payload portion. Similarly to the fixed header length $1/\mu_h$ is modeled as limiting case of Erlang-Y distribution, and exponentially distributed payload length is given by $1/\mu_{se}$. The global balance equation of the above system is as follows [7]

$$\left\{ \begin{array}{l} (\lambda_s + \lambda_p) P (0, 0, 0) = \mu_p P (0, 1, 0) + \mu_{se} P (1, 0, 0), \\ (Y \mu_h + \lambda_p) P (1, 0, 1) = \lambda_s P (0, 0, 0), \\ (Y \mu_h + \lambda_p) P (1, 0, 2) = Y \mu_h P (1, 0, 1), \\ \vdots \\ (Y \mu_h + \lambda_p) P (1, 0, Y) = Y \mu_h P (1, 0, Y-1), \\ (\lambda_s + \mu_{se}) P (1, 0, 0) = Y \mu_h P (1, 0, Y), \end{array} \right. \tag{14}$$

1) Blocking probability: - Solving (14) for P (0; 0; 0) and using (5), the blocking probability of the SU connection can be written as

$$P_B (h,e) (Y) = 1 - \frac{\mu_p \lambda_p}{(\lambda_p + \mu_p) \left(\lambda_s + \lambda_p - \frac{\lambda_s \mu_{se}}{\mu_{se} + \lambda_p} \left(\frac{Y \mu_h}{Y \mu_h + \lambda_p} \right)^Y \right)} \tag{15}$$

In the limit $P_B(h, \epsilon) = \lim_{Y \rightarrow \infty} P_B(h, \epsilon)(Y)$ is the blocking probability of SU connections with exponentially distributed payload length. It can be shown that [7]

$$P_B(h, \epsilon) = 1 - \frac{\mu_p \lambda_p}{(\lambda_p + \mu_p) \left(\lambda_s + \lambda_p - \frac{\lambda_s \mu_{se}}{\mu_{se} + \lambda_p} e^{-\frac{\lambda_p}{\mu_h}} \right)} \quad (16)$$

2) Force termination Probability: The force termination probability can be calculated by using the definition in (9). In this scenario the force termination probability [7]

$$P_F(h, \epsilon)(Y) = \frac{\lambda_p \sum_{y=0}^Y P(1, 0, y)}{\lambda_s P(0, 0, 0)} \quad (17)$$

Using the balance equation (14) the $P_F(h, \epsilon)(Y)$ can be expressed as

$$P_F(h, \epsilon)(Y) = \frac{\lambda_p \left(\sum_{y=1}^Y \left(\frac{Y \mu_h}{Y \mu_h + \lambda_p} \right)^y \right)}{Y \mu_h} + \frac{\lambda_p}{\mu_{se} + \lambda_p} \left(\frac{Y \mu_h}{Y \mu_h + \lambda_p} \right)^Y \quad (18)$$

In the limit, let $P_F(h, \epsilon) = \lim_{Y \rightarrow \infty} P_F(h, \epsilon)(Y)$ be the force termination probability of SU connection with exponentially payload length.

$$P_F(h, \epsilon) = 1 - e^{-\frac{\lambda_p}{\mu_h}} + \frac{\lambda_p}{\mu_{se} + \lambda_p} e^{-\frac{\lambda_p}{\mu_h}} \quad (19)$$

$$P_F(h, \epsilon) = P_F(h, 0) + P_F(0, \epsilon)(1 - P_F(h, 0)) \quad (20)$$

Where $P_F(h, 0) = 1 - e^{-\frac{\lambda_p}{\mu_h}}$ and $P_F(0, \epsilon) = \frac{\lambda_p}{\mu_{se} + \lambda_p}$.

IV. NUMERICAL RESULT

In this section, we have shown some numerical examples of the forced termination probability and blocking probability with fixed and exponentially distributed payload length. The Fig. 4 shows the blocking probability for fixed payload length ($P_B(h; f)$) for different value of mean header duration ($1/\mu_h$). So it is clear from the figure when mean payload length increases ($1/\mu_{sf}$), blocking probability decreases and also analyzed when mean header duration ($1/\mu_h$) increases, blocking probability also increases.

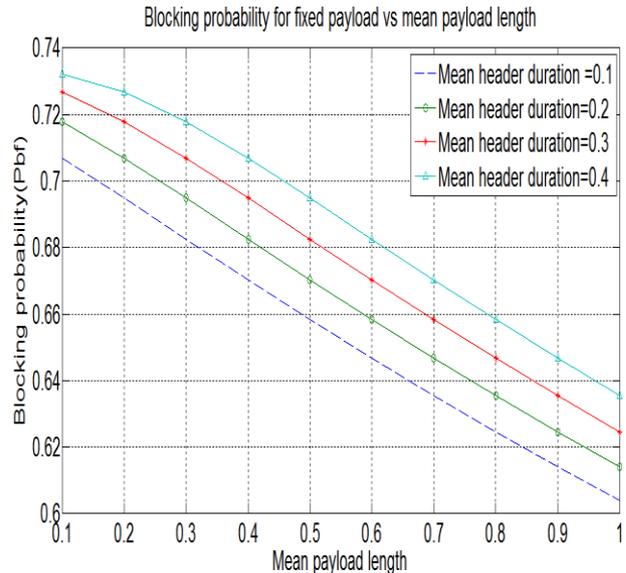


Fig. 4 Blocking probability for fixed payload against mean payload length for different mean header duration

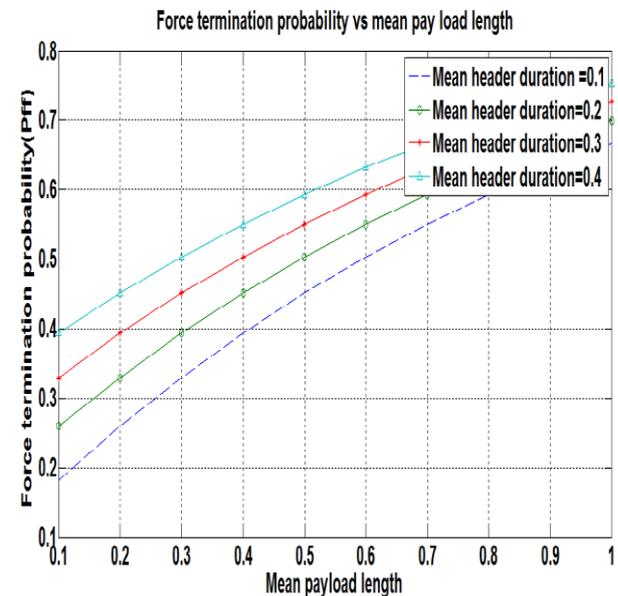


Fig. 5 Force termination probability for fixed payload against mean payload length for different mean header duration

The Fig. 5 shows the force termination probability for fixed payload length ($P_F(h; f)$) for different value of mean header duration ($1/\mu_h$). So it is clear from the figure when mean payload length increases ($1/\mu_{sf}$), force termination probability increases and also analyzed when mean header duration ($1/\mu_h$) increases, force termination probability also increases.

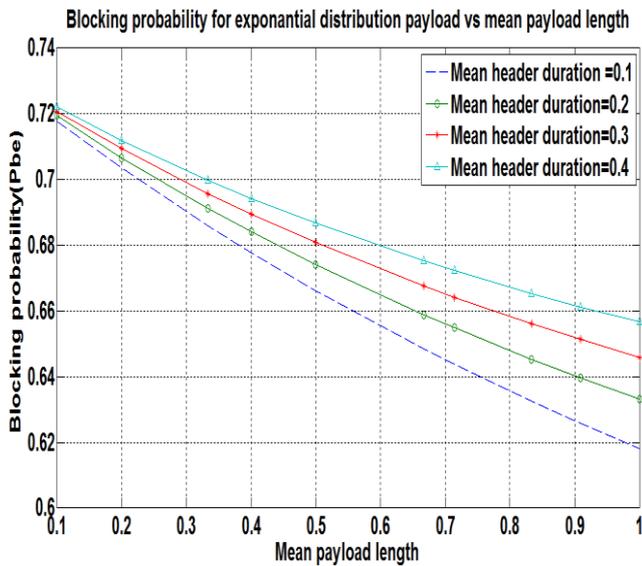


Fig. 6 Blocking probability ($P_{B(h;\epsilon)}$) for exponential distribution payload against mean payload length for different mean header duration

The Fig. 6 shows the blocking probability for exponential distributed payload length ($P_{B(h;\epsilon)}$) for different value of mean header duration ($1/\mu_h$). So it is clear from the figure when mean payload length increases ($1/\mu_{se}$), blocking probability decreases and also analyzed when mean header duration ($1/\mu_h$) increases, blocking probability also increases.

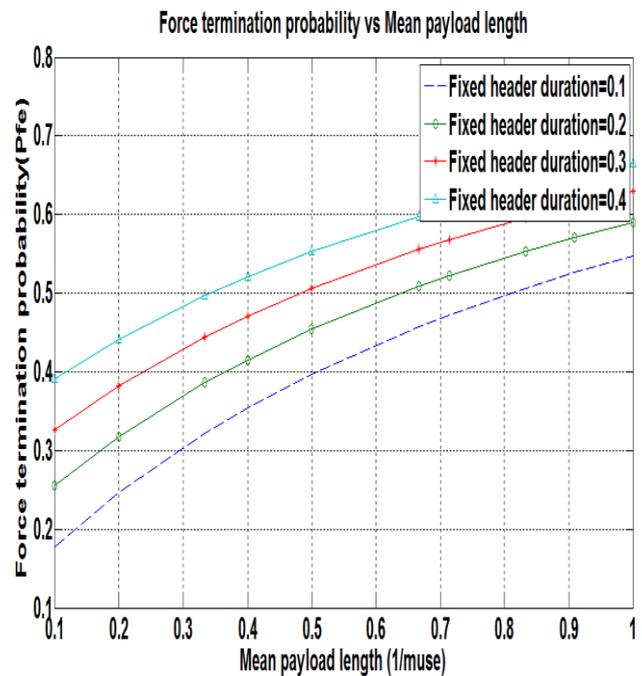


Fig. 7 Force termination probability ($P_{F(h;\epsilon)}$) for exponential distribution against mean payload length for different mean header duration

The Fig. 7 shows the force termination probability for fixed payload length ($P_{F(h;\epsilon)}$) for different value of mean header duration ($1/\mu_h$). So it is clear from the figure when mean payload length increases ($1/\mu_{se}$), force termination probability increases and also analyzed when mean header duration ($1/\mu_h$) increases, force termination probability also increases.

The Fig. 8 shows the blocking probability for fixed payload length ($P_{F(h;f)}$) against arrival rate of SU for different value of mean header duration ($1/\mu_h$). So it is clear from the figure when arrival rate of SU increases (λ_s), blocking probability increases and also analyzed when mean header duration ($1/\mu_h$) increases, blocking probability also decreases.

Fig. 9 shows the blocking probability for exponential distributed payload length ($P_{F(h;\epsilon)}$) against arrival rate of SU for different value of mean header duration. In this figure when arrival rate of SU increases, blocking probability increases, but when mean header length increases blocking probability also increases.

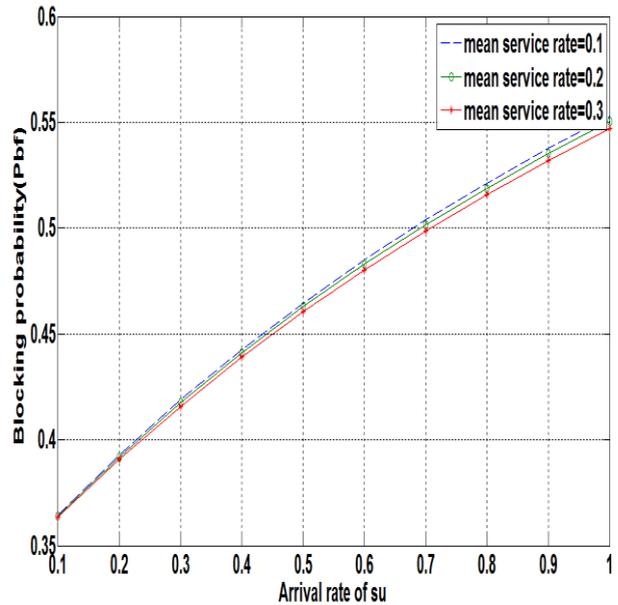


Fig. 8 Blocking probability for fixed payload against arrival rate of SU for different mean header duration

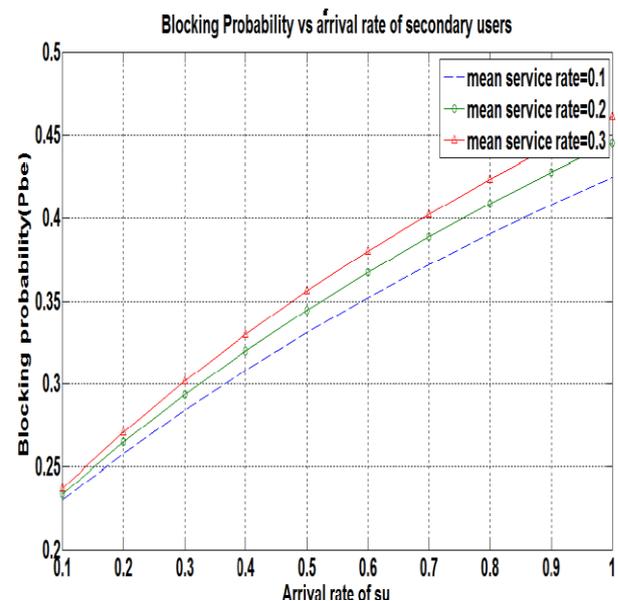


Fig. 9 Blocking probability for exponential distributed payload against arrival rate of SU for different mean header duration

A. Comparison between fixed and exponential payload length

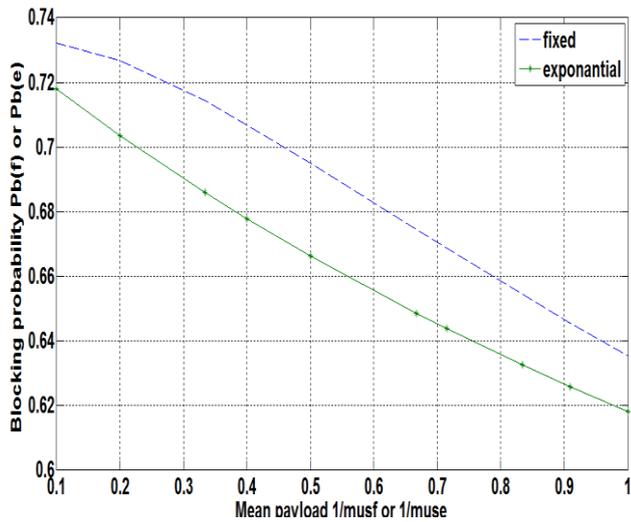


Fig. 10 Comparison between Blocking probability for fixed ($P_{B(f)}$) and exponential payload ($P_{B(e)}$) length against mean payload ($1/\mu_{fp}$ or $1/\mu_{se}$)

In Fig. 10 shows the Comparison between Blocking probability for fixed ($P_{B(f)}$) and exponential payload ($P_{B(e)}$) length against mean payload ($1/\mu_{fp}$ or $1/\mu_{se}$) for mean header duration ($1/\mu_h=0.1$). So it is clear from the figure blocking probability decreases when mean payload length increase and also observe the exponential distribution of blocking probability are lower than the fixed payload blocking probability.

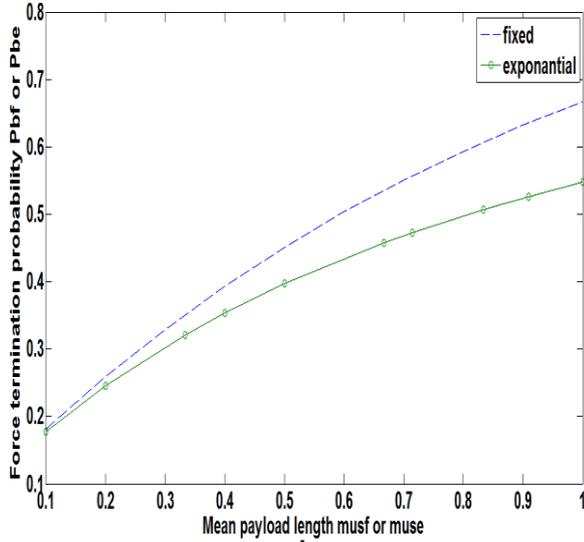


Fig. 11 Comparison between Force termination probability for fixed ($P_{f(h,f)}$) and exponential payload ($P_{e(h,e)}$) length against mean payload ($1/\mu_{sf}$ or $1/\mu_{se}$)

In Fig. 11 shows that the Comparison between Force termination probability for fixed ($P_{f(h,f)}$) and exponential payload ($P_{e(h,e)}$) length against mean payload ($1/\mu_{sf}$ or $1/\mu_{se}$) for mean header duration ($1/\mu_h=0.1$). So it is clear from the figure force termination probability increases when mean payload length increase and also observe the

exponential distribution of force termination probability are lower than the fixed payload force termination probability.

In Fig. 12 shows the Comparison between Blocking probability for fixed ($P_{B(f)}$) and exponential payload ($P_{B(e)}$) length against arrival rate of SU for mean header duration ($1/\mu_h=0.1$). So it is clear from the figure blocking probability increases when arrival rate of SU increase and also observe the exponential distribution of blocking probability are lower than the fixed payload blocking probability.

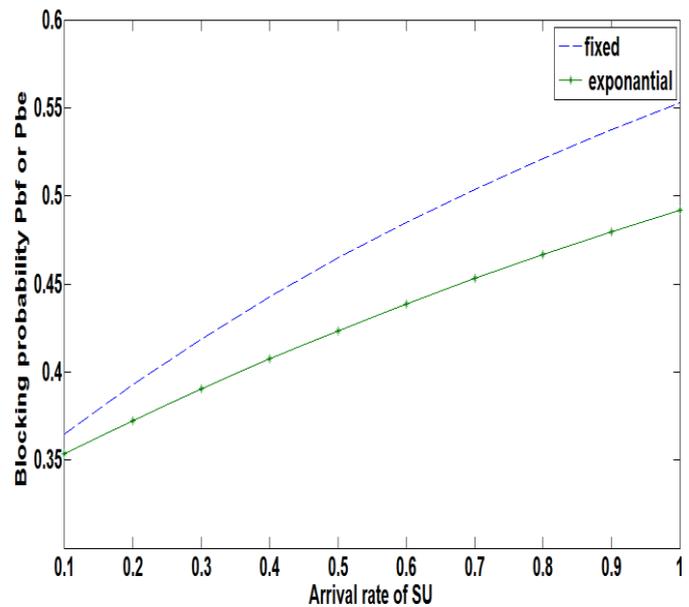


Fig. 12 Comparison between Blocking probability for fixed ($P_{B(f)}$) and exponential payload ($P_{B(e)}$) length against arrival rate of SU

V. CONCLUSION

In this paper, we have used a continuous time markov chain model to calculate QoS parameter of the SU for fixed payload length and exponential distribution payload length with fixed header using a single channel of PU and SU. Numerical results are shown and compared with those of fixed header duration. Results show that the blocking probability and force termination probability is greater in fixed payload length rather than exponential distribution payload length.

This work may further be extended for multiple PU channel and two SU groups with spectrum handoff and channel reservation in interweave access mode.

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